A NEW APPROACH TO ANALYSIS OF CURVILINEAR CONDUCTING SURFACE RADIATORS

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Abstract

We present a new approach to analysis of curvilinear perfect conducting surface radiators. The feature of this approach is a formulation of boundary value problem as a system of integral equations in regard to the complex electrodynamic vector potential and the scalar potential and simultaneous solution of this system for unknown distributions of current density vector and charge density using technique of parametrical mapping for representing curvilinear surface and the Galerkin's method with boundary elements. Advantages of the approach in comparison with the Harrington integral equation on example of surface current distributions at the third order surface (Ferguson's-patches) are demonstrated.

Keywords: Boundary value problem, integral equation, numerical solution, Galerkin's method, parametric surface, curvilinear conducting surface radiator.

1. INTRODUCTION

It is well known, predicting of characteristics of a radiator is easy to carry out, if an amplitude-phase distribution (APD) of current on its surface is known. To determine APD of a current on a radiator surface an appropriate boundary value problem must be solved. Obviously a mathematical formulation of such a problem using electrodynamic potentials is the best way for analysis of a surface radiator of the arbitrary shape. In this case the problem is reduced to obtaining an integral equation.

Application of integral equations for analysis of radiators has already got more than centenary history [1]. It is known that for a thin conductor what the surface is, it is better to apply an electrical field integral equation (EFIE). Pocklington and Harrington EFIE are for a long time known and widely applied in practice of linear antennas [1]. We do not consider Hallen integral equation, since it has no electrical field in a right part in an obvious form [1] that hampers the description of an excitation source.

However, numerical solutions of the integral equations in the case of surface current distribution is rather difficult. As such Pocklington EFIE contains terms in it's kernel, inversely proportional to a quintic degree of distance between a source locus and view point of a field. Therefore, to provide acceptable accuracy of numerical integration an excessively large costs of CPU time is required. As far as Harrington EFIE concerned its kernel contains divergence of a current density vector that in the case of a curvilinear surface gains a rather cumbersome and complicated form. Nevertheless it is necessary to define it analytically since the numerical derivation inside numerical integration can result in divergency process and consequently requires special investigations of stability in each particular case.

The disadvantages of Pocklington and Harrington EFIE arise due to reduction of quantity of unknowns in the integral equation by excluding APD of charge density from the potential part of the equations. It is reached by application either Lorentz calibration, or the law of current continuity. In outcome, the only unknown quantity in the equation is current. Such an approach made sense earlier, when computer powers were insufficient to store in memory a large scale arrays of data and to inverse matrixes of large dimensions. At present such limitations have already not take a place. Nevertheless at the solution of the integral equation by a variational-steady Galerkin's method for the surface case it is necessary to evaluate quadruple integrals and the cost of CPU time of numerical integration with a satisfactory degree of accuracy still remains rather essential. Therefore at a formulation of mathematical description of the boundary value problem for a conducting surface radiator we shall displace accent on obtaining more simple integrand expressions for calculations.

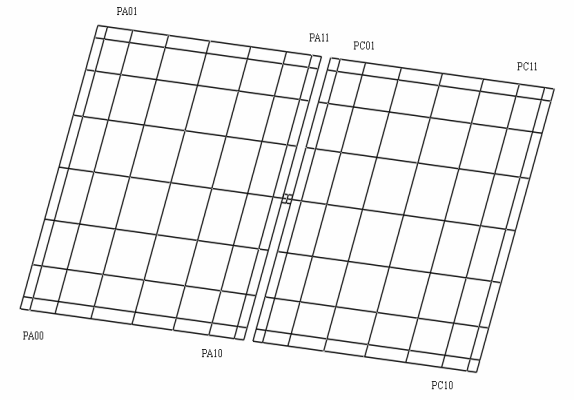
We propose a new approach to analysis of curvilinear ideally conductive surface radiators. The feature of this approach consists in a formulation of the boundary value problem as a set of equations in regard to the complex electrodynamic vector and scalar potentials, simultaneous solution of the set of equations obtained for unknown complex current density vector and charge density, application of a technique of parametric

mapping for representation of computational domain and Galerkin's method with boundary elements.

