International Conference on Antenna Theory and Techniques, 6-9 October, 2009, Lviv, Ukrainepp. 110-112

THE ELECTROMAGNETIC WAVES DIFFRACTION BY BOUNDED AND SEMI-INFINITE SEQUENCES OF PLANE SCREENS WITH SLOT

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Abstract

The diffraction problem of electromagnetic waves by the systems of plane screens with slot was considered. The operator approach was used. The possibility of eigen-waves propagation between screens was took into consideration. The dependencies of reflection and transmission coefficients of structure and excitation coefficient of plane waveguides from the distance between screens were investigated.

Keywords: screen with slot, reflection and transmission coefficients.

1. INTRODUCTION

Diffraction problems by finite or semi-infinite periodical structures are investigated for a long time [1]-[4]. The solution of such problems allows creating of mathematical models of real devises. The diffraction problem by finite and semi-infinite systems of plane screens is solved using operator approach [5] in this work.

2. SINGLE SCREEN WITH SLOT

Let's place the screen with slot in the plane $z=0$ so that slot was cut along OX axis. The slot length is $2d$. For E - polarization case let's represent the E_x component of electric field as superposition of fields, scattered by the left (with spectral function $D_1(\xi)$) and right (with spectral function $D_2(\xi)$) half plane:

$$E^+(y,z)=\int_{-\infty}^{\infty}\left(D_1(\xi)+D_2(\xi)\right)e^{ik\xi y+ik\gamma(\xi)z}d\xi$$

$$+\int_{-\infty}^{\infty}q(\xi)e^{ik\xi y-ik\gamma(\xi)z}d\xi,\ z>0,$$

$$E^-(y,z)=\int_{-\infty}^{\infty}\left(D_1(\xi)+D_2(\xi)\right)e^{ik\xi y-ik\gamma(\xi)z}d\xi$$

$$+\int_{-\infty}^{\infty}q(\xi)e^{ik\xi y-ik\gamma(\xi)z}d\xi,\ z<0.$$

Functions $D_1(\xi)$, $D_2(\xi)$ can be obtained from the following equations:

$$D_1=s^+R_1s^-D_2+s^+R_1s^-q \qquad (1)$$

$$D_2=s^-R_2s^+D_1+s^-R_2s^+q \qquad (2)$$

where operators $s^{\pm}$ determine the amplitudes variation while coordinate system shifts along OY axis in positive or negative directions, operators R_1, R_2 are reflection operators of half plane with $y<0$ and $y>0$, which can be obtained by factorization approach [6]. Integrands in integral equations (1), (2) have singularities. After eliminating singularities we obtain solvable equations:

$$D_1(\xi)=\frac{ie^{-ikd\zeta}}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\sqrt{1-\zeta}}{\sqrt{1-\xi}}D_2(\zeta)-D_2(\xi)\right)\frac{e^{ikd\xi}}{\zeta-\xi}d\zeta$$

$$+D_2(\xi)+\frac{ie^{ikd\xi}}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\sqrt{1-\zeta}}{\sqrt{1-\xi}}q(\zeta)-q(\xi)\right)\frac{e^{-ikd\zeta}}{\zeta-\xi}d\zeta+q,$$

$$D_2(\xi)=-\frac{ie^{-ikd\xi}}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\sqrt{1+\zeta}}{\sqrt{1+\xi}}D_1(\zeta)-D_1(\xi)\right)\frac{e^{ikd\zeta}}{\zeta-\xi}d\zeta$$

$$+D_1(\xi)-\frac{ie^{-ikd\xi}}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\sqrt{1+\zeta}}{\sqrt{1+\xi}}q(\zeta)-q(\xi)\right)\frac{e^{ikd\zeta}}{\zeta-\xi}d\zeta+q.$$

3. FINITE SYSTEM OF SCREENS

Suppose that the reflection r_M and transmission t_M operators of a system of M plane screens with slot and reflection r and transmission t operators of a single screen are known. Let's place the $M+1$-th identical screen with slot in the plane $z=0$ so, that slots were placed strictly one under another. The distance between screens is h. Let's denote amplitudes of incidence, reflected, transmitted field and field between $M+1$-th and M-th screens, propagating in negative and positive directions of $Z-d$ as $q(\xi)$, $a(\xi)$, $D(\xi)$, $C_1(\xi)$, $B_1(\xi)$. They are connected by the following equations:

