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AN OPTIMIZED MAS FOR SOLVING SCATTERING PROBLEMS

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Abstract

A new criterion driving the choice of the locations of the Auxiliary Sourcers (AS) is introduced with the aim to improve the performance of the Method of Auxiliary Sources (MAS) applied to the solution of the integral equations as those encountered in electromagnetic scattering.

The approach is based on the optimization of the singular value behavior of the matrix relating the AS excitations and the scattered field values at the matching points on the scatterer boundary. The ill-conditioning of the problem of determining the AS excitations matching the boundary conditions is then significantly reduced and the accuracy of the estimated scattered field is improved.

The performance of the method is numerically assessed, in a 2D scalar geometry, by discussing in the details the case of a circular perfectly conducting scatterer.

Keywords: Electromagnetic scattering, Auxiliary sources, Singular value optimization.

1. INTRODUCTION

The Method of Auxiliary Sources (MAS) is a numerical technique which can be applied to the solution of elliptic boundary value problems and fruitfully exploited in electromagnetic radiation or scattering problems. The technique aims at expressing the radiated/scattered field as that produced by fictitious sources (the Auxiliary Sources – AS) located somewhere inside the physical radiator/scatterer and whose excitations generate a radiated/scattered field compatible with the boundary conditions [1].

The method is mathematically rigorous, since the basis functions used for the field expansions on the problem boundary (i.e., the field radiated by the AS' on the boundary) are complete [2]. Moreover, it appears more attractive than the standard Method of Moments (MoM) [3], due to its simpler implementation which prevents from singularities associated to the diagonal elements of the so-called "impedance matrix" used in MoM, and thus avoid time consuming numerical integrations.

Nevertheless, the MAS is generally considered less robust than MoM, due to the missing of a reliable, general criterion driving a correct location of the AS'. In other words, as reported in the literature [4], the unsuccessful selection of AS locations can lead to poor accuracies or even to completely unacceptable solutions.

In this paper, we propose a criterion to determining the optimal number and locations of the AS'. These are obtained by optimizing the singular value behavior [5,6] of the relevant matrix connecting the AS excitations to the field values at the matching points on the scatterer boundary. Accordingly, the ill-conditioning of the problem of determining the AS excitations matching the boundary conditions is strongly mitigated and the reliability and the accuracy of the solution improved. It is worth noting that the approach enables a fully automatic selection of the number and location of the AS', without any user's intervention.

The performance of the method is numerically assessed, in a 2D scalar geometry, by discussing in the details the case of a circular perfectly conducting scatterer, so that a comparison against theoretical solution is possible [1].

2. OPTIMIZED MAS

Let us consider a 2D, perfectly conducting scatterer, illuminated by a known impinging electromagnetic z-directed field E_i, as depicted in Fig. 1.

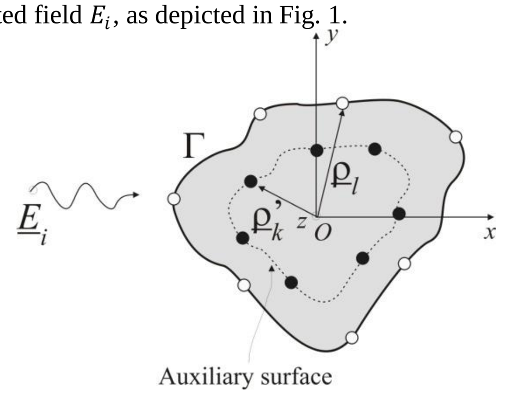

Fig. 1. Geometry of the problem.

According to the MAS, the z-component of the scattered field E_s outside the object is represented as the superposition of the fields radiated by fictitious current filaments located in the object's interior, that is (apart from unessential constant factors) [1-5]

$$E_s\left(\underline{\rho}\right)=\sum_{k=1}^{K} c_k H_0^{(2)}\left(\beta\left(\left|\underline{\rho}-\underline{\rho}_k'\right|\right)\right), \qquad (1)$$

where $\underline{\rho}$ denotes a generic observation point, $\underline{\rho}_k'$ locates the k-th AS, β is the wavenumber, $H_0^{(2)}(\cdot)$ is the Hankel function of zero-th order and second kind, c_k is the excitation coefficient of the k-th AS, and K is the overall number of sources.