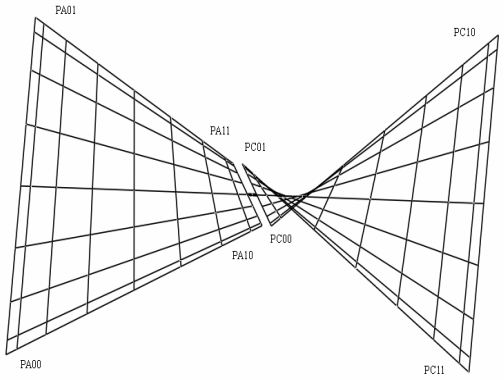
2. MATHEMATICAL DESCRIPTION OF THE PROBLEM

For the mathematical description and numerical solution of the boundary value problem for curvilinear conductive surface radiator we shall construct the generalized model of the antenna on the basis of two Ferguson's patches [2], connected by a narrow strip at the middle (Fig. 1). Such a structure allows, on the one hand, to describe flexibly a wide class of radiators, and, on the other hand, provides a capability to evaluate features of application of different versions of mathematical description of the boundary value problem for numerical researches, as far as Ferguson's patch is a surface of the third order.

We offer not to solve Pocklington or Harrington EFIE, but to form the following sets of equations, using the charge conservation law in the integral and differential form [3]:



a)



b)

Fig. 1. Model of an antenna with Ferguson's patches.

a) model1 – planar case

b) model2 – curvilinear case

$$\begin{cases} -j\omega\dot{\vec{A}} - \operatorname{grad}\dot{\Phi} = \dot{\vec{E}} \\ \operatorname{div}\dot{\vec{A}} + j\omega\varepsilon_0\mu_0\dot{\Phi} = 0 \end{cases}, \tag{1}$$

$$\begin{cases} -j\omega\dot{\vec{A}} - \operatorname{grad}\dot{\Phi} = \dot{\vec{E}} \\ \operatorname{div}\dot{\vec{J}} + j\omega\dot{\rho} = 0 \end{cases} \tag{2}$$

and to solve one of them. In (1) and (2):

$\dot{\vec{E}}$ is a complex vector of electric field intensity;

j is the imaginary unit;

ω is a circle frequency;

μ_0 is the free space magnetic constant $4\pi\cdot 10^{-7}$ (H/m);

ε_0 is the free space electrical constant $(36\pi)^{-1}\cdot 10^{-9}$ (F/m);

$\dot{\vec{A}}$ is the following complex electrodynamical vector potential

$$\dot{\vec{A}} = \frac{\mu_0}{4\pi}\int_{V'}\dot{\vec{J}}\frac{\exp(-jkr)}{r}dV', \tag{3}$$

$\dot{\Phi}$ is the following complex electrodynamical scalar potential

$$\dot{\Phi} = \frac{1}{4\pi\varepsilon_0}\int_{V'}\dot{\rho}\frac{\exp(-jkr)}{r}dV'. \tag{4}$$

In (3) and (4):

V' is a volume in free space, where there are sources of a field;

$\dot{\vec{J}}$ is a complex vector of a current density;

$\dot{\rho}$ is a complex charge density;

$k = \omega\sqrt{\varepsilon_0\mu_0}$ is the free space wave number;

r is a distance between a source locus and view point of the field.

It enables one to apply a Galerkin's method to integrals with much more simple integrands than in Pocklington and Harrington EFIE.

