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THE ELECTROMAGNETIC WAVES DIFFRACTION ON FINITE APERTURE ARRAY

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Abstract

The diffraction both plane monochromatic wave and ultra short electromagnetic impulse is discussed. At the heart of the solution the method of integral equation in space-frequency and time-space representations is used. Integral equations are solved by the Galerkin's method with Tchebyshev basis which is everywhere except corner points takes into account the field singularity at the metal edges. Moreover in time-space equations splines are used for approximation of time dependence solution. The methods of effective calculation of matrix elements in the system of linear algebraic equations are discussed. The C# program is worked out. The investigation results are given.

Keywords: diffraction, impuls, integral equation, Galerkin's method, splines, metal screen.

1. Introduction

The structure under investigation is an ideally conductive infinitely thin metal screen with arbitrary number of rectangular apertures. The screen lies on the boundary of dielectrics. The diffraction both plane monochromatic wave and ultra short electromagnetic impulse is discussed. At the heart of the solution the method of integral equation in space-frequency and time-space representations is used. Integral equations are solved by the Galerkin's method with Tchebyshev basis which is everywhere except corner points takes into account the field singularity at the metal edges. Moreover in time-space equations splines are used for approximation of time dependence solution. The methods of effective calculation of matrix elements in the system of linear algebraic equations are discussed. The C# program is worked out. The investigation results are given.

2. Target settings

The screen with N rectangular apertures lies on the boundary of dielectrics with parameters ε_1, μ_1 for $y \geq 0$ and ε_2, μ_2 for $y \leq 0$. The aperture axes are parallel to *OX* and *OZ* axes. The dimensions of ν's aperture are $2l_\nu$ on *OX* axis and $2L_\nu$ on *OZ* axis, the center coordinates – $\overline{x}_\nu, \overline{z}_\nu$. The diffraction problem of monochromatic wave solution is added up to vector integral-differential equation (IDE).

$$\begin{aligned} & grad \int_S \sigma(x', z') g_1(x - x', z - z') ds' + \\ & + k^2 \int_S \vec{J}(x', z') g_2(x - x', z - z') ds' = \\ & = ikZ_0 \vec{H}^e(x, z) \quad x, z \in S \end{aligned} , \qquad (1)$$

where $\vec{J}$, $\sigma = \dfrac{\partial J_x}{\partial x} + \dfrac{\partial J_z}{\partial z}$ - magnetic current and charge density on the aperture S,

$g_{1,2}(x - x', z - z') = V^{-1}$

$$f_{1,2} = \frac{1}{2\pi R} \sum_{j=1}^{2} \zeta_j^{1,2} \exp(-ik_j R),$$

$$R = \sqrt{(x - x')^2 + y^2 + (z - z')^2}, \zeta_j^1 = \frac{1}{\mu_j}, \ \zeta_j^2 = \varepsilon_j .$$

IDE (1) is solved by the Galerkin's method. The unknown functions $E_{x,z}(x,z)$ are decomposed in series on Tchebyshev polynomials with unknown coefficients $X_{mn}^{(\mu)}$, $Z_{mn}^{(\mu)}$:

$$E_x(x,z) = \sum_{\mu=1}^{N} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} X_{mn}^{(\mu)} W_{mn}^{(x,\mu)}(x,z), \qquad (2)$$

$$E_z(x,z) = \sum_{\mu=1}^{N} \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} Z_{mn}^{(\mu)} W_{mn}^{(z,\mu)}(x,z), \qquad (3)$$

where

$$W_{mn}^{(x,\mu)}(x,z) = \overline{T}_m\left(\frac{\tilde{x}_\mu}{l_\mu}\right) \overline{U}_n\left(\frac{\tilde{z}_\mu}{L_\mu}\right),$$

$$W_{mn}^{(z,\mu)}(x,z) = \overline{U}_m\left(\frac{\tilde{x}_\mu}{l_\mu}\right) \overline{T}_n\left(\frac{\tilde{z}_\mu}{L_\mu}\right),$$

$$\tilde{x}_\mu = x - \bar{x}_\mu, \tilde{z}_\mu = z - \bar{z}_\mu,$$

$$\bar{T}_n\left(\frac{z}{L}\right) = \frac{T_n\left(\frac{z}{L}\right)}{\sqrt{L^2 - z^2}},$$

$$\bar{U}_n\left(\frac{z}{l}\right) = -\frac{1}{n+1}U_n\left(\frac{z}{l}\right)\sqrt{1-\left(\frac{z}{l}\right)^2},$$

T_n, U_n - Tchebyshev polynomials. The polynomial choice is done including conditions on metal edge. The basis functions (2), (3) fulfills the condition at the edge on the aperture everywhere, except small neighborhood of edge points. Moreover

$$\frac{d}{dz}\bar{U}_n\left(\frac{z}{L}\right) = \bar{T}_{n+1}\left(\frac{z}{L}\right) = \frac{T_{n+1}\left(\frac{z}{L}\right)}{\sqrt{L^2 - z^2}}.$$

