THIN VIBRATORS WITH ARBITRARY EXCITATION AND SURFACE IMPEDANCE

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Abstract

The approximate analytical solution of the integral equation concerning the current in a thin straight vibrator with variable complex surface impedance has been obtained. The vibrator is located in unlimited space and is excited by the arbitrary field of impressed sources. For the case of vibrator's excitation in the distinct from the centre point along its length the calculations have been carried out and the plots of electrodynamic characteristics of the vibrator, depending on the magnitude and the type of its surface impedance and location of the point of excitation, are presented.

Keywords: vibrator, surface impedance, arbitrary point of excitation, electrodynamic characteristics.

1. INTRODUCTION

At present thin vibrators have wide application as different resonant elements of antenna-waveguide devices. The variety of available vibrator structures and also creation of new constructions on their basis stipulate constant interest of investigators to the problems of their analysis and synthesis. A special place among vibrator antennas is occupied by vibrators with the distributed along their length surface impedance (variable in a general case) which creates additional possibilities for shaping of the given electrodynamic characteristics of vibrator scatterers and radiators [1-4]. In the proposed report the approximate analytical solution of the integral equation concerning the current in a thin straight vibrator with the arbitrary excitation and surface impedance has been obtained. In order to check reliability of the obtained solution the comparison with the experimental values and calculated results, obtained by the method of moments, has been made. The results, showing the efficiency and possibilities of the vibrator's use with the distributed surface impedance in practical application, are also represented.

2. PROBLEM FORMULATION AND SOLUTION OF THE INTEGRAL EQUATION FOR THE CURRENT

For the vibrator representing a thin straight impedance cylinder of radius r and length $2L$ ($r/(2L)$ <<1), located in the free space and excited by the set field of impressed sources, the quasi-one-dimensional integral equation, concerning an electric current, has the following form [4]:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\int\limits_{-L}^{L} J(s')G_s(s,s')\,\mathrm{d}s' = -i\omega E_{0s}(s)+i\omega z_i(s)J(s). \quad (1)$$

Here $z_i(s)$ is the complex internal impedance per unit length, Ohm/m; $E_{0s}(s)$ is the component of the impressed sources electric field along the vibrator axis; $G_s(s,s')=\frac{e^{-ik\sqrt{(s-s')^2+r^2}}}{\sqrt{(s-s')^2+r^2}}$; $k=\frac{2\pi}{\lambda}$, λ is the wavelength (r/λ <<1); ω is the circular frequency; $s(s')$ is the longitudinal coordinate, coupled with the axis (surface) of the vibrator; $J(s)$ is the unknown current, submitted to the boundary conditions: $J(\pm L)=0$.

In a most general case the field of impressed sources, also as internal impedance of the vibrator, can be represented as sum of two components – symmetrical (upper index "*s*") and antisymmetrical (upper index "*a*") relatively to its geometrical centre: $E_{0s}(s)=E_{0s}^s(s)+E_{0s}^a(s)$, $z_i(s)=z_i^s(s)+z_i^a(s)$. At this, naturally, the vibrator current will also consist of two parts $J(s)=J^s(s)+J^a(s)$, and the equation (1) will transform into the system of two connected integral equations, concerning the unknown currents $J^s(s)$ and $J^a(s)$:

$$\begin{cases} \left(\dfrac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\displaystyle\int_{-L}^{L} J^s(s')G_s(s,s')\,\mathrm{d}s' = -i\omega E_{0s}^s(s) \\ \qquad + i\omega[z_i^s(s)J^s(s)+z_i^a(s)J^a(s)], \\ \left(\dfrac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\displaystyle\int_{-L}^{L} J^a(s')G_s(s,s')\,\mathrm{d}s' = -i\omega E_{0s}^a(s) \\ \qquad + i\omega[z_i^s(s)J^a(s)+z_i^a(s)J^s(s)]. \end{cases}$$

$$\begin{cases} \left(\dfrac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\displaystyle\int_{-L}^{L} J^s(s')G_s(s,s')\,\mathrm{d}s' = -i\omega E_{0s}^s(s) \\ \qquad + i\omega[z_i^s(s)J^s(s)+z_i^a(s)J^a(s)], \\ \left(\dfrac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\displaystyle\int_{-L}^{L} J^a(s')G_s(s,s')\,\mathrm{d}s' = -i\omega E_{0s}^a(s) \\ \qquad + i\omega[z_i^s(s)J^a(s)+z_i^a(s)J^s(s)]. \end{cases} \qquad (2)$$