Analysis of numerical solutions of the boundary value problems using (2) has shown this set of equations can be applied to a segment of a flat straight-line nonparametric and distortionless surface only. The attempts to apply (2) for other surface types were not successful. Besides, despite of more simple form, the system (2) has no advantages in CPU time as compared to system (1), since the calculation of Lorentz calibration (second equation in (1)) is carried out from 200 up to 800 times faster than calculation of integrals of the first equation in (1) due to application of the points matching method [1]. Therefore for further investigations we shall use the system of integral equations (1).

Besides we shall compare CPU time of calculations and distributions of surface current density obtained using system (1) with Harrington EFIE (in spite of that the solutions Pocklington and Harrington EFIE by Galerkin's method are completely equivalent), as far as computing time for a diagonal element of the matrix of the generalized impedances for Pocklington integral equation is on the average from 7 up to 20 times more,

than the computing time for the similar element in Harrington equation at the same value.

To transform the equation (1) from the case of volumetric distribution of currents and charges to a surface case, we shall enter the parametric description of a surface. Let's consider in free space a simple connected, simple, smooth and non-closed surface (S) of a perfect indefinitely thin conductor. This surface represents a regular mapping in the metric three-dimensional space of a parametric rectangular flat segment (T) [4]:

$$xm = xm(xt, yt),\ ym = ym(xt, yt),\ zm = zm(xt, yt), \quad (5)$$

where xt, yt are parametric coordinates of a parametric plane (T);

xm, ym, zm are metric coordinates of a metric surface (S).

By derivation of electrodynamical potentials on coordinates of a view point of a field, after cumbersome, but not complicated transformations, we obtain the following new record of Harrington integral equation and the set of equations (1) in a parametric and compact vector form:

$$\begin{aligned}\dot{\vec{E}} = &\frac{-j\omega\mu_0}{4\pi}\iint_{T'}\dot{\vec{J}}\frac{\exp(-jkr)}{r}Ydxt'dyt' \\ &-\frac{1}{j\omega 4\pi\varepsilon_0}\iint_{T'}\operatorname{div}\dot{\vec{J}}\frac{\exp(-jkr)}{r^2}(1+jkr)\vec{r}_0\,Ydxt'dyt'\end{aligned} \quad ,(6)$$

$$\begin{cases}\dfrac{-j\omega\mu_0}{4\pi}\iint_{T'}\dot{\vec{J}}\dfrac{\exp(-jkr)}{r}Ydxt'dyt' \\ +\dfrac{1}{4\pi\varepsilon_0}\iint_{T'}\dot{\rho}\dfrac{\exp(-jkr)}{r^2}(1+jkr)\vec{r}_0\,Ydxt'dyt' = \dot{\vec{E}} \\ -\iint_{T'}\dot{\vec{J}}\dfrac{\exp(-jkr)}{r^2}(1+jkr)\vec{r}_0\,Ydxt'dyt' \\ +j\omega\iint_{T'}\dot{\rho}\dfrac{\exp(-jkr)}{r}Ydxt'dyt' = 0\end{cases} \quad ,(7)$$

where

$$r = \sqrt{(x-xm)^2+(y-ym)^2+(z-zm)^2}\ ; \quad (8)$$

$$\vec{r}_0 = \frac{x-xm}{r}\vec{i} + \frac{y-ym}{r}\vec{j} + \frac{z-zm}{r}\vec{k}\ ; \quad (9)$$

Y is the Jacobian of transition from parametric to metric coordinates [5];

$\vec{r}_0$ is a unit vector of direction from a field source locus with parametric coordinates (xt', yt') to a view point of a free-space field with coordinates (x, y, z).

