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ACCURACY ESTIMATION OF CROSS POLAR RADIATION PREDICTION OF OPEN-ENDED THIN-WALL CIRCULAR WAVEGUIDE BY APPROXIMATE METHODS

Dubrovka F. F. and Piltyay S. I.

National Technical University of Ukraine “Kyiv Polytechnic Institute”,
Department of Theoretical Foundations of Radio Engineering, Kyiv, Ukraine
E-mail: fedor.dubrovka@gmail.com
E-mail: crosspolar@ukr.net

Abstract

Approximate methods and exact factorization method have been used to predict peak crosspolar of radiation levels of open-ended thin-wall circular waveguides. The ranges of minimal discrepancy with factorization method are estimated for each approximate method. Optimal diameters that minimize peak crosspolar levels in front half-space, back half-space and the whole space have been defined.

1. INTRODUCTION

Radiation characteristics of an aperture antenna can be predicted approximately by Fourier Transform method and Kirchhoff-Huygens method [1]. Also, a modification of Fourier Transform method is proposed in [2] for the calculation of circular aperture’s radiation pattern in front half-space. Exact formulae for the calculation of radiation pattern of an open-ended thin-walled circular waveguide are introduced in [3]. These formulae are obtained by factorization method.

Theoretical and practical interest presents finding out how much peak levels of crosspolar radiation differ if they are calculated by approximate methods and exact factorization method. Such results are given in this paper they indicate the ranges of approximate methods’ applicability for crosspolar radiation prediction.

2. PREDICTION FORMULAS

Let’s consider that only fundamental mode H_{11} is propagating in the circular waveguide (Fig. 1). Mathematically cross polar field $E_q(\theta,\varphi)$ can be expressed in such a way [2]:

$$E_q(\theta,\varphi) = 0{,}5\cdot[E_\theta(\theta) - E_\varphi(\theta)]\cdot\sin(2\varphi),$$

where $E_\theta(\theta)$ and $E_\varphi(\theta)$ are far-field radiation patterns in E and H planes accordingly.

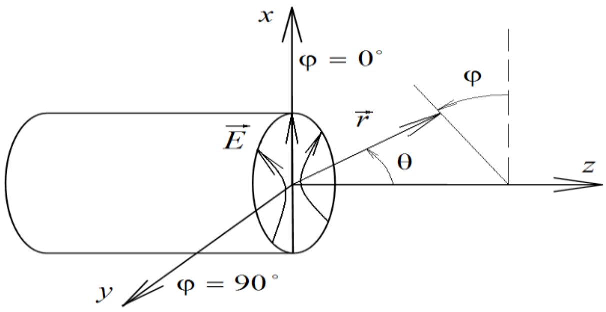

Fig. 1. Open-ended circular waveguide.

Far-field radiation patterns of an open-ended circular waveguide in E and H planes can be expressed by formulae given below.

Fourier Transform method

This method is based on the Fourier transform of electric field at the aperture into far-field. It is meant that the space behind the aperture is shadowed by the infinite flange. Far-field is a product of the aperture’s and elementary radiator’s factors:

$$E_\theta(\theta) = \left| \int_0^{2\pi}\int_0^{a} E(r,\psi)\cdot\exp(ikr\sin(\theta)\cos(\psi))\cdot r dr d\psi \right| \cdot \frac{1+\cos(\theta)}{2};$$

$$E_\varphi(\theta) = \left| \int_0^{2\pi}\int_0^{a} E(r,\psi)\cdot\exp(ikr\sin(\theta)\cos(\pi/2-\psi))\cdot r dr d\psi \right| \cdot \frac{1+\cos(\theta)}{2},$$

where $E(r,\psi) = \sqrt{[E_r(r,\psi)]^2 + [E_\phi(r,\psi)]^2}$ is the amplitude electric field’s distribution at the aperture, a is an inner radius of the circular waveguide, k is a wave number of generator.

