International Conference on Antenna Theory and Techniques, 6-9 October, 2009, Lviv, Ukraine pp. 143-144

THE FAST CONVERGING METHOD OF CALCULATION OF WIRE RADIATORS IN INFINITE PLANAR PHASED ANTENNA ARRAY

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Abstract

The effective fast converging method of calculation of wire radiators in infinite planar phased antenna array is developed. The method is based on use of Green's function submitted as double series with accelerated convergence. The moment method is applied and piecewise sine basic and weight functions automatically satisfied to Kirchhoff's current law are used. Expressions for impedance matrix elements consist of two series. Members of the series generated by spectral part of Green's function are represented as the closed analytical expressions. Members of series generated by spatial part of Green's function are calculated numerically by one-fold integration.

Keywords: Infinite phased array, numerical methods, integral equations, thin wire radiator, Green's function.

1. INTRODUCTION

The model of an infinite array is often used for an estimation of characteristics of radiators in big array. In this case the problem is reduced to one radiator, and Green's function is represented in the form of double series in which at calculations it is necessary to keep to several thousand summands. In work [1] the method of accelerating the convergence of series representing the Green's function is offered**.**

In the given work the effective method for calculation the elements of a matrix of mutual impedances for thin-wire radiators with use the mentioned Green's function is developed.

2. THEORY

Let's consider an infinite planar array of wire structures of the arbitrary form located in units of an oblique-angled grid defined by vectors $\mathbf{d}_1$ and $\mathbf{d}_2$.

The resultant formula in [1] contains Green's function in spectral (the first row in the formula (1)) and spatial representations (the second row in the formula (1)), including the one for imaginary wave number:

$$G^{\infty}(\mathbf{r}-\mathbf{r}') = \frac{1}{2A}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\left[\frac{\exp\left(-\gamma_{pq}|z-z'|\right)}{\gamma_{pq}} - \frac{\exp\left(-\nu_{pq}|z-z'|\right)}{\nu_{pq}}\right]\exp\left[-i\boldsymbol{\kappa}_{pq}^{\perp}(\mathbf{r}-\mathbf{r}')\right] + \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\exp(-uR_{mn})}{R_{mn}}\exp(-im\psi_1 - in\psi_2), \quad (1)$$

where

$$\gamma_{pq} = \sqrt{\boldsymbol{\kappa}_{pq}^{\perp\,2} - k^2}, \quad \nu_{pq} = \sqrt{\boldsymbol{\kappa}_{pq}^{\perp\,2} + u^2},$$

$$\boldsymbol{\kappa}_{pq} = \boldsymbol{\kappa}_{pq}^{\perp} + \mathbf{z}\kappa_3,$$

$$\boldsymbol{\kappa}_{pq}^{\perp} = \frac{2\pi p + \psi_1}{A}\mathbf{d}_2\times\mathbf{z} + \frac{2\pi q + \psi_2}{A}\mathbf{z}\times\mathbf{d}_1.$$

In these formulas $\mathbf{z}$ is unit vector, orthogonal to the plane of array, vectors $\mathbf{d}_1$, $\mathbf{d}_2$ and $\mathbf{z}$ are right-handed, $A = \mathbf{z}(\mathbf{d}_1\times\mathbf{d}_2)$ is an area of the parallelogram representing an individual cell, ψ_1 and ψ_2 are phase shifts between the adjacent radiators. The number u is real.

In [2] the method of calculation of wire radiators in infinite planar phased array is offered. Each radiator consists of the piece-wise linear thin wire elements arbitrarily located in space. The functions introduced in [3] which automatically satisfy to Kirchhoff's current law are used as basis and weight. It has been found in [2] that in such a case the elements of impedance matrix in array are expressed as double series of linear combinations of integrals

$$I = \int_{-\infty}^{\infty}\frac{\exp(iat)}{t^2+b^2}\,\frac{\sin(c_1t-t_1)}{c_1t-t_1}\,\frac{\sin(c_2t-t_2)}{c_2t-t_2}\,dt,$$

in which the numbers a, b, c_1, c_2, t_1, t_2 are functions of summation indexes p, q, m, n.