For further calculations we describe representation of distribution of a tangential to the surface vector quantity $\vec{a}$ in terms of components (a_{xt}, a_{yt}) of local basis $(\vec{\tau}_{xt}, \vec{\tau}_{yt})$ (Fig. 2). We set an origin point Q of the vector through parametric coordinates (xt, yt) creating Gaussian coordinate system on the surface (S). In each point Q the local coordinate system τ_xt, τ_yt is defined, which is characterized by the unit vectors $\vec{\tau}_{xt}, \vec{\tau}_{yt}, \vec{n}$. They create local basis that enables one to describe distribution of the vector quantity tangential to the surface. The unit vectors of local basis are described using of unit vectors $(\vec{i}, \vec{j}, \vec{k})$ of the rectangular cartesian coordinate system (x, y, z). Thus, by analogy with the vector $\vec{a}$ the surface vector $\dot{\vec{J}}$ or $\dot{\vec{E}}_\tau$ can be described.

The expressions (5) give all necessary information to formulate the boundary value problem on the surface (S) for vector of electric field intensity on an arbitrary ideally conductive surface presented in the parametric manner. The surface current, which is characterized by the density vector $\dot{\vec{J}}$, is arranged on a surface in such a way to create of a component of the electrical field $\dot{\vec{E}}_\tau$ tangential to a surface that completely compensates tangential component of an extraneous source field $\dot{\vec{E}}^e_\tau$:

$$\dot{\vec{E}}_\tau + \dot{\vec{E}}^e_\tau = 0\ . \quad (10)$$

If $\dot{\vec{E}}^e_\tau$ is equal to zero on all surface, except a segment, that can be named as a segment of excitation, we obtain a classical way of introduction of an extraneous source into the electromagnetic system [6]. As the surface is in free space and its thickness is approached to zero, we shall set and search the electrical field only on one side of the surface.

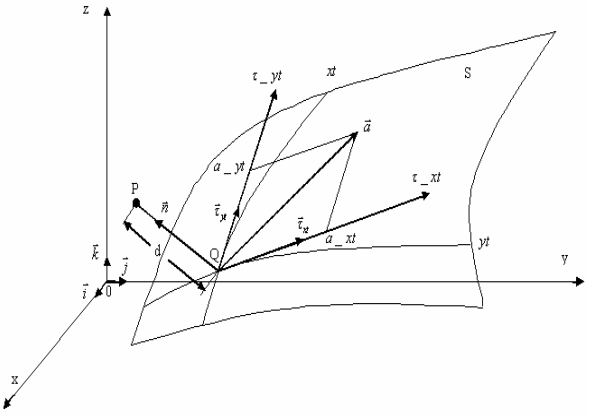


Fig. 2. Local vector basis for source locus and field observation point.

However we should remark, that for a tangential component of the electric field intensity in a view point (P) with parametric surface coordinates (xt, yt) we shall search not directly on the surface (S), but at some small distance d from it (fig. 2), what is normal to the surface in this point [7]. It also will help to avoid a singularity, when the view point of the field coincides with the source locus.

Now, according to the boundary condition (10) for the electric field intensity in regard to the surface components, we shall proceed from vector equations (6) and (7) to scalar equations circumscribing distribution of tangential components of the electrical field on the surface (S). We obtain the unknown quantities using known unit vectors of local basis:

$$\dot{E}\tau_{xt} = \dot{\bar{E}} \cdot \vec{\tau}_{xt} \tag{11}$$

$$\dot{E}\tau_{yt} = \dot{\bar{E}} \cdot \vec{\tau}_{yt} . \tag{12}$$

We can represent distribution of surface current density using unit vectors of local basis record as follows:

$$\dot{\bar{J}}(xt, yt) = \dot{J}xt(xt, yt) \cdot \vec{\tau}_{xt} + \dot{J}yt(xt, yt) \cdot \vec{\tau}_{yt} , \tag{13}$$

where $\dot{J}xt(xt, yt)$ and $\dot{J}yt(xt, yt)$ are complex amplitudes of components of the surface current density vector in directions $\vec{\tau}_{xt}$ and $\vec{\tau}_{yt}$ accordingly.

As it is known, an application of Galerkin's method with boundary elements for solution of integral equations consists in transforming an integral equation to a system of linear algebraic equations with subsequent solution [1]. For this purpose it is necessary to select a proper system of basis functions, which successfully describe a quantity to be approximated and are convenient for application in computational domain of a particular configuration.

