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SYNTHESIS OF 2D DIRECTIONAL DIAGRAMS OF ANTENNAS WITH ARBITRARY APPERTURE BASED ON ATOMIC AND R-FUNCTIONS

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Abstract

In this report problems of synthesis 2D directional diagram (DD) of antennas with arbitrary apertures on atomic and R-functions (V.L. Rvachov functions) are considered. A new approach for 2D DD synthesis based on generalized N-th dimensional Kravchenko-Kotel'nikov Theorems is offered. The theory of R-functions allows edge effects are appear in singularity points by synthesis and analysis of 2D antenna arrays with arbitrary forms. In this work was carried out comparison with well-known the following methods: Whittaker-Kotel'nikov-Shannon – series expansion, Fourier – series expansion, Chebyshev – series with series generalized Kravchenko-Kotel'nikov-Levitan.

Keywords: Synthesis and analisys, antennas appertura, atomic and R-functions, directional diagram.

1. INTRODUCTION

In according to [1] the problem of synthesis of 2D antennas consist in finding such antennas aperture dimensions which provide obtaining optimal DD with respect to some criterion. The basic difference from one dimensional case is that with synthesis of 2D antennas aperture form is formed which provide optimal DD. The theory of R-functions using (V.L. Rvachov functions) allows leading such synthesis and obtaining analytical expression for DD of arbitrary antennas aperture. The theory of R-functions [4,5] is used to antenna synthesis from the positions of N-th dimensional generalized WKS [6] and atomic functions theory [2-6].

2. THE SYNTHESIS PROBLEM OF 2D DD ANTENNA

Consider two-dimensional function $f(x) = f(x_1, x_2) \in L_2(\mathbb{R}^2)$ with Fourier transform (FT) $\tilde{f}(\omega) = \tilde{f}(\omega_1, \omega_2) \in L_2(\mathbb{R}^2)$. Will be assume that function $f(x)$ satisfy Plancherel-Polya Theorem [1]. According to this Theorem in order that $\tilde{f}(\omega)$ vanished outside of bounded domain $A \subset \mathbb{R}^2$ is necessary and sufficient that function $f(x)$ is could be continued on all complex variables $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ as entire function finite degree. Moreover least bounded domain B outside of $\tilde{f}(\omega) = 0$ is defined a condition $K_f(\alpha_1, \alpha_2) = h_f(\alpha_1, \alpha_2)$, where $K_f(\alpha_1, \alpha_2)$ is supported function of the domain B and $h_f(\alpha_1, \alpha_2)$ is P indicator of entire function $f(z_1, z_2)$ that is continuation $f(x_1, x_2)$ [1]. In antennas theory $\tilde{f}(\omega)$ is a field distribution in antenna aperture, where

$$\tilde{f}(\omega) = \int_A f(x) e^{-i(x,\omega)} dx. \qquad (1)$$

Here, $f(x)$ is two-dimensional DD of antenna and $(x, \omega) = x_1\omega_1 + x_2\omega_2$. Existence of supported function is sufficient to construct a domain in which $\tilde{f}(\omega) \neq 0$. In limits of visibility domain of DD $f(x)$ is continuous function. Because its FT is not concentrated if finite domain $A \subset \mathbb{R}^2$. The problem of synthesis is consist of definition a domain B and obtaining in visibility domain approximation function $f(x)$ by a function $g(x)$ with satisfactory accuracy.

3. GENERALIZATION OF 2D WHITTAKER-KOTEL'NIKOV-SHANNON THEOREM

Bring an special case of generalized WKS Theorem for 2D case. N-th dimensional case was considered in [6]. Let $A \subset \mathbb{R}^2$ is open domain which contained in rectangle $D = [-\omega_{\partial_1}, \omega_{\partial_1}] \times [-\omega_{\partial_2}, \omega_{\partial_2}]$. The Theorem takes place.

Theorem 1.