Let us represent the vibrator currents in the form of the product of the $J_n^{s,a}$ unknown amplitudes on the given distribution functions $f_n^{s,a}(s)$ ($n=0,1$)

$$J^{s,a}(s) = J_0^{s,a}f_0^{s,a}(s) + J_1^{s,a}f_1^{s,a}(s), f_n^{s,a}(\pm L)=0, \qquad (3)$$

and let us use “the induced electromotive forces method” to solve the equations system (2), which is analogical to “the induced magnetomotive forces method”, approbated by us to solve the problems of electromagnetic coupling of the electrodynamic volumes via narrow slots in their common walls [5]. Then the equations (2) are transformed into the system of algebraic equations of the fourth order:

$$\begin{cases} J_0^s Z_{00}^{s\Sigma} + J_1^s Z_{01}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{00}^{sa} + J_1^a \tilde{\tilde{Z}}_{01}^{sa} = -\left(\dfrac{i\omega}{2k}\right)E_0^s, \\ J_0^s Z_{10}^{s\Sigma} + J_1^s Z_{11}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{10}^{sa} + J_1^a \tilde{\tilde{Z}}_{11}^{sa} = -\left(\dfrac{i\omega}{2k}\right)E_1^s, \\ J_0^a Z_{00}^{a\Sigma} + J_1^a Z_{01}^{a\Sigma} + J_0^s \tilde{\tilde{Z}}_{00}^{as} + J_1^s \tilde{\tilde{Z}}_{01}^{as} = -\left(\dfrac{i\omega}{2k}\right)E_0^a, \\ J_0^a Z_{10}^{a\Sigma} + J_1^a Z_{11}^{a\Sigma} + J_0^s \tilde{\tilde{Z}}_{10}^{as} + J_1^s \tilde{\tilde{Z}}_{11}^{as} = -\left(\dfrac{i\omega}{2k}\right)E_1^a. \end{cases} \qquad (4)$$

The following designations are accepted in (4) ($m=0,1; n=0,1$):

$$Z_{mn}^{s,a} = \frac{1}{2k}\left\{ \begin{array}{l} -\dfrac{\mathrm{d}f_m^{s,a}(s)}{\mathrm{d}s}A_n^{s,a}(s)\Bigg|_{-L}^{L} \\ +\displaystyle\int_{-L}^{L}\left[\dfrac{\mathrm{d}^2 f_m^{s,a}(s)}{\mathrm{d}s^2} + k^2 f_m^{s,a}(s)\right]A_n^{s,a}(s)\mathrm{d}s \end{array} \right\},$$

$$A_n^{s,a}(s) = \int_{-L}^{L} f_n^{s,a}(s')G_s(s,s')\mathrm{d}s',$$

$$Z_{mn}^{(s,a)\Sigma} = Z_{mn}^{s,a} + \tilde{Z}_{mn}^{s,a},$$

$$\tilde{Z}_{mn}^{s,a} = -\frac{i\omega}{2k}\int_{-L}^{L} f_m^{s,a}(s)f_n^{s,a}(s)z_i^s(\mathrm{s})\mathrm{d}s,$$

$$\tilde{\tilde{Z}}_{mn}^{\left\{ sa \atop as \right\}} = -\frac{i\omega}{2k}\int_{-L}^{L} f_m^{\left\{ s \atop a \right\}}(s) f_n^{\left\{ a \atop s \right\}}(s)z_i^a(\mathrm{s})\mathrm{d}s, \qquad (5)$$

$$E_m^{s,a} = \int_{-L}^{L} f_m^{s,a}(s)E_{0s}^{s,a}(s)\mathrm{d}s.$$

The equations system (4) is essentially simplified for the vibrator with constant along its length impedance ($z_i^s(s)=const$, $z_i^a(s)=0$):