The boundary condition on Γ can be then written as

$$\sum_{k=1}^{K} c_k H_0^{(2)}\left(\beta\left(\left|\underline{\rho}-\underline{\rho}_k'\right|\right)\right)=-E_i\left(\underline{\rho}\right), \qquad \underline{\rho}\epsilon\Gamma. \qquad (2)$$

By sampling eq. (2) at a number of L points $\underline{\rho_l}$ on Γ, $l=1,\dots,L$, it can be recast in the matrix form

$$\underline{\underline{Z}}\,\underline{I}=-\underline{V} \qquad (3)$$

where $\underline{\underline{Z}}$ is the matrix whose generic element is $Z_{lk}=H_0^{(2)}(\beta\left(\left|\underline{\rho_l}-\underline{\rho}_k'\right|\right))$, $\underline{I}$ is the vector containing the c_k's and $\underline{V}$ contains the incident field samples $E_i\left(\underline{\rho_l}\right)$.

The excitations c_k of the current filaments can be determined by solving eq. (3) by a Singular Value Decomposition (SVD) approach [7]. On denoting by σ_n the singular values of $\underline{\underline{Z}}$, the ill-conditioning of the problem of determining the c_k's can be faced by a Truncated SVD (TSVD), i.e., by cutting all the singular values falling below $\tau\sigma_1$, where τ is a threshold fixed according to the expected amount of noise (due, e.g., to the finite machine precision) and σ_1 is the greatest singular value. By this approach, the condition number of the regularized $\underline{\underline{Z}}$ is σ_N/σ_1, where $N=dim\{\sigma_n|\sigma_n\geq\tau\sigma_1\}$.

Obviously, the inversion of the regularized $\underline{\underline{Z}}$ should be performed under the "best" possible conditions. This occurs when N is large, i.e., when the number of singular vectors employed to expand $\underline{I}$ is as large as possible. Finally, it should be noticed that the singular values behavior depends on the locations of the AS', which defines a family of problems parameterized on the AS' locations.

These observations provides a criterion to determining the optimal AS' locations, for a fixed number K.

Indeed, for a fixed K, the number N of singular values falling above $\tau\sigma_1$ can be optimized by minimizing the functional

$$\Phi(\underline{\rho}_1',\underline{\rho}_2',\dots,\underline{\rho}_K')=\frac{1}{dim\{\sigma_n|\sigma_n>\sigma_1\tau\}}. \qquad (4)$$

It is worth noting that the outlined approach does not depend neither on the particular scatterer's geometry nor on the characteristics of impinging field, so that it can be applied to the general case.

3. OPTIMIZED MAS FOR PERFECTLY CONDUCTING CIRCULAR CYLINDERS

In this Section, we apply the previously discussed optimized MAS approach to the case of a circular cylinder with radius a under plane wave incidence, so that comparisons with theoretical results can be achieved. For this simple situation, and due to the problem symmetry, the AS' are located over a circle of radius $b<a$ with uniform angular spacing, so that $\Phi=\Phi(b)$ [4].

A threshold $\tau=-50dB$ has been assumed throughout the simulations. Furthermore, a number of AS' $K=[2\beta a]+1$, $[x]$ denoting the integer part of x, has been considered.

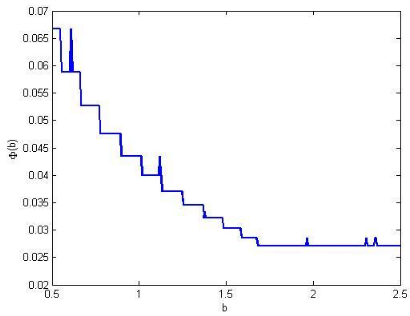

Fig. 2. Behaviour of the functional Φ(b) for a=2.5λ.

Circular cylinder with a=2.5λ

We consider first the case of a circular cylinder with radius a=2.5λ, i.e., for a cylinder with a radius "difficult" to be dealt with [4]. Fig. 2 shows the behavior of the optimized functional against the radius b. The optimal radii b_{opt} are represented by any $b>1.6\lambda$.