Let function $f(x) \in L_2(\mathbb{R}^2)$ have a finite spectrum (supp $\tilde{f}(\omega) = A$). Then in any discretization step

$x_\partial = \left(x_{\partial_1}, x_{\partial_2}\right)$ such as $A \subset D$ an expansion $f\left(x_1, x_2\right) =$

$$= \sum_{k_1,k_2 \in \mathbb{Z}} x_{\partial_1} x_{\partial_2} f\left(k_1 x_{\partial_1}, k_2 x_{\partial_2}\right) g_A\left(x_1 - k_1 x_{\partial_1}, x_2 - k_2 x_{\partial_2}\right), \quad (2)$$

is true, where

$$g_A\left(x_1, x_2\right) = \frac{1}{4\pi^2} \iint_A e^{i(x_1\omega_1 + x_2\omega_2)} d\omega_1 d\omega_2 . \quad (3)$$

Proof.

Expand $\tilde{f}\left(\omega\right)$ into Fourier series in rectangle D ($x_{\partial_i} = \dfrac{2\pi}{\omega_{\partial_i}}$)

$$\tilde{f}\left(\omega_1, \omega_2\right) = \sum_{k_1,k_2 \in \mathbb{Z}} D_{k_1,k_2} e^{-i\left(k_1 x_{\partial_1}\omega_1 + k_2 x_{\partial_2}\omega_2\right)}, \quad (4)$$

where

$D_{k_1,k_2} =$

$$= \frac{1}{4\omega_{\partial_1}\omega_{\partial_2}} \int_{-\omega_{\partial_1}}^{\omega_{\partial_1}} \int_{-\omega_{\partial_2}}^{\omega_{\partial_2}} \tilde{f}\left(\omega_1, \omega_2\right) e^{i\left(k_1 x_{\partial_1}\omega_1 + k_2 x_{\partial_2}\omega_2\right)} d\omega_1 d\omega_2 =$$

$$= x_{\partial_1} x_{\partial_2} f\left(k_1 x_{\partial_1}, k_2 x_{\partial_2}\right). \quad (5)$$

Expansion (5) is obtained subject to

$$f\left(x_1, x_2\right) = \frac{1}{4\pi^2} \iint_A \tilde{f}\left(\omega_1, \omega_2\right) e^{i\left(k_1 x_{\partial_1}\omega_1 + k_2 x_{\partial_2}\omega_2\right)} d\omega_1 d\omega_2 . (6)$$

Substituting (5) in (4) and executing inverse Fourier transform obtaining an interpolation formula (2).

Let consider different corollaries of this Theorem [6].

Corollary 1.

Let $A = D = \left[-\omega_{\partial_1}, \omega_{\partial_1}\right] \times \left[-\omega_{\partial_2}, \omega_{\partial_2}\right]$. Then (2) take a form

$f\left(x_1, x_2\right) =$

$$= \sum_{k_1,k_2 \in \mathbb{Z}} f\left(k_1 x_{\partial_1}, k_2 x_{\partial_2}\right) \prod_{i=1}^{2} \operatorname{sinc}\left(\omega_{\partial_i}\left(x_i - k_i x_{\partial_i}\right)\right) \quad (7)$$

Corollary 2.

Let $D = \left[-u_{\partial_1}, u_{\partial_1}\right] \times \left[-u_{\partial_2}, u_{\partial_2}\right]$ is a square but domain A is a circle with radius $w_0 = u_{\partial_1} = u_{\partial_2}$. In that case (2) will be write thus

$$f\left(x_1, x_2\right) == \sum_{k_1,k_2 \in \mathbb{Z}} f\left(k_1 \frac{\pi}{w_0}, k_2 \frac{\pi}{w_0}\right) \frac{y_1\left(\sqrt{a}\right)}{a}, \quad (8)$$

$$a = \left(w_0 x_1 - k_1 \pi\right)^2 + \left(w_0 x_2 - k_2 \pi\right)^2 ,$$

where $y_1\left(x\right)$ is first-order Bessel function of the first kind.