$$\begin{cases} J_0^s(Z_{00}^s + \tilde{Z}_{00}^s) + J_1^s(Z_{01}^s + \tilde{Z}_{01}^s) = -\left(\dfrac{i\omega}{2k}\right)E_0^s, \\ J_0^s(Z_{10}^s + \tilde{Z}_{10}^s) + J_1^s(Z_{11}^s + \tilde{Z}_{11}^s) = -\left(\dfrac{i\omega}{2k}\right)E_1^s, \\ J_0^a(Z_{00}^a + \tilde{Z}_{00}^a) + J_1^a(Z_{01}^a + \tilde{Z}_{01}^a) = -\left(\dfrac{i\omega}{2k}\right)E_0^a, \\ J_0^a(Z_{10}^a + \tilde{Z}_{10}^a) + J_1^a(Z_{11}^a + \tilde{Z}_{11}^a) = -\left(\dfrac{i\omega}{2k}\right)E_1^a. \end{cases} \qquad (6)$$

The expression for the current has the form in this case ($J(s)=J^s(s)+J^a(s)$):

$$J^{s,a}(s) = -\frac{i\omega}{2k}\left[\begin{array}{l} \dfrac{E_0^{s,a}Z_{11}^{(s,a)\Sigma} - E_1^{s,a}Z_{01}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}} f_0^{s,a}(s) \\ + \dfrac{E_1^{s,a}Z_{00}^{(s,a)\Sigma} - E_0^{s,a}Z_{10}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}} f_1^{s,a}(s) \end{array} \right], \qquad (7)$$

As an example we will consider the problem about radiation of electromagnetic waves by the vibrator with the constant impedance, excited in the $s=-s_\delta$ point by the V_0 voltage generator (Fig. 1). Then

$$E_{0s}(s) = V_0\delta(s+s_\delta) = E_{0s}^s(s) + E_{0s}^a(s),$$
$$E_{0s}^{\left\{ s \atop a \right\}}(s) = \frac{V_0}{2}\left[\delta(s+s_\delta) \pm \delta(s-s_\delta)\right], \qquad (8)$$

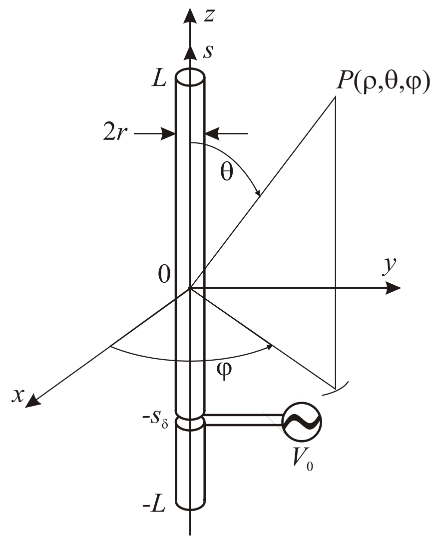

Fig. 1. The problem geometry and the symbols used.

It is natural to suppose, that the induced electromotive forces method gives more precise integral equation solution, if the approximating $f_n^{s,a}(s)$ current functions are chosen righter. Hence let us choose the functions, obtained in [4] at solution of the integral equation (1) for the current in the impedance vibrator by the asymptotic method of averaging as $f_0^{s,a}(s)$:

$$\begin{aligned} f_0^s(s) &= \cos\tilde{k}s_\delta \sin\tilde{k}L\cos\tilde{k}s \\ &-(1/2)\cos\tilde{k}L(\sin\tilde{k}\mid s-s_\delta\mid + \sin\tilde{k}\mid s+s_\delta\mid), \\ f_0^a(s) &= \sin\tilde{k}s_\delta \cos\tilde{k}L\sin\tilde{k}s \\ &+(1/2)\sin\tilde{k}L(\sin\tilde{k}\mid s-s_\delta\mid - \sin\tilde{k}\mid s+s_\delta\mid), \end{aligned} \quad (9)$$

where $\tilde{k} = k - \dfrac{i\overline{Z}_S}{r\Omega}$; $\overline{Z}_S = 2\pi r z_i = \overline{R}_S + i\overline{X}_S$ is the normalized on 120π Ohm surface impedance, $\Omega = 2\ln(2L/r)$. We use the expressions, obtained from [6] when investigating the integral equation (1) characteristics for the function $f_1^{s,a}(s)$ in the case, when $z_i = 0$:

$$\begin{aligned} f_1^s(s) &= \cos\tilde{k}s - \cos\tilde{k}L, \\ f_1^a(s) &= \sin\tilde{k}s - (s/L)\sin\tilde{k}L. \end{aligned} \quad (10)$$

Now substituting the expressions for the $f_n^{s,a}(s)$ into the formulas (5), we obtain all coefficients in the equations system (6), defining, in their turn, the current formulas (7).

