KRAVCHENKO-RVACHOV PROBABILITY DISTRIBUTION IN THE PROBLEMS OF ANALYSIS AND SYNTHESIS FOR LINEAR ARRAYS

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Abstract

For the first time an applications of probability densities constructed on the atomic functions (AF) theory for analysis and synthesis for linear arrays are considered. Average characteristics of linear arrays with random phase errors are studied such as average directional diagram (DD), average directive gain (DG), beam width (BW), and side lobe level. Introduced errors correlation coefficient based on AF and the law of errors distribution is Kravchenko-Rvachov distribution. Correlation field characteristics are constructed for the cases when the probability densities of envelopes DD are Kravchenko-Rayleigh and Kravchenko-Gauss distributions. In this case a far-field boundary of antenna with random errors in gain-phase source distributions is defined. Thus a far-field is defined for two characteristics of antennas which are average DD and correlation fluctuations function of complex field An executed numerical experiment for the Gaussian and Kravchenko-Rvachov distribution as well as Kravchenko-Gauss has shown efficiency of introduced approach.

Keywords: Synthesis and analisys, antennas appertura, atomic and R-functions, directional diagram.

1. INTRODUCTION

In paper on the first stage a field statistic of linear system is considered for the case when phase errors of sources conditioned a few simultaneously independent mechanisms. It is known [1] such situation is typical for multielement antenna of huge dimensions with branched feed system and commutation of individual elements. Errors correlation coefficient are studied constructed on atomic function theory [2-5] introduced in [2]. A comparison with the case of exponential and gaussian errors correlation coefficient is made. On the second stage a linear system with atomic distribution laws of sources amplitude and atomic distribution laws of phase dispersion errors is researched. On the third stage a statistical characteristics of linear equispaced array are studied.

2. LINEAR SYNPHASED ANTENNA WITH ATOMIC FUNCTION ERROR CORRELATION COEFFICIENT

Let phase errors in linear system $\varphi(x)$ caused by N simultaneously and statistically independent mechanisms

$$\varphi(x)=\sum_{i=1}^{N}\varphi_i(x).$$

According to [1] consider that $\varphi_i(x)$ are normal random functions with $\overline{\varphi_i(x)}=0$ dispersions $\overline{\varphi_i^2(x)}=\alpha_i$ and correlation coefficients $r_i(x,x_1)=\overline{\varphi_i(x)\varphi_i(x_1)}/\alpha_i$.

From [1] follows that average field value and DD over the field have such view

$$\overline{f(\psi)}=e^{-\alpha/2}\frac{\sin\psi}{\psi},\quad \alpha=\sum_{i=1}^{N}\alpha_i,$$

$$\overline{\left|f(\psi)\right|^2}=e^{-\alpha}\left[\left(\frac{\sin\psi}{\psi}\right)^2+\frac{1}{4}\sum_{m=1}^{\infty}\frac{\alpha^m}{m!}I_m(\psi)\right].$$

Here $I_m(\psi)=\int_{-1}^{1}\int_{-1}^{1}r^m e^{i\psi(x-x_1)}dxdx_1$. In case $m=1$ $I_1(\psi)=\sum_i\frac{\alpha_i}{\alpha}I(c_i,\psi,\psi)$ where

$$I(c_i,\psi,\psi)=\int_{-1}^{1}\int_{-1}^{1}r_i\left(\frac{x-x_1}{c_i}\right)e^{i\psi(x-x_1)}dxdx_1. \qquad (1)$$

Formula (1) is a generalization of the following integral which have main meaning in statistical antennas theory

$$I(c_i,\psi,\psi_1)=\int_{-1}^{1}\int_{-1}^{1}r_i\left(\frac{x-x_1}{c_i}\right)e^{i\psi x-i\psi_1 x_1}dxdx_1. \qquad (2)$$

Integral (2) follows from (1) if $\psi_1=\psi$. Integral values $I(c_i,\psi,\psi)$, $I(c_i,\psi,-\psi)$, $I(c_i,0,\psi)$ for exponential and gaussian correlation coefficients of phase errors are tabulated in [1]. We will be interested in correlation coefficient which have such view [2-5]

$$r_{Ki}(x) = up(x) = \frac{1}{2\pi}\int_{\mathbb{R}}\prod_{j=1}^{\infty}\operatorname{sinc}\left(\frac{t}{2^j}\right)e^{itx}dt. \qquad (3)$$