Modified Fourier Transform method

$$E_{\theta}(\theta)=\left|\int_0^{2\pi}\int_0^{a}E(r,\psi)\cdot\exp\left(ikr\sin(\theta)\cos(\psi)\right)\cdot rdrd\psi\right|\cdot\cos(\theta);$$

$$E_{\varphi}(\theta)=\left|\int_0^{2\pi}\int_0^{a}E(r,\psi)\cdot\exp\left(ikr\sin(\theta)\cos(\pi/2-\psi)\right)\cdot rdrd\psi\right|\cdot\cos(\theta).$$

Kirchhoff-Huygens method

The basis of this method is the Kirchhoff-Huygens principle. The far-field can be expressed using the fields at the aperture. The formulae obtained by the method are [1, 3]:

$$E_{\theta}(\theta)=\left|\left(1+\frac{h_{H11}}{k}\cos(\theta)\right)\cdot\frac{J_1(ka\sin(\theta))}{\sin(\theta)}\right|;\quad E_{\varphi}(\theta)=\left|J_1'(ka\sin(\theta))\Big/\left[\frac{h_{H11}}{k}-\cos(\theta)\right]\right|,$$

where h_{H11} is longitudinal wave number, $J_1(x)$ is Bessel function of the first kind, $J_1'(x)$ is its derivative.

Factorization method

It is supposed that the waveguide's length goes to infinity. The thickness of circular waveguide's walls is infinitesimal. The factorization method is based on the solution of integral equation obtained from boundary conditions on the structure's surface. The solution is being found as a product of two holomorphic functions. The results are as follows:

$$E_{\theta}(\theta)=\begin{cases}\sqrt{\dfrac{J_1(ka\sin(\theta))}{|H_1(ka\sin(\theta))|}}\cdot\dfrac{\exp(X_E(-ka\cos(\theta))/2)}{|\sin(\theta)|}, & \text{if } \mu_1\le ka\le\nu_1\\[2ex] \sqrt{\dfrac{J_1(ka\sin(\theta))}{|H_1(ka\sin(\theta))|}}\cdot\dfrac{\exp(X_E(-ka\cos(\theta))/2)}{|\sin(\theta)|}\cdot\sqrt{\displaystyle\prod_{i=1}^{N_E}\dfrac{\cos(\theta)-h_{Ei}/k}{\cos(\theta)+h_{Ei}/k}}, & \text{if } \nu_1\le ka\end{cases}$$

where $X_E(w)=\dfrac{-2w}{\pi}\displaystyle\int_0^{ka}\dfrac{\left[\left(\arg(H_1(v))+\dfrac{\pi}{2}\right)\cdot v-\left(\arg(H_1(\sqrt{(ka)^2-w^2}))+\dfrac{\pi}{2}\right)\cdot\sqrt{(ka)^2-w^2}\right]}{\left(v^2-(ka)^2+w^2\right)\cdot\sqrt{(ka)^2-v^2}}dv$, $H_1(x)$ is Hankel function of the first kind, μ_1 is the first root of $J_1'(x)$, ν_1 is the first root of $J_1(x)$, h_{Ei} is longitudinal wave number of E_{1i} mode, N_E is number of E_{1i} propagating modes.

$$E_{\varphi}(\theta)=\begin{cases}\sqrt{\dfrac{J_1'(ka\sin(\theta))}{|H_1'(ka\sin(\theta))|}}\cdot\dfrac{\exp(X_H(-ka\cos(\theta))/2)}{|\sin(\theta)|\sqrt{\cos^2(\theta)-\left(\dfrac{h_{H11}}{k}\right)^2}}\cdot A(\theta), & \text{if } \mu_1\le ka\le\mu_2\\[2ex] \sqrt{\dfrac{J_1'(ka\sin(\theta))}{|H_1'(ka\sin(\theta))|}}\cdot\dfrac{\exp(X_H(-ka\cos(\theta))/2)}{|\sin(\theta)|\sqrt{\cos^2(\theta)-\left(\dfrac{h_{H11}}{k}\right)^2}}\cdot A(\theta)\cdot\sqrt{\displaystyle\prod_{i=2}^{N_H}\dfrac{\cos(\theta)-h_{Hi}/k}{\cos(\theta)+h_{Hi}/k}}, & \text{if } \mu_2\le ka\end{cases}$$

