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INVESTIGATION OF SOLUTIONS OF THE NON-LINEAR SYNTHESIS PROBLEM FOR LINEAR ARRAY BY THE GENERALIZED NEWTON METHOD

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Abstract

The synthesis problem for linear array according to the prescribed amplitude radiation pattern is considered in the variational statement. The functional representing the weighted inner product of prescribed and synthesized amplitude radiation patterns is used. The problem reduces to solving the non-linear integral equation. The generalized Newton procedure is used for search of solutions. The results of numerical calculations are given for several types of prescribed amplitude radiation pattern.

Keywords: Amplitude radiation pattern, variational statement, non-linear integral equation, generalized Newton method.

1. INTRODUCTION

In the process of solving the synthesis problems according to the prescribed amplitude radiation pattern appear the non-linear integral or matrix equations. The solution of such equations is realized most effectively by the method of successive approximations. These methods allow to solve the corresponding equation at fixed value of characteristic electrophysical parameter of antenna, which is contained in the equation kernel [1].

Since the corresponding equations have, as rule, the non-unique solution, that investigation of the number of solutions and their properties is carried out for fixed type of initial approximation and for separate values of characteristic parameter. This is disadvantage of these methods.

The proposed modified Newton procedure allows to investigate the properties of solution for various types of initial approximation and to use the information of previous steps to receive the solution in wide range of characteristic parameter.

2. GENERALIZED SCHEME OF NEWTON METHOD

The phase optimization problem according to prescribed amplitude radiation pattern F in the variational statement consists in the maximization of the functional

$$\chi(\psi) = (|A[|u|\exp(i\psi)]|, F)_f , \qquad (1)$$

where $|u|$ and F are given real positive functions. The sought optimizing function is ψ representing the phase distribution of current in antenna elements.

The various modifications of the Newton method have been used for solving the nonlinear equations [2]. The corresponding Lagrange-Euler equation for (1) can be written as the system of two nonlinear equations [3]

$$\Phi(w, f, c) = 0 , \qquad (2a)$$

$$\Psi(w, f, c) = 0 , \qquad (2b)$$

where

$$\Phi(w, f, c) = w - A^*[F \exp(i \arg f)] , \qquad (3a)$$

$$\Psi(w, f, c) = f - A[|u|\exp(i \arg w)] . \qquad (3b)$$

Functions w, f are unknown in these equations. If they are found, the sought real function ψ calculated as $\psi = \arg w$.

Following [3], we denote the increment to w, f, and c (c is the characteristic electrophysical parameter of antenna, which contains in kernel of operator A) at the p-th step of the Newton method as $\delta_w = w_{p+1} - w_p$, $\delta_f = f_{p+1} - f_p$, $\varepsilon = c_{p+1} - c_p$, and write the equations for δ_w, δ_f, and ε in the form

$$\Phi_c\varepsilon + \Phi_w(w_p, f_p, c_p, \delta_w) + \Phi_f(w_p, f_p, c_p, \delta_f) = -\Phi(w_p, f_p, c_p) , \qquad (4a)$$

$$\Psi_c\varepsilon + \Psi_w(w_p, f_p, c_p, \delta_w) + \Psi_f(w_p, f_p, c_p, \delta_f) = -\Psi(w_p, f_p, c_p) , \qquad (4b)$$

where functions Φ_c, Φ_w, Φ_f, Ψ_c, Ψ_w, Ψ_f depend on the operators A, A^*, and array parameters.

The procedure of determination of the "distance" h between the branches of various solutions consists in accounting of all constituents of general solution. The next formula for the increment of solution is used

$$h^2 = |\Delta c|^2 + \|\Delta w\|^2 + \|\Delta f\|^2 , \qquad (5)$$

where $\Delta c = c_m - c_{m-1}$, $\Delta w = w_m - w_{m-1}$, $\Delta f = f_m - f_{m-1}$, m is the point number on the branch of solution. By virtue of (5), the linear equation system (4) should be supplemented by the equation

$$\Delta c_p \cdot \varepsilon + (\Delta w_p, \delta_w) + (\Delta f_p, \delta_f) = 0 . \qquad (6)$$

In the matrix form the linear equation system (4) can be written as

$$\begin{Bmatrix} \Phi_c & \Phi_w & \Phi_f \\ \Psi_c & \Psi_w & \Psi_f \\ \Delta c & \Delta w & \Delta f \end{Bmatrix} \times \begin{Bmatrix} \varepsilon \\ \delta_w \\ \delta_f \end{Bmatrix} = \begin{Bmatrix} -\Phi \\ -\Psi \\ 0 \end{Bmatrix} . \qquad (7)$$

The matrix blocks in (7) are denoted by the same letters as the functions generated them.