Substituting (3) in (2) obtain (The index i is lowered)

$$I_K(c,\psi,\psi_1) = \int_{-1}^{1}\int_{-1}^{1} up\left(\frac{x-x_1}{c}\right)e^{i\psi x - i\psi_1 x_1}dxdx_1. \qquad (4)$$

Consider the following partial cases (4):

$$I_{K1}(c,\psi,\psi) = \int_{-1}^{1}\int_{-1}^{1} up\left(\frac{x-x_1}{c}\right)e^{i\psi(x-x_1)}dxdx_1, \qquad (5)$$

$$I_{K2}(c,\psi,-\psi) = \int_{-1}^{1}\int_{-1}^{1} up\left(\frac{x-x_1}{c}\right)e^{i\psi(x+x_1)}dxdx_1, \qquad (6)$$

$$I_{K3}(c,0,\psi) = \int_{-1}^{1}\int_{-1}^{1} up\left(\frac{x-x_1}{c}\right)e^{-i\psi x_1}dxdx_1. \qquad (7)$$

In Tables 1-3 are brought tabulated integral values (5)-(7).

According to [1] define the main physical parameters of antennas with correlation coefficient such view (3). Make a comparison with exponential and Gaussian cases of certain statistical characteristics.

1) Average DD

$$\overline{|f(\psi)|^2} = \left(\frac{\sin\psi}{\psi}\right)^2\left(1-\sum_{i=1}^{N}\alpha_i\right)+\frac{1}{4}\sum_{i=1}^{N}\alpha_i I(c_i,\psi,\psi).$$

2) Average DG decrease

$$\Delta = \sum_{i=1}^{N}\left[1-\frac{1}{4}I(c_i,0,0)\right].$$

3) Extension of the main beam

$$2\Delta\psi = \sum_{i=1}^{N}0,92\cdot\left[I(c_i,\psi_0,\psi_0)-0,5\cdot I(c_i,0,0)\right],$$

where $\psi_0 = 1,38$.

4) Dispersion of amplitude and phase of field

$$\left\{\begin{matrix}\overline{(\Delta R)^2}\\ R_0^2\overline{(\Delta\Phi)^2}\end{matrix}\right\} = \sum_{i=1}^{N}\frac{\overline{(\Delta R)_i^2}}{R_0^2\overline{(\Delta\Phi)_i^2}},$$

where $\left\{\begin{matrix}\overline{(\Delta R)_i^2}\\ R_0^2\overline{(\Delta\Phi)_i^2}\end{matrix}\right\} = \frac{\alpha_i}{8}\left[I(c_i,\psi,\psi)\mp I(c_i,\psi,-\psi)\right].$

5) Dispersion of main maximum direction

$$\overline{\psi^2_{макс}} = \frac{9}{4}\int_{-1}^{1}\int_{-1}^{1}xx_1\overline{\varphi(x)\varphi(x_1)}dxdx_1 =$$

$$= \sum_{i=1}^{N}\frac{9}{4}\alpha_i\int_{-1}^{1}\int_{-1}^{1}r_i(x,x_1)xx_1dxdx_1.$$

Figure 1 shows a comparison of the representations $I_{K1}(1,\psi,\psi)$ and $I_G(1,\psi,\psi)$. On Figure 2 is brought a comparison DG decrease for gaussian $\Delta_G(c,0,0)$ and atomic $\Delta_K(c,0,0)$ distributions from which is following that quantity $\Delta_G(c,0,0)$ decrease quickly than $\Delta_K(c,0,0)$. Characteristics (3)-(5) are analyzed. It is shown that when correlation coefficient is (3) then in comparison with Gaussian correlation coefficient an expansion of main side lobe of average DD occurs.

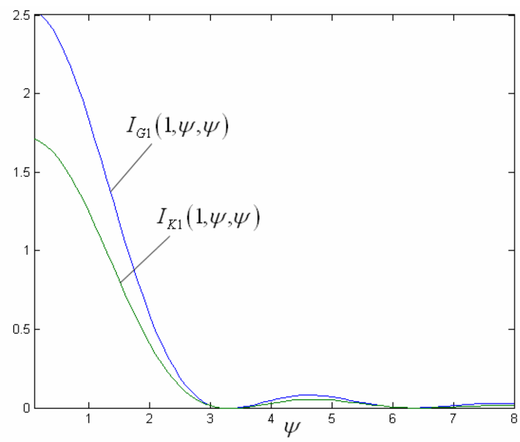

Fig. 1. Comparison of function $I(1,\psi,\psi)$ graphics with Gaussian and atomic forms of errors correlation coefficient.