where $X_H(w)=\dfrac{-2w}{\pi}\displaystyle\int_0^{ka}\dfrac{\left[\left(\arg(H_1'(v))-\dfrac{\pi}{2}\right)\cdot v-\left(\arg(H_1'(\sqrt{(ka)^2-w^2}))-\dfrac{\pi}{2}\right)\cdot\sqrt{(ka)^2-w^2}\right]}{\left(v^2-(ka)^2+w^2\right)\cdot\sqrt{(ka)^2-v^2}}dv$, $H_1'(x)$ is Hankel function's derivative, μ_2 is the second root of $J_1'(x)$, $A(\theta)=\left|1+\cos(\theta)+\dfrac{2\Delta^2}{1+\Delta^2}\cdot\dfrac{\dfrac{h_{H11}}{k}-\cos(\theta)}{1-\dfrac{h_{H11}}{k}}\right|$, h_{Hi} is longitudinal wave number of H_{1i} mode, N_H is number of H_{1i} propagating modes.

3. Front half-space

We have selected a range of the normalized waveguide's diameter D/λ = 0,6...1,5. All calculations have been made for the plane of maximal crosspolar levels $\varphi = 45^\circ$. The location of peak crosspolar level depends on the normalized diameter. The dependences of peak crosspolar levels versus the normalized waveguide's diameter in front half-space are shown in Fig. 2.

As one can see in Fig. 2 peak crosspolar levels obtained by Fourier method increase with the growth of the normalized diameter. The similar dependence is predicted by modified Fourier method. Both methods predict that peak crosspolar level is located in front half-space.

Peak levels of cross polarization dependence versus D/λ obtained by Kirchhoff-Huygens method has got an extreme character with a minimum at D/λ = 0,75. In range of the normalized waveguide's diameter D/λ = 0,6...0,75 peak crosspolar level decreases with the growth of diameter. In range D/λ = 0,75...1,5 peak crosspolar level increases with the growth of diameter. An analogous extreme character of peak crosspolar levels gives the factorization method, but with minimal peak level of cross polarization in front half-space at the waveguide's diameter $\underline{D = 0{,}99\lambda}$. So this diameter can be considered as optimal in front half-space. In passing we should note that the results obtained differ from results published in [4].

We can see that Kirchhoff-Huygens method describes the behavior of peak cross polar level correctly (compared with factorization method). The maximal discrepancy of Kirchhoff-Huygens method with factorization method is 17.7 dB. In the range D/λ = 1,0...1,5 the results obtained by Kirchhoff-Huygens and Fourier methods are almost the same. The maximal discrepancy of Fourier method from factorization method is 18.2 dB. In the range D/λ = 0,9...1,5 modified Fourier method gives results that are the closest to the results of factorization method. The maximal discrepancy of modified Fourier method from factorization method is 15.8 dB.

It is interesting fact that the normalized diameters corresponding to minimal crosspolar levels are almost the same as the diameters of higher modes' cutoffs. The diameter $D = 0{,}75\cdot\lambda$ corresponds to minimal crosspolar level which is calculated by Kirchhoff-Huygens method. The diameter $D = 0{,}77\cdot\lambda$ corresponds to mode's E_{01} cutoff. The diameter $D = 0{,}97\cdot\lambda$ corresponds to mode's H_{21} cutoff.

4. Back half-space

The dependence of peak cross polar level versus normalized waveguide's diameter in back half-space is shown in Fig. 3.

In general, as one can see in Fig. 3, peak levels of cross polarization calculated by Kirchhoff-Huygens method decrease with the growth of diameter besides the range D/λ = 1,1...1,2 where peak crosspolar level increases with the growth of diameter. In contrary to Kirchhoff-Huygens method a peak crosspolar dependence calculated by factorization method has got extreme character. The waveguide's diameter $\underline{D = 1{,}22\lambda}$ corresponds to the minimal peak level of cross polarization in back half-space. This diameter corresponds to modes' H_{01} and E_{11} cutoff. The diameter is optimal in back half-space.