It should be noticed that, fixing a threshold τ for the singular values amounts to fixing a desired condition number, as done in [4]. It has been also verified that, lowering τ (i.e., accepting a higher degree of ill-conditioning) brings the auxiliary circle radius b, close to a, which matches the observations of [4].

On the other side, Fig. 3 shows the optimal locations of the AS' when $b=1.2\lambda$. Finally, Figs. 4 and 5 show the reconstructed scattered field amplitude and phase, respectively, as compared to those evaluated according to a cylindrical wave expansion.

Circular cylinder with a=6λ

We finally show the case when a=6λ. The considered optimal radius has been $b_{opt}=4.9\lambda$. Figs. 6 and

7 compare the recovered far-field pattern amplitude and phase, respectively, with that evaluated according to the cylindrical wave expansion.

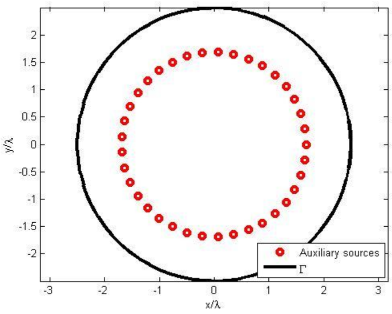

Fig. 3. Optimized AS' positions for the case a=2.5λ.

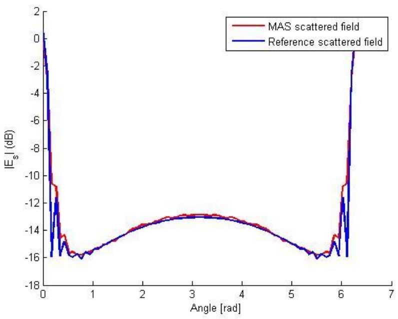

Fig. 4. Amplitude of the scattered field for the case a=2.5λ.

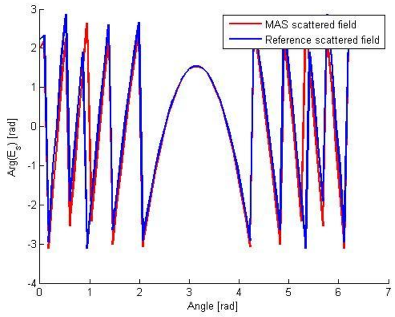

Fig. 5. Phase of the scattered field for the case a=2.5λ.

5. CONCLUSIONS AND FUTURE DEVELOPMENTS

We have proposed an approach to optimize the choice of the locations of the AS for the MAS for solving the electromagnetic scattering direct problem. The method has been herein tested against the case of perfectly conducting circular cylinders under a plane wave incidence.

As future developments, we foresee the evaluation of the method in the case of elliptical cylinders, for which wave expansions in terms of Mathieu functions are available and applications to the case of arbitrarily shaped scatterers.

The presented approach is amenable to extensions to other numerical techniques for the solution of electromagnetic radiation/scattering problems, e.g., MoM.

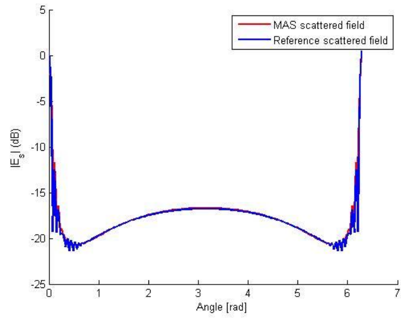

Fig. 6. Amplitude of the scattered field for the case a=6λ.

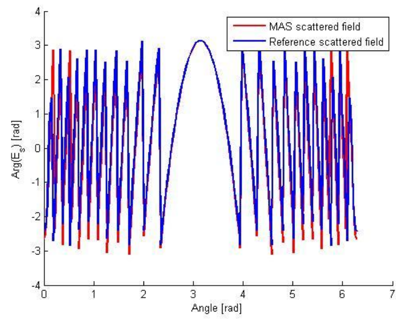

Fig. 7. Amplitude of the scattered field for the case a=6λ.

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