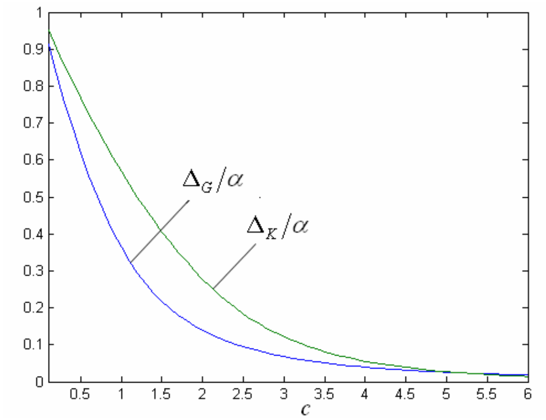

Fig. 2. Comparison of average DG decrease if gaussian and atomic correlation coefficients.

3. LINEAR SYSTEM WITH ATOMIC DISTRIBUTION LAW OF SOURCES AND PHASE ERRORS DISPERSION

Following [1,2] study a statistical parameters of antennas in case when distribution laws of sources amplitudes $A_0(x)$ and errors dispersion $\sigma^2(x)$ have the following view:

$$p_{KR}(x) = up(x) \text{ - Kravchenko-Rvachov,} \qquad (8)$$

$$p_{KG}(x) = \frac{1}{c}up(x)e^{-\frac{x^2}{a}} \text{ - Kravchenko-Gauss,} \qquad (9)$$

$$p_{KG}(x) = \frac{1}{c}up(x)e^{-\frac{|x|}{c}} \text{ -Kravchenko-Poisson} \qquad (10)$$

Constants c are chosen thus $\int_{\mathbb{R}}p(x)dx = 1$.

Consider that errors are small. Radius of correlation errors is much smaller then system dimension. The functions $A_0(x)$ and $\sigma^2(x)$ are symmetric that automatically complete for distributions (8)-(10). In this case will obtain main characteristics.

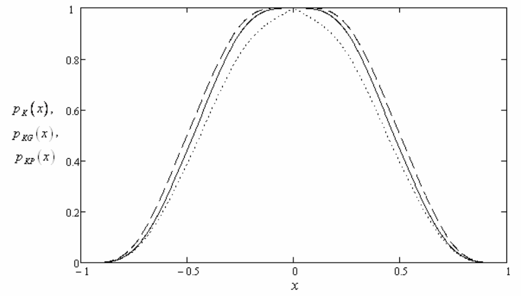

Fig. 3. Kravchenko-Rvachov distribution (solid line), Kravchenko-Gauss (dotted line), and Kravchenko-Poisson (dashed line).

3.1. AVERAGE DD

According to [1] searched out that average DD with random distribution laws of sources amplitudes and phase errors dispersion has such view

$$\overline{|f(\psi)|^2} = \int_{-1}^{1}\int_{-1}^{1} A_0(x)A_1(x_1)e^{-\frac{1}{2}\left[\sigma^2(x)-2r(x_1,x_2)\sigma(x)\sigma(x_1)+\sigma^2(x_1)\right]}\cdot e^{i\psi(x-x_1)}dxdx_1. \quad (11)$$

In consideration [1] of the formula (11) will be written thus

$$\overline{|f(x)|^2} = f_0^2(\psi) - \left[I_1(\psi) - I_2(\psi)\right],$$

where $f_0(\psi) = \int_{-1}^{1} A_0(x)e^{i\psi x}dx$,

$$I_1(\psi) = f_0(\psi)\int_{-1}^{1} A_0(x)\sigma^2(x)e^{i\psi x}dx,$$

$$I_2(\psi) = \sqrt{\pi}c_\rho e^{-\frac{\psi^2 c_\rho}{4}}\int_{-1}^{1} A_0^2(x)\sigma^2(x)dx,$$

where $c_\rho = \dfrac{2\rho}{L}$.