Last integrals are calculated in the closed form by means of residue theory [2].

The second row in the formula (1) represents the set of Green's functions $G(r) = \exp(-ur)/4\pi r$ for imaginary wave number $k = -iu$. It is established, that in this case the fields of linear electric current $I(z) = A\cos kz + B\sin kz$ are

$$E_z = \frac{i}{\omega\varepsilon_a'}\left[\frac{dI(z')}{dz'}G(r) + I(z')\frac{\partial G(r)}{\partial z}\right]_{z_0}^{z_1},$$

$$E_\rho = \frac{i}{\omega\varepsilon_a'}\frac{1}{u\rho}\left\{\left[\frac{dI(z')}{dz'}\frac{\partial g(r)}{\partial z} + I(z')\frac{\partial^2 g(r)}{\partial z^2}\right]_{z_0}^{z_1} + \left(k^2+u^2\right)\left[-I(z')g(r)\Big|_{z_0}^{z_1} + \int_{z_0}^{z_1}\frac{dI(z')}{dz'}g(r)dz'\right]\right\}, \quad (2)$$

where $g(r) = rG(r) = \exp(-ur)/4\pi$. The formula (2) contains integration the slowly varying function and hence this procedure demands small expenses of processor time.

Summarizing results and applying a reduction to calculation of infinite series, we will have for elements of impedance matrix

$$Z_{ij}^{\infty} = \sum_{p=-P}^{P}\sum_{q=-Q}^{Q} Zspec_{ij}^{pq} + \sum_{m=-M}^{M}\sum_{n=-N}^{N} Zspat_{ij}^{mn}. \quad (3)$$

The first series contains only analytical expressions; the second one contains well computed integrals.

3. Numerical results

As an example we will consider a radiator with geometry shown on fig. 1. The sense of designations is clear from drawing, we will notice only, that all conductors are located in one plane. The voltage δ-generator is placed in the middle of a crosspiece connecting wire. Radiators are over the infinite perfect conducting ground screen. The antenna has parameters: L=0,27 m, H=0,33 m, D=0,05 m, β =40°, and diameters of all conductors are equal 16 mm. The radiators are located in the nodes of a square grid, distance between radiators d_1=d_2=0,52 m, planes of vibrators are placed diagonally. Note that radiator contains wires of various orientations relative to array plane: horizontal, vertical and inclined, that gives the chance to investigate influence of an inclination of conductors on convergence of series (3).

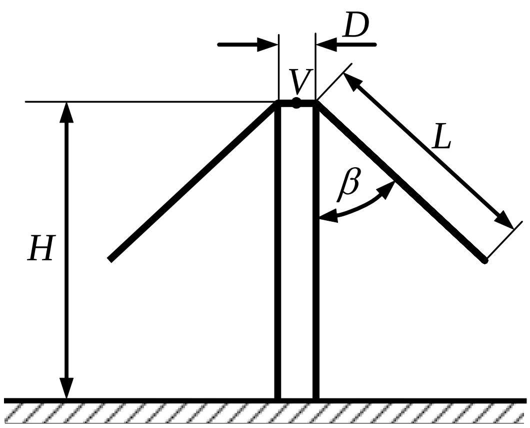

Fig. 1. The geometry of a radiator.

The analysis shows, that already at P, Q, M, N=3 (49 terms in each series (3)), are observed convergence of results when curves graphically coincide.

For comparison we will specify, that some thousand terms are necessary to keep the satisfactory convergence when using the spectral or element by element approach.

4. Conclusions

Thus, in the given work it is developed the effective fast converging method of calculation of wire radiators in infinite planar phased antenna array. Efficiency of a method is based on use fast converging Green's function in a combination with effective way of calculation of matrix of mutual impedances when they are defined analytically or by means of unitary integration of slowly varying function. In most cases it is enough to keep some tens terms for satisfactory convergence.

Acknowledgments

The author expresses acknowledgments to all authors, whose results have been used in the given work.

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