$$a=tq-q+teB_1,$$

$$C_1=tq+teB_1-eB_1, \qquad (4)$$

$$B_1 = r_M e C_1 = \tilde{r}_M e C_1 - e C_1, \qquad (5)$$
$$D = t_M e^M C_1,$$

where $\tilde{r}_M(\xi,\zeta) = r_M(\xi,\zeta) + \delta(\xi-\zeta)$, operator e allowing to define amplitude variation while coordinate system shifting along OZ axis in positive direction on the value h, t is a transmission operator of a unity layer. After denoting

$$C_1(\xi) = \frac{C(\xi)}{1-\exp(2ikh\gamma(\xi))},$$

from equations (4), (5) we obtain the following integral equation

$$C(\xi) = \int_{-\infty}^{\infty} t(\xi,\zeta) q(\zeta) d\zeta + \int_{-\infty}^{\infty} \frac{P(\xi,\zeta)C(\zeta)}{1-\exp(2ikh\gamma(\zeta))} d\zeta . (6)$$

In the case, if plane waveguide eigenwaves propagation possible between screens, i.e. $kh > 2\pi$, the integrand in (6) have poles of the first order. For elimination of singularities the regularization procedure is needed, after which we obtain operator equation

$$C = tq + PGC , \qquad (7)$$

where the kernel of operator P is described by the expression

$$P(\xi,\zeta) = \int_{-\infty}^{\infty} t(\xi,\eta) \tilde{r}_M(\eta,\zeta) e^{ikh(\gamma(\eta)+\gamma(\zeta))} d\eta$$
$$- t(\xi,\zeta) e^{2ikh\gamma(\zeta)} - e^{ikh(\gamma(\xi)+\gamma(\zeta))} \tilde{r}_M(\xi,\zeta),$$

and operator G acts on arbitrary function $q(\zeta)$ by the formula

$$(Gq)(\xi) = \frac{\chi(\xi)q(\xi)}{1-e^{2ikh\gamma(\xi)}}$$
$$+ \left(\frac{\bar{\chi}(\xi)q(\xi)}{1-e^{2ikh\gamma(\xi)}} - \sum_{\substack{p=-N \\ p\neq 0}}^{N} \delta(\xi-\beta_p) \int_{M_1}^{M_2} \frac{\sigma_p q(\beta_p)}{\zeta-\beta_p} d\zeta \right)$$
$$+ \sum_{\substack{p=-N \\ p\neq 0}}^{N} \delta(\xi-\beta_p) \sigma_p q(\beta_p) \left\{ \ln\left(\frac{M_2-\beta_p}{\beta_p - M_1} \right) + \pi i \,\mathrm{sgn}(p) \right\},$$

$$\chi(\xi) = \begin{cases} 1, x \notin [M_1, M_2], \\ 0, x \in (M_1, M_2), \end{cases} \quad \bar{\chi}(\xi) = \begin{cases} 0, x \notin [M_1, M_2], \\ 1, x \in (M_1, M_2), \end{cases}$$

$$\beta_p = \mathrm{sgn}(p) \sqrt{1 - \left(\frac{\pi p}{kh} \right)^2}, \qquad p = -N,\ldots,N, \qquad p \neq 0,$$

$N = \left[\frac{kh}{\pi} \right]$, $[M_1; M_2] \supset [-1;1]$. In the case of double screen with slot, $M = 1$ and it is necessary to assign $\tilde{r}_M = t$.

4. Semi-Infinite System of Screens

The operator equations relatively unknown amplitudes C_n, B_n, a (see Fig.1) of the electrical field component E_x for the semi-infinite system of screens with slot can be written in the form:

$$a = \tilde{R} q , \qquad (8)$$
$$a = rq + teB_1, \qquad (9)$$
$$C_1 = tq + reB_1, \qquad (10)$$
$$B_1 = \tilde{R} e C_1, \qquad (11)$$
$$B_1 = reC_1 + teB_2,$$
$$B_n = reC_n + teB_{n+1},$$
$$C_{n+1} = teC_n + reB_{n+1}, \; n = 1, 2, \ldots,$$

where $R(\xi,\zeta) = \tilde{R}(\xi,\zeta) + \delta(\xi-\zeta)$ is the reflection operator of the semi-infinite structure. After some transformations of equations (8)-(11) we obtain the operator equation for operator R determination

$$R = t + te\mathrm{Re}G(I + e\mathrm{Re}G)^{-1} R - te^2 G (I + e\mathrm{Re}G)^{-1} R ,$$

which can be solved by the iterative procedure.