For distribution $p_K(x)$ an expression for $f_0(\psi)$ is possible to write [3] in analytical view [3]

$$f_0(\psi) = \prod_{j=1}^{\infty} \text{sinc}\left(\frac{\psi}{2^j}\right).$$

3.1. AVERAGE DG

Define in according to [1] DG decrease value

$$\Delta = \frac{I_1(0) - I_2(0)}{f_0^2(\psi)} - \frac{\int_{-a_\rho}^{a_\rho}(I_1 - I_2)d\psi}{\int_{-a_\rho}^{a_\rho} f_0^2(\psi)d\psi},$$

where $a_\rho = \dfrac{\pi L}{\lambda}$

3.3. FLUCTUATIONS OF DIRECTION OF MAIN MAXIMUM

From [1] follows that fluctuations of directions of main maximum have such view

$$\delta\psi_{0,5\rho}^2 \cong \frac{\sqrt{\pi}c_\rho e^{-\frac{\psi^2 c_\rho^2}{4}}\int_{-1}^{1} A_0^2(x)\sigma^2(x)\sin^2(\psi_0 x)dx}{\int_{-1}^{1} xA_0(x)\sin(\psi_0 x)dx}.$$

A comparison of statistical characteristics for distributions (8)-(10) with equal gaussian and rayleigh distributions.

4. CONCLUSION

Statistic of linear system field is considered in case when phase errors of sources caused by several simultaneous mechanisms. Besides correlation coefficients of phase errors have a view of atomic Kravchenko-Rvachov distribution (3). A comparison with distributions which are used in statistical antennas theory i.e. gaussian and exponential is made. Computed and tabulated integrals (5)-(7) in Tables 1-3. Numerical experiment and comparison a new approach with well-known results has shown its efficiency.

5. ACKNOWLEDGMENTS

The investigations were supported by grant NSh – 5708.2008.9.

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Table 1. Value of function $I_{K1}(c,\psi,\psi)$ under atomic form of errors correlation coefficient.

c	ψ					
	0	2	4	6	8	10
0,05	0.0974	0.0974	0.0972	0.097	0.0965	0.0961
0,10	0.1970	0.1966	0.1953	0.193	0.1902	0.1864
0,15	0.2936	0.2921	0.2878	0.280	0.2711	0.2591
0,20	0.3887	0.3853	0.3754	0.359	0.3374	0.3109
0,25	0.4825	0.4760	0.4569	0.426	0.3864	0.3391
0,30	0.5748	0.5638	0.5316	0.481	0.4167	0.3435
0,35	0.6658	0.6485	0.5988	0.522	0.4281	0.3259
0,40	0.7554	0.7300	0.6578	0.549	0.4215	0.2902
0,45	0.8436	0.8081	0.7084	0.563	0.3989	0.2415
0,50	0.9304	0.8825	0.7502	0.563	0.3631	0.1853
0,60	1.0999	1.0200	0.8072	0.529	0.2650	0.0729
0,70	1.2638	1.1418	0.8299	0.457	0.1551	-0.011
0,80	1.4221	1.2473	0.8212	0.363	0.0579	-0.052
0,90	1.5749	1.3364	0.7856	0.262	-0.010	-0.053
1,0	1.7221	1.4094	0.7288	0.166	-0.046	-0.034
1,2	2.0000	1.5094	0.5760	0.024	-0.042	0.0047
1,4	2.2556	1.5544	0.4095	-0.03	-0.004	0.0075
1,6	2.4890	1.5546	0.2638	-0.03	0.0148	0.0032
1,8	2.7002	1.5216	0.1562	-0.00	0.0151	0.0100
2,0	2.8891	1.4674	0.0877	0.017	0.0136	0.0187
2,5	3.2648	1.3030	0.0282	0.038	0.0270	0.0197
3,0	3.5147	1.1631	0.0312	0.043	0.0376	0.0158
4,0	3.7785	0.9929	0.0691	0.036	0.0470	0.0144
5,0	3.8924	0.9110	0.1002	0.026	0.0534	0.0127
6,0	3.9444	0.8714	0.1183	0.019	0.0565	0.0121

Table 2. Value of function $I_{K2}(c,\psi,-\psi)$ under atomic form of errors correlation coefficient.