Let's take rectangular piecewise linear basis functions, which are ideally fitting for approximating a surface current [8], and are convenient for application in quadrangular computational domain and successfully applied to practical calculations [9]. For the problem under consideration it is an optimum selection, because such functions allow to meet naturally boundary conditions for the surface current, namely: a component, normal to an edge of the surface is equal zero, has continuous derivative at the element, and provide a satisfactory smoothness of the current approximation. Incidentally we should remark, that the approximations of larger orders increase essentially computing load, as the number of quadruple integrals in a surface case grows proportionally $(m+1)^2$, where m is the order of approximation. The accuracy of the solution can be adjusted by the number of boundary elements in the computational area. Approximation of a surface charge is naturally performed by step functions that correspond to a derivatives of basis current functions. Also it is necessary to note that, owing to a curvilinear surface (in general case), the natural parametrization [4] should be used for proper approximation of current distribution on the surface. Plots of these basis functions are schematically shown in Fig. 3 for curvilinear surface segment.

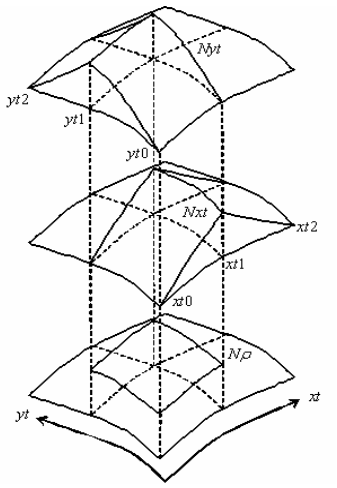


Fig. 3. Two-dimensional basis functions for approximation of current and charge distribution.

Now, having obtained character of discretization of computational area on boundary elements and kind of approximation of unknown current and charge distributions, we can write down equations (6) and (7), after application of Galerkin's process to them, in a matrix form as follows:

$$\begin{bmatrix} \mathbf{Zxx} & \mathbf{Zxy} \\ \mathbf{Zyx} & \mathbf{Zyy} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{Jx} \\ \mathbf{Jy} \end{bmatrix} = \begin{bmatrix} \mathbf{Ex} \\ \mathbf{Ey} \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} \mathbf{Mxx} & \mathbf{Mxy} & \mathbf{Rx} \\ \mathbf{Myx} & \mathbf{Myy} & \mathbf{Ry} \\ \mathbf{Lx} & \mathbf{Ly} & \mathbf{Fr} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{Jx} \\ \mathbf{Jy} \\ \boldsymbol{\rho} \end{bmatrix} = \begin{bmatrix} \mathbf{Ex} \\ \mathbf{Ey} \\ \mathbf{0} \end{bmatrix}, \tag{15}$$

where vectors $\mathbf{Jx}$, $\mathbf{Jy}$, $\boldsymbol{\rho}$ are unknown complex amplitudes of current and charge distributions, to be determinated in result of solving the system of matrix equations by any appropriate way. The elements of submatrixes, which characterize properties of the electrodynamical object under research are evaluated through quadruple and double integrals. The last are not put here for their cumbersome.

It should be noted the problem of simulation of extraneous excitation source is rather difficult and represents a particular area of investigation. The way we use is a direct field representation of the source in computational domain. It gives necessary flexibility in simulation of extraneous excitation and capability for such investigation. However we shall take advantage of the fact that for the analysis problem of the radiator an excitation region has rather small electrical and geometrical sizes as compared to the radiator. Therefore we consider, that the simple enough representation of an extraneous field along a narrow strip (Fig. 1) will not introduce essential errors to calculations of the current and charge distributions at all remaining surface of the radiator. The mathematical formulation of the thesis, taking into account (10), looks like this:

$$\dot{E}xt^{e}_{nx} = \begin{cases} -1, \text{ in the middle of the narrow strip} \\ 0, \text{ otherwere} \end{cases} \tag{16}$$

$$\dot{E}yt^{e}_{ny} = 0 . \tag{17}$$

Thus, we have completely described the boundary value problem for electrical field at any curvilinear ideally conductive surface presented in the parametric form. Now we can start numerical research in order to estimate efficiency of the proposed EFIE system in comparison with Harrington EFIE.