This condition makes easier the IDE solution, because allows the integration by parts. After some standard transformations for the Galerkin's method and integration by parts the infinite system of linear algebraic equations (SLAE) is got out

$$\sum_{\mu=1}^{N}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\left(X_{mn}^{(\mu)}A_{mn,m'n'}^{11,\nu\mu} + Z_{mn}^{(\mu)}A_{mn,m'n'}^{12,\nu\mu}\right) = -i\omega\mu_0 B_{m'n'}^{(z)}$$

$$\sum_{\mu=1}^{N}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\left(X_{mn}^{(\mu)}A_{mn,m'n'}^{21,\nu\mu} + Z_{mn}^{(\mu)}A_{mn,m'n'}^{22,\nu\mu}\right) = i\omega\mu_0 B_{m'n'}^{(x)}$$

$$\nu = 1,2,...N, m' = 0,1...., n' = 0,1....$$

where

$$B_{m'n'}^{(z,x)} = \int_{S_\nu} W_{mn}^{(x,z,\mu)}(x,z)H_{z,x}^{e}(x,z)dxdz,$$

$$A_{mn,m'n'}^{11,\nu\mu} = k^2 D_{mn,m'n'}^{\nu\mu} - C_{m,n+1,m',n'+1}^{\nu\mu},$$

$$A_{mn,m'n'}^{12,\nu\mu} = A_{m'n',mn}^{12,\mu\nu} = C_{m+1,n,m',n'+1}^{\nu\mu},$$

$$A_{mn,m'n'}^{22,\nu\mu} = k^2 E_{nm,n'm'}^{\nu\mu} - C_{m+1,n,m'+1,n'}^{\nu\mu},$$

$$C_{mn,m'n'}^{\nu\mu} = \int_{S_\nu}\bar{T}_{m'}\left(\frac{\tilde{x}_\nu}{l_\nu}\right)\bar{T}_{n'+1}\left(\frac{\tilde{z}_\nu}{L_\nu}\right)dxdz \times \times\int_{S_\mu}\bar{T}_m\left(\frac{\tilde{x}'_\mu}{l_\mu}\right)\bar{T}_{n+1}\left(\frac{\tilde{z}'_\mu}{L_\mu}\right)g_1(x-x',z-z')dx'dz', \quad (4)$$

$$D_{mn,m'n'}^{\nu\mu} = \int_{S_\nu}\bar{T}_{m'}\left(\frac{\tilde{x}_\nu}{l_\nu}\right)\bar{U}_{n'}\left(\frac{\tilde{z}_\nu}{L_\nu}\right)dxdz \times \times\int_{S_\mu}\bar{T}_m\left(\frac{\tilde{x}'_\mu}{l_\mu}\right)\bar{U}_n\left(\frac{\tilde{z}'_\mu}{L_\mu}\right)g_2(x-x',z-z')dx'dz'. \quad (5)$$

$$E_{mn,m'n'}^{\nu\mu} = \int_{S_\nu}\bar{U}_{m'}\left(\frac{\tilde{x}_\nu}{l_\nu}\right)\bar{T}_{n'}\left(\frac{\tilde{z}_\nu}{L_\nu}\right)dxdz \times \times\int_{S_\mu}\bar{U}_m\left(\frac{\tilde{x}'_\mu}{l_\mu}\right)\bar{T}_n\left(\frac{\tilde{z}'_\mu}{L_\mu}\right)g_2(x-x',z-z')dx'dz'. \quad (6)$$

If $\nu \neq \mu$, the integrals in matrix elements $C_{mn,m'n'}^{\nu\mu}$, $D_{mn,m'n'}^{\nu\mu}$, $E_{mn,m'n'}^{\nu\mu}$ are found numerically by the quadrature of highest precision [1]. These calculations are simple and effective – the quadrature nods number is only by 2-3 higher then the corresponding Tchebyshev polynomials order. However for $\nu = \mu$ the subintegral functions in (4) - (6) have the singularity of $1/R$ type, when the source and observation points are coincident.

So, for $\nu = \mu$ it is necessary to extract analytically and transform the singular part of the equation core [2], [3]. For the rectangular apertures the Fourier transformation of basis functions is known, so it is much easier to calculate the matrix elements in spectral domain. The corresponding methods for matrix elements calculation are developed.