The unknown functions δ_w, δ_f in the system (7) are complex. We can reduce two upper equation blocks of this system to those for the real constant ε and functions only. In particular, let us consider this possibility for the case if the parameter c is fixed ($\varepsilon = 0$). There, the last equation in (7) is absent, and (7) becomes the form

$$\begin{Bmatrix} \widehat{\Phi}_w & \widehat{\Phi}_f \\ \widehat{\Psi}_w & \widehat{\Psi}_f \end{Bmatrix} \times \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -\widehat{\Phi} \\ -\widehat{\Psi} \end{Bmatrix} . \qquad (8)$$

where $\delta_1 = \mathrm{Im}(\delta_w / w)$, $\delta_2 = \mathrm{Im}(\delta_f / f)$, and the new elements of the block-matrices are formed by the functions

$$\widehat{\Phi}_w = |w|^2 \delta_1 , \qquad (9a)$$

$$\widehat{\Phi}_f = -\mathrm{Re}(w \cdot A^* F \exp(i \arg f_0)\delta_2]) , \qquad (9b)$$

$$\widehat{\Psi}_w = -\mathrm{Re}(\overline{f} \cdot A[|v| \exp(i \arg w_0)\delta_1]) , \qquad (9c)$$

$$\widehat{\Psi}_f = |f|^2 \delta_2 , \qquad (9d)$$

$$\widehat{\Phi} = \mathrm{Im}(\overline{w}\Phi) , \qquad (9e)$$

$$\widehat{\Psi} = \mathrm{Im}(\overline{f}\Psi) . \qquad (9f)$$

The system (9) can be simplified if even one of the functions w and f has no zeros in its definition domain owing the diagonality of the matrices $\widehat{\Phi}_w$ and $\widehat{\Psi}_f$. In this case one of the functions δ_1, δ_2 can be eliminated from the system, and solution to (8) is reduced to solution of one equation only.

3. Detailed Elaboration for Linear Array

We consider the case, when the operator A describes the radiation pattern (array factor) of linear array.

$$f(\xi) = Av \equiv \sqrt{\frac{c}{2\pi}} \sum_{n=-M}^{M} v_n \exp(icn\xi) , \qquad (10)$$

where $c = ka \sin \alpha$, $N = 2M + 1$ it total number of array elements, ξ is generalized angular coordinate. The operator A^* acts in the following way

$$A^* f = \sqrt{\frac{c}{2\pi}} \int_{-\pi/c}^{\pi/c} f(\xi) \exp(-icn\xi) d\xi , \qquad (11)$$

In this case, the maximization of (1) cab be reduced to the simplified functional

$$\chi(\psi) = \int_{-\pi/c}^{\pi/c} |f(\xi)| F(\xi) d\xi . \qquad (12)$$

The Lagrange-Euler equations for the functional (12) have the form (7) with functions

$$\Phi(w, f, c) = w(x) - \\ -\sqrt{\frac{c}{2\pi}} \int_{-\pi/c}^{\pi/c} F(\xi) \exp(i \arg f) \exp(-icnx\xi) d\xi , \qquad (13a)$$

$$\Psi(w, f.c) = f(\xi) - \\ -\sqrt{\frac{c}{2\pi}} \sum_{n=-N}^{N} |v_n(x)| \exp(i \arg w) \exp(icnx\xi) . \qquad (13b)$$

The sought function of problem are $\psi = \arg(w)$ and $\varphi = \arg(f)$. The properties of these functions essentially depend on the evenness of the given functions $|u|$ and F. The investigation of solutions is most easy in the case when they both are even functions of their arguments: $|u(-x)| = |u(x)|$ and $F(-\xi) = F(\xi)$. Unfortunately, any analytical representation of the solutions to (7) like [4] did not found for the phase synthesis problem, and all these solutions and their branching should be investigated numerically.

For the case of symmetrical data, these solutions can be classified into the following groups:

1. Both the functions $u(x)$ defined by action of operator (11), and $f(\xi)$ defined by operator (10) are real at $x \in [-a, a]$, where $2a$ - longitudinal size of array, and $\xi \in [-\pi/c, \pi/c]$.
2. One of these functions, say $u(x)$ is real, and other $f(\xi)$ has an odd (with accuracy to a constant addend) phase $\varphi(-\xi) = -\varphi(\xi)$.
3. Both functions $u(x)$ and $f(\xi)$ have even phases $\psi(-x) = \psi(x)$, $\varphi(-\xi) = \varphi(\xi)$.

The solutions of such three types are investigated in the process of numerical calculations.

4. Numerical Results

The numerical calculations are carried out for the several types of the prescribed amplitude radiation pattern. The results for the prescribed amplitude radiation pattern $F(\xi) = \cos(\pi\xi/2)$ are shown in Figs. 1-4. In Fig. 1, the value of functionals χ and σ is shown for various types of initial approximation of the current phase $\psi^{(0)}(x)$. The maximization problem of χ is equivalent in some sense to minimization problem of functional σ which in our case has the form

$$\sigma(\psi) = \int_{-\infty}^{\infty} [|f(\xi)| - F(\xi)]^2 d\xi . \qquad (14)$$

The solid lines correspond to χ values, and the dashed lines correspond to σ values. The number of array elements $N = 11$, parameter c changes from

$c = 0$ to $c = 10$. For the values of Nc which do not exceed $Nc = 5$ all types of solutions give the same values of χ and σ. At $c \approx 2\pi$ the branching of solutions appears, and optimal value for χ and σ functionals gives the solution with even phase ψ.