Knowledge of real distribution of the $J(s)$ current permits us to calculate all electrodynamic characteristics of the impedance vibrator. So, the input impedance $Z_{in} = R_{in} + iX_{in}$ or the input admittance $Y_{in} = G_{in} + iB_{in}$ in the excitation point are defined by means of the following expressions:

$$Z_{in}[\text{Ohms}] = \frac{60i}{J_0^s f_0^s(s_\delta) + J_1^s f_1^s(s_\delta) + J_0^a f_0^a(s_\delta) + J_1^a f_1^a(s_\delta)}, \quad (11)$$
$$Y_{in}[\text{millimhos}] = 10^3 / Z_{in}.$$

Then the VSWR voltage standing wave ratio into the antenna feeder with the W wave impedance equals:

$$\text{VSWR} = \frac{1+\mid S_{11}\mid}{1-\mid S_{11}\mid}, \quad S_{11} = \frac{Z_{in} - \text{W}}{Z_{in} + \text{W}}, \quad (12)$$

where S_{11} is the reflection coefficient in the feeder.

3. NUMERICAL RESULTS

In order to check reliability of the obtained approximate expression for the current (7) the dependencies of real and imaginary parts of the input admittance of the perfectly conducting vibrator, excited in the distinct from its centre point, from its electrical length (Fig. 2) have been calculated.

Here the calculated values, obtained by Galerkin's method at approximation of the current by trigonometric functions of the whole region, are plotted

$$J(s) = \sum_{n=1}^{N} J_n \sin\frac{n\pi(L+s)}{2L}, \quad (13)$$

what is more, to reach necessary accuracy the functions number in formula (13) are chosen to be equal $N = 24$. The comparison of calculation curves in Fig. 2 allows us to make conclusion of adequacy of the current chosen approximating functions (9) and (10) to real physical process.

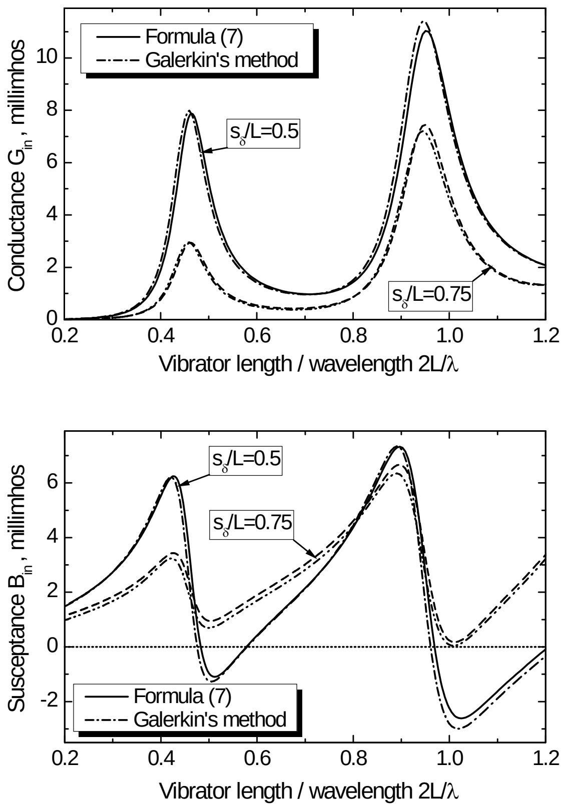

Fig. 2. The input admittance of the perfectly conducting vibrator at $\Omega = 10$.