5. Numerical Results

Using the algorithm stated above the series of numerical simulation for reflection, transmission and excitation of plane waveguides coefficients was realized. In Fig.2 the dependences of reflection ρ (straight line) and transmission coefficient τ (dash line) of double screen with slot of kh for the normal incidence are shown. Vertical lines correspond to the plane waveguide eigenwaves frequencies excitation. As one can see dependencies are practically periodical with period $kh \approx \pi$. The reflection and transmission coefficients are reduced while distance between screens is increased. It is explained by the eigenwaves excitation between screens.

The dependence of reflection coefficient ρ_∞ of semi-infinite structure as a function of kh for the normal incidence are shown in Fig.1. Maxima of ρ_∞ are greater then maxima of ρ on the interval $kh \in (0;2\pi)$, because of there is no excitation of plane waveguides eigenwaves at semi-infinite structure and two layered structure has a transmission field. During $kh > 2\pi$ maxima of ρ_∞ are less than corresponding maxima for the finite structure on the interval $kh \in (m\pi; (m+1)\pi)$, $m = 2, 3, \ldots$, because of all waveguides are become excited in semi-infinite structure. The reflection coefficient of semi-infinite structure approaches to the reflection coefficient of finite structure while the number of layers is increased. Due to the waveguides excitation coefficients increasing the maxima of reflection coefficient are reduced. Fig.3 shows the dependence of left waveguide of the first layer excitation coefficient as a function of kh for the semi-infinite structure for the normal incidence.

6. CONCLUSIONS

The possibility of a common approach using for the diffracted fields of a such structures obtaining was demonstrated on the examples of solving of diffraction problems by finite and semi-infinite periodical systems of screens with slot was demonstrated. Such approach can be used for solving of a number of problems.

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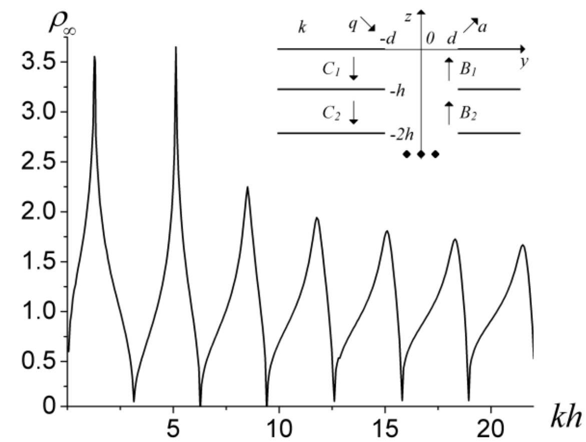

Fig. 1. The dependence of reflection coefficient of semi-infinite structure with slot of kh, $kd = 1.28$.

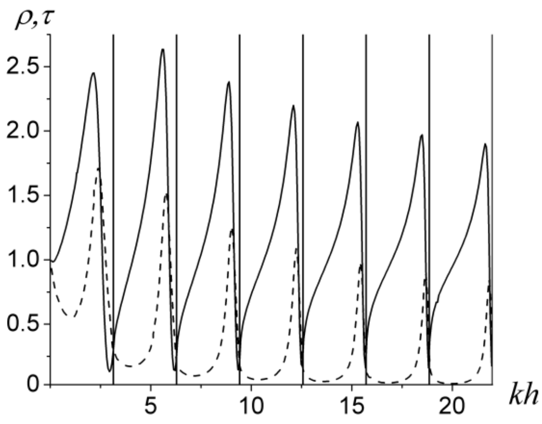

Fig. 2. The dependences of reflection (straight line) and transmission coefficient (dash line) of double screen with slot of kh, $kd = 1.28$.

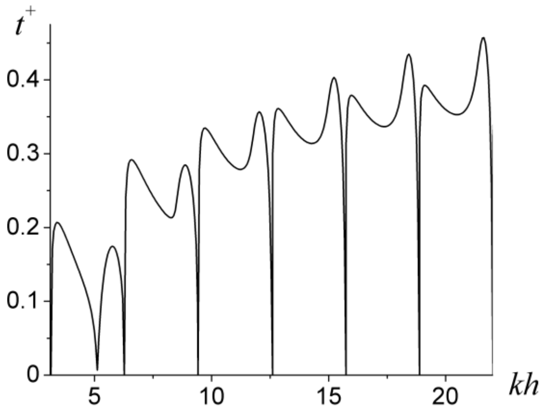

Fig. 3. The dependence of excitation coefficient of semi-infinite structure of kh, $kd = 1.28$.