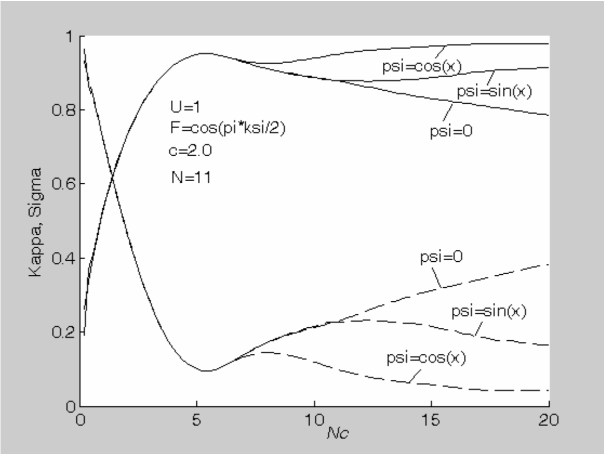

Fig. 1. The χ and σ values versus the electrical size of array Nc

The prescribed amplitude radiation pattern F and synthesized $|f|$ are shown in Fig. 2. The amplitudes $|f|$ in the considerable extent differ from the amplitude F in consequence of small value of c parameter ($c = 1.6$ only).

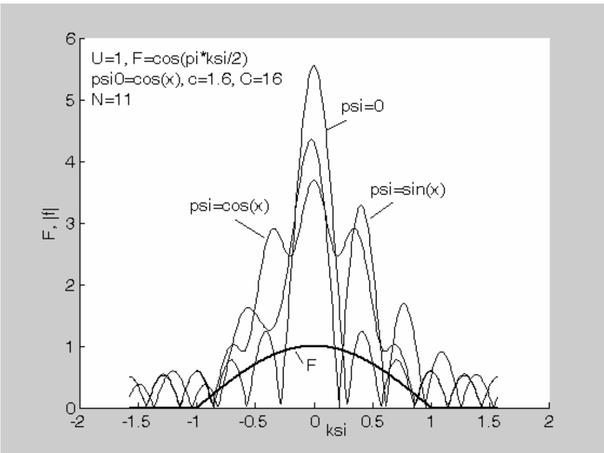

Fig. 2. The synthesized amplitude radiation patterns for various types of current phase

The optimal values of sought phase distributions ψ are shown in Fig.3, the given current amplitude distribution is $|u| = 1$. The optimal phase distributions ψ keep the parity properties of corresponding initial approximations ψ_0. The optimal values of χ and σ provides the solution with phase distribution $\psi_0(x) = \cos(x)$ (see Fig. 1).

The quality of approximation to prescribed amplitude pattern F too much depends on the parameter c. At $c = 3.14$ the χ and σ values are noticeably smaller than for $c = 1.6$, although the value of N is greater in case of Fig. 1.

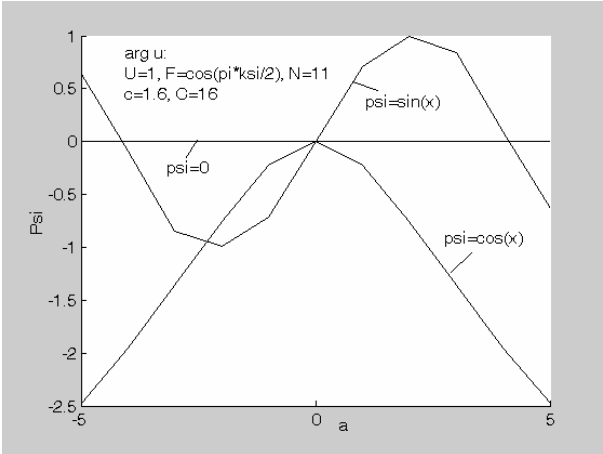

Fig. 3. The optimal phase distributions of current for various initial approximations

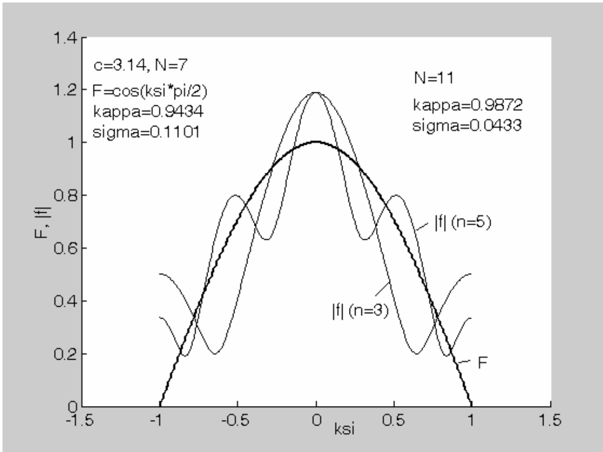

Fig. 4. Amplitude radiation pattern $|f|$ for various number N of array elements at $c = 3.14$

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