In [4] there are no results about crosspolar radiation in back half-space. Therefore there is need to carry out accurate experimental investigation in anechoic chamber. Is general the numerical results demonstrate more intensive crosspolar radiation in back half-space than in front half-space.

Complete space

Joining the numerical results in front and back half-spaces we can obtain the dependence of peak cross polar levels in complete space on the normalized diameter. Numerical results are shown in Fig. 4. For each value of the normalized diameter Fig. 4 shows the maximal peak cross polar level in complete space.

The results of Fourier method and modified Fourier method are the results in front half-space. Another situation is with Kirchhoff-Huygens and factorization methods. There are two ranges. In the first range peak cross polar level is located in back half- space.

For Kirchhoff-Huygens method this range is D/λ = 0,6...1,0. For factorization method this range is D/λ = 0,6...1,15. In the second range peak cross polar level is located in front half-space. For Kirchhoff-Huygens method this range is D/λ = 1,0...1,5. For factorization method this range is D/λ = 1,15...1,5.

The circular waveguide's diameter $\underline{D = 1{,}14\lambda}$ corresponds to the minimal peak level of cross polarization in complete space. This optimal diameter corresponds to the point of movement of peak cross polar level from back to front half-space. This result is in good agreement with [5].

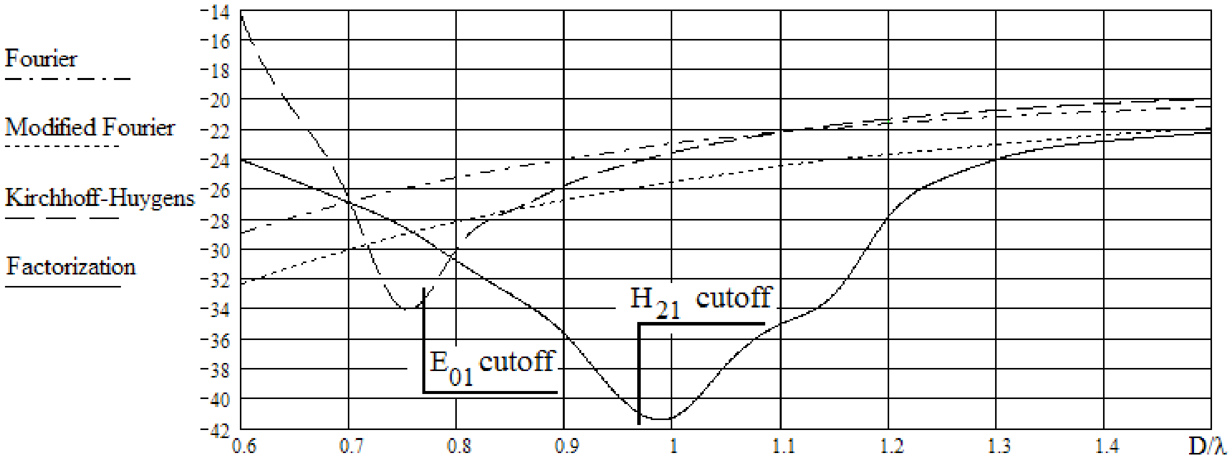

Fig. 2. Peak crosspolar level (dB) in plane φ = 45º in front half-space.