c	ψ				
	0	2	4	6	8
0,05	0.0974	-0.018	0.0122	-0.0050	-0.0009
0,10	0.1970	-0.035	0.0248	-0.0111	-0.0007
0,15	0.2936	-0.052	0.0372	-0.0179	0.0008
0,20	0.3887	-0.067	0.0492	-0.0254	0.0036
0,25	0.4825	-0.081	0.0608	-0.0332	0.0075
0,30	0.5748	-0.094	0.0717	-0.0412	0.0122
0,35	0.6658	-0.106	0.0820	-0.0491	0.0174
0,40	0.7554	-0.117	0.0913	-0.0566	0.0228
0,45	0.8436	-0.126	0.0997	-0.0636	0.0281
0,50	0.9304	-0.134	0.1071	-0.0699	0.0331
0,60	1.0999	-0.146	0.1182	-0.0796	0.0411
0,70	1.2638	-0.153	0.1243	-0.0849	0.0455
0,80	1.4221	-0.154	0.1254	-0.0858	0.0461
0,90	1.5749	-0.150	0.1216	-0.0826	0.0435
1,0	1.7221	-0.141	0.1135	-0.0763	0.0392
1,2	2.0000	-0.107	0.0875	-0.0597	0.0310
1,4	2.2556	-0.056	0.0550	-0.0458	0.0284
1,6	2.4890	0.0077	0.0240	-0.0399	0.0301
1,8	2.7002	0.0819	0.0008	-0.0413	0.0312
2,0	2.8891	0.1611	-0.0114	-0.0452	0.0301
2,5	3.2648	0.3517	-0.0021	-0.0456	0.0291
3,0	3.5147	0.4981	0.0273	-0.0358	0.0350
4,0	3.7785	0.6673	0.0762	-0.0190	0.0452
5,0	3.8924	0.7463	0.1058	-0.0077	0.0515
6,0	3.9444	0.7840	0.1219	-0.0010	0.0554

Table 3. Value of function $I_{K3}(c,0,\psi)$ under atomic form of errors correlation coefficient.

c	ψ					
	0	2	3	4	6	8
0,05	0.0974	0.0449	0.0054	-0.0180	-0.0053	0.0121
0,10	0.1970	0.0920	0.0122	-0.0357	-0.0119	0.0246
0,15	0.2936	0.1386	0.0203	-0.0520	-0.0196	0.0364
0,20	0.3887	0.1856	0.0296	-0.0671	-0.0281	0.0475
0,25	0.4825	0.2329	0.0401	-0.0809	-0.0374	0.0576
0,30	0.5748	0.2803	0.0518	-0.0933	-0.0473	0.0665
0,35	0.6658	0.3279	0.0645	-0.1044	-0.0574	0.0742
0,40	0.7554	0.3754	0.0782	-0.1142	-0.0675	0.0807
0,45	0.8436	0.4229	0.0929	-0.1226	-0.0774	0.0859
0,50	0.9304	0.4702	0.1083	-0.1297	-0.0869	0.0901
0,60	1.0999	0.5641	0.1410	-0.1403	-0.1039	0.0959
0,70	1.2638	0.6564	0.1756	-0.1465	-0.1172	0.1001
0,80	1.4221	0.7465	0.2111	-0.1492	-0.1263	0.1047
0,90	1.5749	0.8340	0.2465	-0.1496	-0.1310	0.1112
1,0	1.7221	0.9181	0.2809	-0.1487	-0.1321	0.1205
1,2	2.0000	1.0749	0.3435	-0.1477	-0.1276	0.1463
1,4	2.2556	1.2141	0.3932	-0.1544	-0.1225	0.1755
1,6	2.4890	1.3340	0.4268	-0.1736	-0.1237	0.2020
1,8	2.7002	1.4342	0.4433	-0.2066	-0.1323	0.2254
2,0	2.8891	1.5154	0.4444	-0.2510	-0.1448	0.2491
2,5	3.2648	1.6498	0.4051	-0.3798	-0.1695	0.3132
3,0	3.5147	1.7215	0.3526	-0.4890	-0.1801	0.3664
4,0	3.7785	1.7838	0.2772	-0.6224	-0.1882	0.4293
5,0	3.8924	1.8048	0.2360	-0.6876	-0.1892	0.4603
6,0	3.9444	1.8126	0.2145	-0.7194	-0.1885	0.4756