4. GENERALIZATION WKS THEOREM BASED ON ATOMIC FUNCTIONS

Consider the corollaries of Theorem 1 using different AF expanded in [2-6]. Will be suppose that interpolation kernel $g_A\left(x_1, x_2\right)$ in (2) could be introduced in multiplication form $g_A\left(x_1, x_2\right) = g_A\left(x_1\right) g_A\left(x_2\right)$. Write down formula (2) for the case when $A = D = \left[-\omega_{\partial_1}, \omega_{\partial_1}\right] \times \left[-\omega_{\partial_2}, \omega_{\partial_2}\right]$, and signal is undergone additional filtration according to [6]

$$f\left(x_1, x_2\right) = \sum_{k_1,k_2 \in \mathbb{Z}} f\left(k_1 x_{\partial_1}, k_2 x_{\partial_2}\right) g_A\left(x_1 - k_1 x_{\partial_1}\right) g_A\left(x_2 - k_2 x_{\partial_2}\right).$$

Consider a row of particular cases which are differs by interpolation kernels form

$g_A\left(x_i\right), i = 1, 2$.

- Generalized Kravchenko-Kotel’nikov series based on AF $h_a\left(\omega\right)$

$$g_A\left(x_i\right) = \prod_{j=1}^{\infty} \operatorname{sinc}\left(\frac{x_i}{a^{j-1}}\right). \quad (9)$$

- Generalized Kravchenko-Kotel’nikov series based on AF fup_n

$$g_A\left(x_i\right) = \operatorname{sinc}^n\left(\frac{x_i}{2^{j-1}}\right) \prod_{j=1}^{\infty} \operatorname{sinc}\left(\frac{x_i}{2^{j-1}}\right). \quad (10)$$

- Generalized Kravchenko-Kotel’nikov series based on Levitan polynomials

$$g_A\left(x_i\right) = \prod_{j=1}^{\infty} \operatorname{sinc}^n\left(\frac{x_i}{2^{j-1}}\right). \quad (11)$$

In Table 1 introduced an estimate of accuracy order for serieses (7), (9)-(11) with difference terms of series amount.

Table 1. Comparison of accuracy order for difference sampling Theorems

Term of series amount	10^3	10^4	10^5
WKS	$3.183 \cdot 10^{-4}$	$3.183 \cdot 10^{-5}$	$3.183 \cdot 10^{-6}$
On AF $h_a\left(\omega\right)$	$1.824 \cdot 10^{-6}$	$1.824 \cdot 10^{-8}$	$1.824 \cdot 10^{-10}$
On AF $fup_n\left(\omega\right)$	$9.119 \cdot 10^{-7}$	$9.119 \cdot 10^{-9}$	$9.119 \cdot 10^{-11}$
On Levitan polynomials	$5.113 \cdot 10^{-8}$	$5.113 \cdot 10^{-10}$	$5.113 \cdot 10^{-12}$

5. THE THEORY OF R-FUNCTIONS IN PROBLEMS OF TWO DIMENSIONAL ANTENNAS

The theory of R-functions allows to construct an amplitude distribution of arbitrary antennas aperture [4,5]. After constructing function $\omega\left(x_1, x_2\right)$ which takes into consider a supporting area geometry a function $w\left(x_1, x_2\right)$

is constructed the following view:

$$w\left(x_1, x_2\right) = \begin{cases} 1, \ \omega\left(x_1, x_2\right) > 0, \\ 0, \ \omega\left(x_1, x_2\right) \le 0. \end{cases} \quad (12)$$

The function (12) is substitute in integral (3). Then a representation (2) is built on constructed 2D interpolation kernel $g_A\left(x_1, x_2\right)$ which is the same (7)-

(11). It is consider a concrete antennas aperture geometry form.

6. CONCLUSION

In report the applications of generalized WKS and Kravchenko-Kotel'nikov Theorems to the problems of antennas synthesis are considered. The theory of R-functions application mold which are considered above is allow to consider sampling Theorem for antenna DD with complex antenna aperture.

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