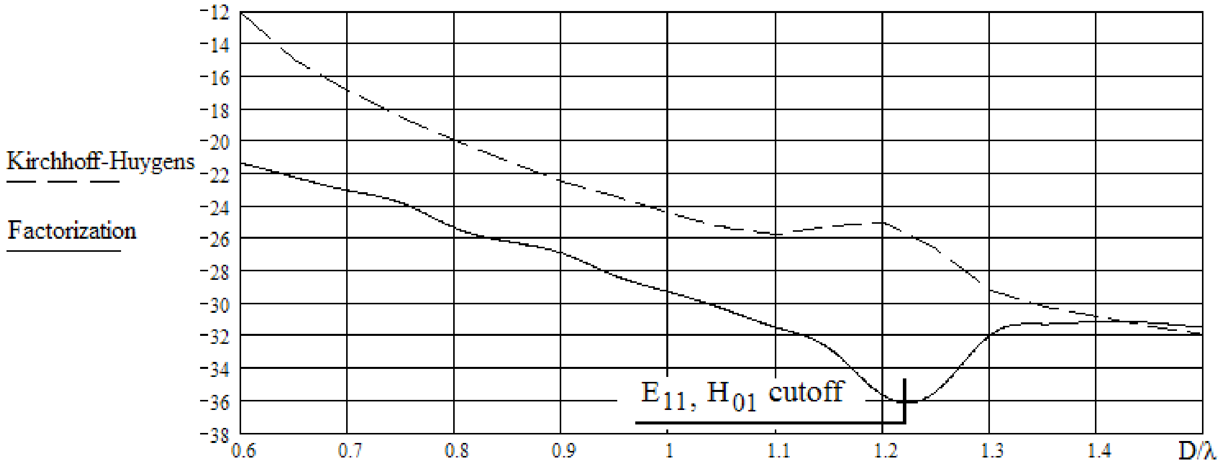

Fig. 3. Peak crosspolar level (dB) in plane φ = 45º in back half-space.

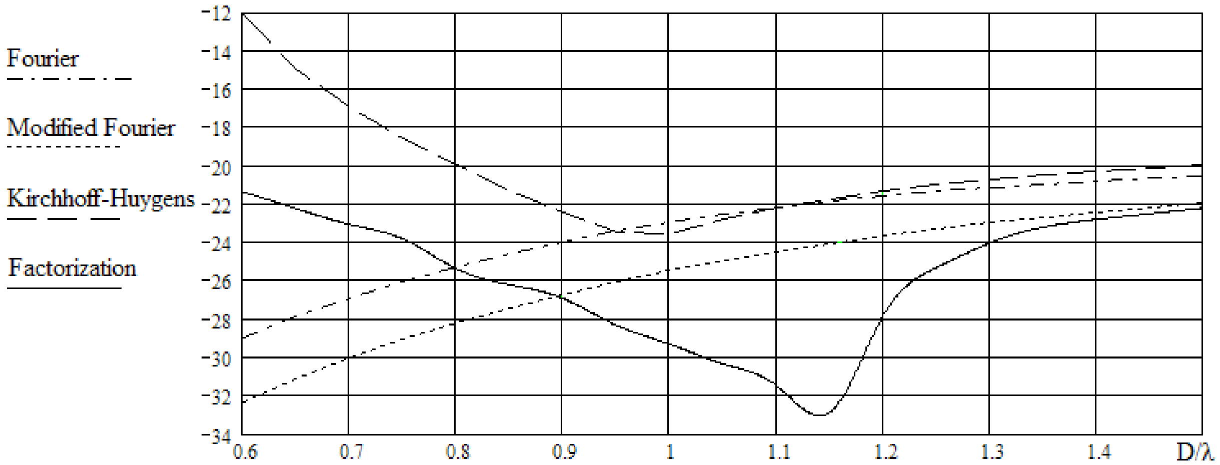

Fig. 4. Peak crosspolar level (dB) in plane φ = 45º in complete space.

TRANSFORMATION OF OPEN-ENDED THIN-WALL CIRCULAR WAVEGUIDE'S RADIATION PATTERN

Let us investigate the alteration of crosspolar radiation pattern with increase of waveguide's diameter. Fig. 5 show circular waveguide's cross polar radiation patterns for D = 0,8λ; D = 1,0λ; D = 1,14λ; D = 1,25λ. These radiation patterns are computed by factorization method. For D = 0,8λ peak crosspolar level in front half-space is greater in back half-space. There are 4 lobes of radiation pattern at cross polarization. For D = 1,0λ peak crosspolar level in front half-space corresponds to the angle θ = 50º. The peak level is greater in back half-space. There are 6 lobes of radiation pattern at cross polarization. There appears a dip of radiation pattern at the angle θ = 81º. For D = 1,14λ peak crosspolar level in front half-space corresponds to the angle θ = 58º. The peak levels are equal in front and back half-space. There are 6 lobes of radiation pattern at cross polarization. The dip of radiation pattern moves to the angle θ = 103º. For D = 1,25λ peak crosspolar level in front half-space corresponds to the angle θ = 65º. The peak level is greater in front half-space. There are 6 lobes of radiation pattern at cross polarization. Peak cross polar level moves from back to front half-space.