It is necessary, that the main lobe of the antenna radiation pattern (RP) deviates from the direction $\theta = 90°$ in series of important practical applications. Such a situation takes place, for example, when antennas base stations for cellular mobile communication [7] are exploited. It is proposed to use the vibrator with the length $2L = 0.8\lambda$ and with a shifted relatively to the vibrator's centre excitation point $s_\delta = -0.2\lambda$ in [8]. In Fig. 3 we show radiation patterns for the perfectly conducting vibrator, calculated by the method of the moments from [8] (circles) and due to the formulas (7), (9), (10). Here it is also shown how the vibrator's availability of the active component of surface impedance $\overline{R}_S$ influences RP. As it is seen, when $\bar{R}_S$ increases the side lobe level sufficiently decreases in the vibrator's RP.

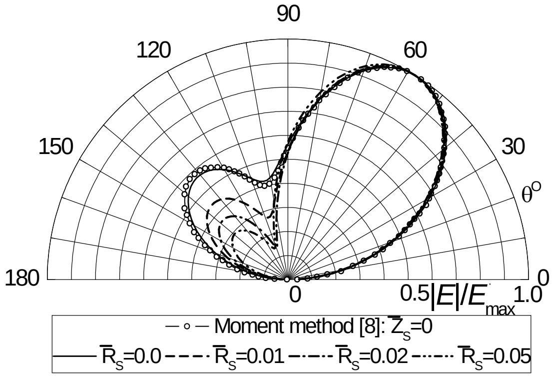

Fig. 3. The vibrator's radiation patterns at f =900 MHz, $2L = 0.8\lambda$, $r = 0.005\lambda$, $s_\delta = -0.2\lambda$.

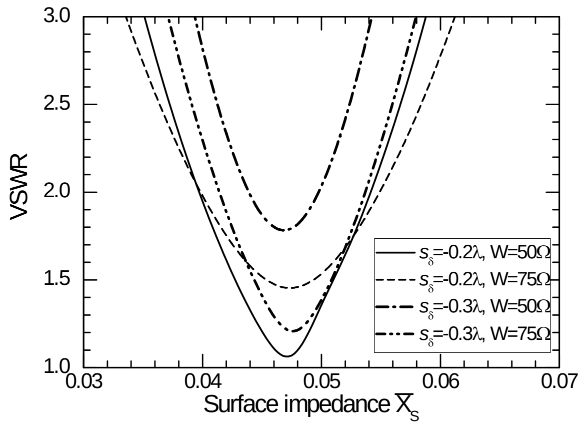

Fig. 4. The dependences of the VSWR from the vibrator's surface impedance at f =900 MHz, $2L = 0.8\lambda$, $r = 0.005\lambda$, $\bar{R}_S$ =0.0.

In [8] it is suggested to use concentrated impedance reactive load, included into the vibrator between the points $s = s_\delta$ and $s = 0$ for the antenna's agreement with the feeding line. At this the minimal values of VSWR at the frequency of f =900 MHz are: VSWR =2.2 for W =50 Ohm, VSWR =1.46 for W =75 Ohm. We proposed to use non-concentrated but distributed along the vibrator surface impedance for the purposes of agreement. As it is seen from the plots in Figs. 4, 5, a definite values of the surface impedance ($\bar{R}_S$ =0.0, $\bar{X}_S$ =0.0475) exists, at which VSWR are minimal for different W and s_δ the antenna's set parameters. The bandwidth on a requirement VSWR ≤2.0 for W =75 Ohm and $s_\delta = -0.2\lambda$ is 27 MHz when using concentrated load X_L =203.8 Ohm, located in the point $s_L = -0.175\lambda$ [8]. Availability of vibrator's distributed impedance sufficiently widens the bandwidth (to 50 MHz for level VSWR ≤2.0 and 28 MHz for level VSWR ≤1.5), what is more, we can attain an acceptable agreement for different values of the W wave impedance of the feeder by changing the excitation point position.

So, availability of the vibrator's distributed impedance of a definite kind (inductive) and the value ($\bar{X}_S$ =0.0475) practically leads to the increase of its "efficient" electrical length (to $2L_{eff} \approx \lambda$) and, as a result, to tuning in the resonance ($B_{in} \approx 0$ in Fig. 2), the field radiation pattern is also changed in comparison with the case $2L = 0.8\lambda$ (Fig. 6). However, the possibility of lowering of the pattern side lobe level by including the $\bar{R}_S$ active impedance exists, as in the case, represented in Fig. 3, though at this the agreement with the feeder for some W values is degraded (Fig. 7).