For time-domain transition the inverse Fourier transformation on ω is applied to (1), using the convolution theorem and the integral

$$g(r,t) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\frac{e^{i(-kr+\omega t)}}{r}d\omega = \frac{1}{2\pi}\delta(t-r/c)/r,$$

where δ – delta function. As a result it is found

$$grad\int_S \sigma(x',z',t-r/c)/r\,ds' - -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_S \vec{J}(x',z',t-r/c)/r\,ds' = \frac{Z_0}{c}\frac{\partial \vec{H}^e(x,z,t)}{\partial t} \quad (6)$$

The basis (2), (3) is used to solve (6) in spatial value. However in this case the unknowns $Z_{m,n}, X_{m,n}$ are the functions of time $Z_{m,n}(t), X_{m,n}(t)$. After the Galerkin's method application not the SLAE but the system of linear integral-differential equations is got out. To solve these equations two method where used. In the first one the functions $Z_{m,n}(t), X_{m,n}(t)$ are approximated by basis functions depending on time $t \in (-\infty, \infty)$. In the second case it was used that in t time the subintegral expressions are the functions of $t' = t - r/c \le t$.

3. Calculation results

Let us show some calculation results for electromagnetic pulse diffraction on the system of rectangular apertures. On fig. 1 the diffracted perpendicular falling impulse with duration T=0.1 ns on two similar apertures with dimensions l_x = 1.0 mm, l_z = 5.0 mm and centers in points X_{c1} = -3.0 mm, Z_{c1} = 0.0б mm, X_{c2} = 3.0 mm, Z_{c2} = 0.0 mm is shown. The observation point is situated under the screen.

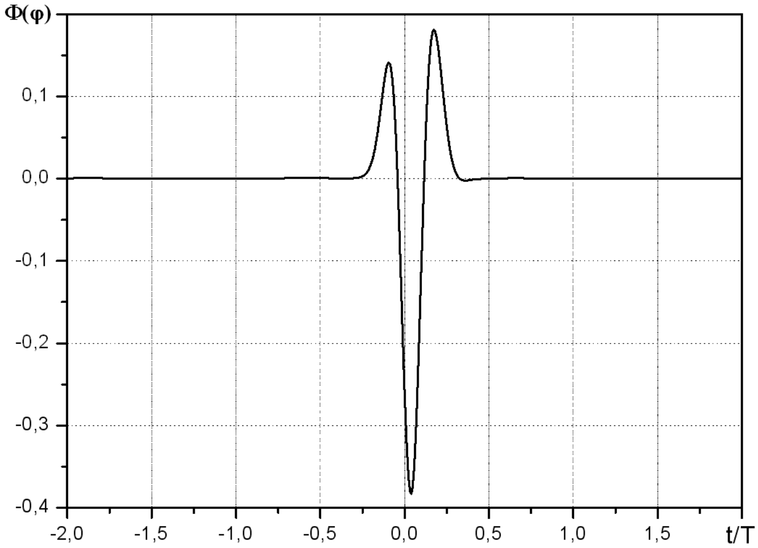

Fig. 1

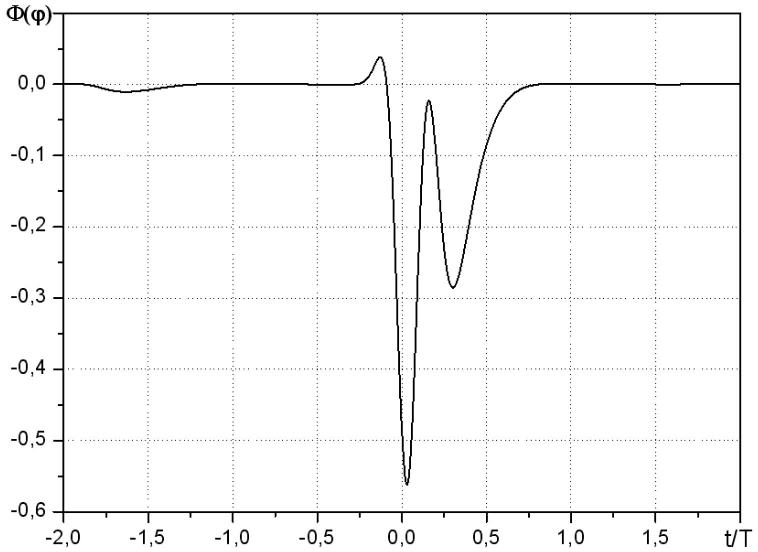

Fig. 2

On fig. 2 the diffracted perpendicular falling impulse with duration T=0.1 ns on two different apertures with dimensions l_{x1} = 1.0mm, l_{z1} = 5.0 mm, l_{x2} = 1.0 mm, l_{z2} = 4.0 mm and central points X_{c1} = -2.0 mm, Z_{c1} = 0.0б mm, X_{c2} = 2.0 mm, Z_{c2} = 0.0 mm is shown. The observation point is also situated under the screen.

So, this results are allows to say about the possibility of shown algorithms and calculation programs application for the investigations of diffracted electromagnetic pulses on the system of apertures in metal screens, situated also in dielectrics.

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