5. CONCLUSIONS

The waveguide's diameter D = 0,99λ corresponds to the minimal peak level of cross polarization in front half-space. In back half-space the optimal diameter D = 1,22λ. The circular waveguide's diameter D = 1,14λ corresponds to the minimal peak level of cross polarization in complete space. In the range D/λ = 0,6...1,14 peak cross polar level is located in back half-space. With the increase of the normalized diameter it moves in front half-space.

Kirchhoff-Huygens method and factorization method describe the behavior of peak cross polar level in complete space in a similar. The maximal discrepancy of Kirchhoff-Huygens method is about 11.0 dB. In the range D/λ = 1,0...1,5 results obtained by Kirchhoff-Huygens and Fourier methods are almost the same. The maximal discrepancy of Fourier method from factorization method is about 11.0 dB as well. In the range D/λ = 0,9...1,5 modified Fourier method gives results that are the closest to the results of factorization method in complete space. The maximal discrepancy is less than 9.0 dB.

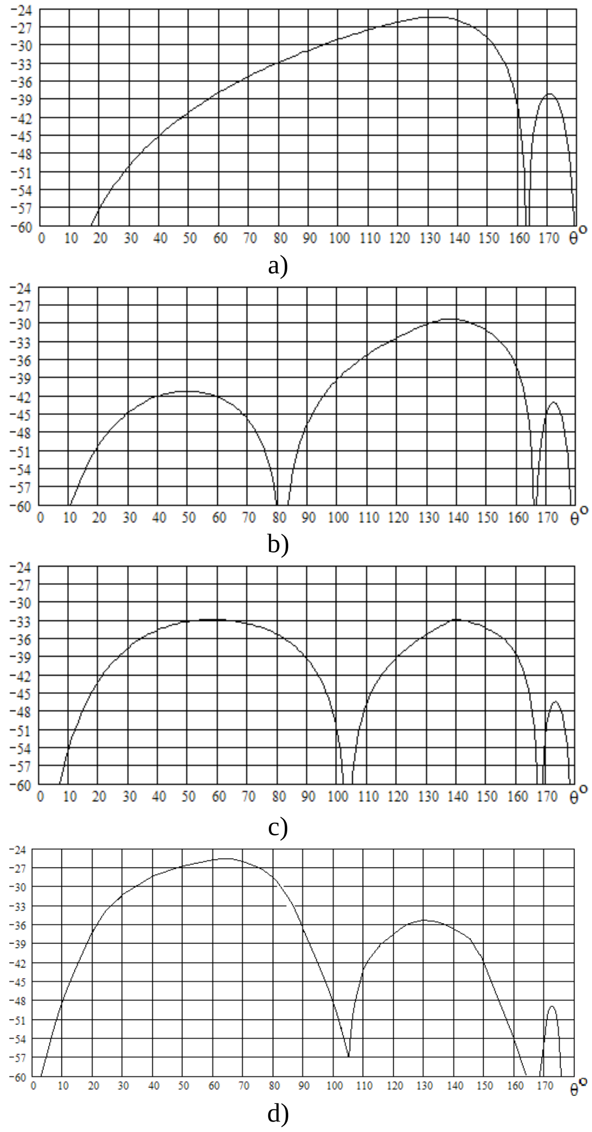

Fig. 5. Crosspolar radiation patterns of circular waveguide in plane φ = 45º: a) D = 0,8λ; b) D = 1,0λ; c) D = 1,14λ; d) D = 1,25λ.

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