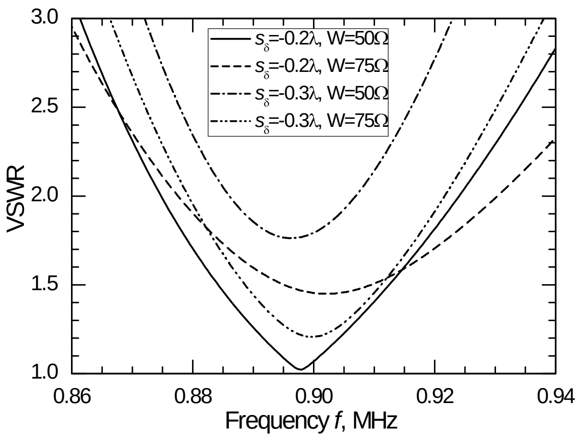

Fig. 5. The dependences of the VSWR from the frequency at $2L = 0.8\lambda$, $r = 0.005\lambda$ and $\bar{X}_S$ =0.0475 on frequency f =900 MHz.

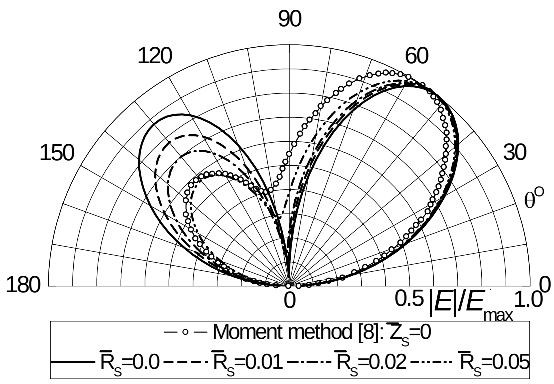

Fig. 6. The vibrator's radiation patterns at f =900 MHz, $2L = 0.8\lambda$, $r = 0.005\lambda$, $\bar{X}_S$ =0.0475, $s_\delta = -0.2\lambda$.

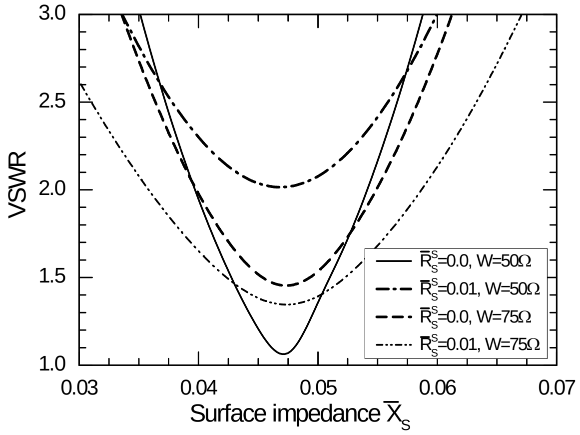

Fig. 7. The dependences of the VSWR from the vibrator's surface impedance at f =900 MHz, $2L = 0.8\lambda$, $r = 0.005\lambda$, $s_\delta = -0.2\lambda$.

4. CONCLUSION

The proposed problem solution of the electromagnetic waves scattering (radiation) by the thin impedance vibrators with the arbitrary excitation and surface impedance, based on the induced electromotive forces method with choice of the adequate approximating functions to represent the current distribution, permits to calculate all electrodynamic characteristics of the vibrator with the help of simple formulas rather precise. The approximating functions for the symmetrical and antisymmetrical components of the current are chosen both from the approximate analytical solution of the integral equation for the current in the impedance vibrator and from physical considerations when investigating the properties of this equation. The given for a concrete example numerical results allow to conclude the efficiency and wide possibilities of vibrators application with the distributed surface impedance in antenna's devices of this type. The suggested approach to the problem solution of the single impedance vibrator can be used to analyze multi-elements vibrator structures, for example, Yagi-Uda's antennas or phased vibrator antenna arrays.

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