3. NUMERICAL SOLUTION

Numerical researches under obtained formulas have been performed using the personal IBM-compatible

computer with the processor AMD Phenom X4 9550 and volume of the RAM equal 8 Gb, under the control of an operating system FreeBSD amd64 7.2-RELEASE. Using the compiler gcc-4.2.1 and the programming language C ++ four software packages crystal_poc, crystal_har, crystal_tol1, crystal_tol2, realizing accordingly algorithmizations of Pocklington and Harrington EFIE, set of equations (1) and (2) for the solution of the boundary value problem by Galerkin's method of adopted model of the surface (Fig. 1) have bean written.

For realization of calculations we have taken two versions of the radiator model consisting of two Ferguson's patches joint at the middle by a narrow strip. The first version of the radiator (model №1) represents two rectangular flat sheets, (fig. 1, a). The second version (model №2) consisted of two curved sheets like a propeller, (fig. 1, b). In the first case, we can receive classical current distributions on the flat radiator and possibility to be convinced in correctness of the solution, and in the second case - new results of investigations of such complicated structure as "propeller".

As one of the purposes of the calculations is a comparison methods realized in software packages crystal_har and crystal_tol1, parameters will be identical to all versions:

- field viewing distance from a surface d = 0.002 m;
- wavelength λ = 4 m;
- the number of discretization points of patch of the axis τ_xt Nx1 = 11;
- the number of discretization points of patch of the axis τ_xt Ny1 = 13.
- relative error of definite integrals calculation ERR = 0.00005.

Plots in Fig. 4 – 8 illustrate modulus's of the calculated current and charge distributions. Dimensions of abscissa and ordinate axes are in meters, dimension of a current density vector $\left[A \cdot m^{-1}\right]$, dimension of charge density $\left[C \cdot m^{-2}\right]$.

One can observe the calculated current distributions using Harrington integral equation (6) and the set of equations (7) almost completely coincide. However it should be to noted that current distributions obtained from the solution of the set of equations (7) mach more smoother and therefore more exact than current distribution obtained from the solution of Harrington EFIE. Furthermore calculation of current distribution under the program crystal_tol1 is 2.7 times faster for model №1 and 2.8 times faster for model №2, than calculation under the program crystal_har. In spite of all this a rather good conformity in power balance and input impedance obtained for model №1 and №2 calculated by both programs take place.

Radiation characteristics of the structures are almost the same in general to symmetrical vibrator antenna. However if we transform the model № 2 in a symmetrical manner to obtain mirror symmetry plane (model №3) we obtain interesting characteristics of axial ratio that illustrated in polar diagram in Fig. 9. One can see the mode №2 have almost uniform elliptical polarization but the model №2 have gaps in symmetrical plane.

For planar case of radiator structure (model №1) axial ratio approached to zero.

4. CONCLUSION

We propose a new approach to numerical solution of a boundary value problem for curvilinear ideally conductive surface. The feature of the approach is an alternative to classical Pocklington and Harrington integral equations, namely a set of equations for boundary value problem solution through electrodynamic potentials.

For the numerical solution of the boundary value problem the parametric mapping technique for representation of a computational domain, Galerkin's method with boundary elements, points matching method for Lorentz calibration, and also piecewise linear basis functions for current approximation and piecewise constant basis functions for charge approximation are used.

As an example of Ferguson-patch's radiators we have shown that the offered alternative mathematical formulation of the boundary value problem allows to predict current distributions much faster and more precisely than the classical Harrington integral equation.

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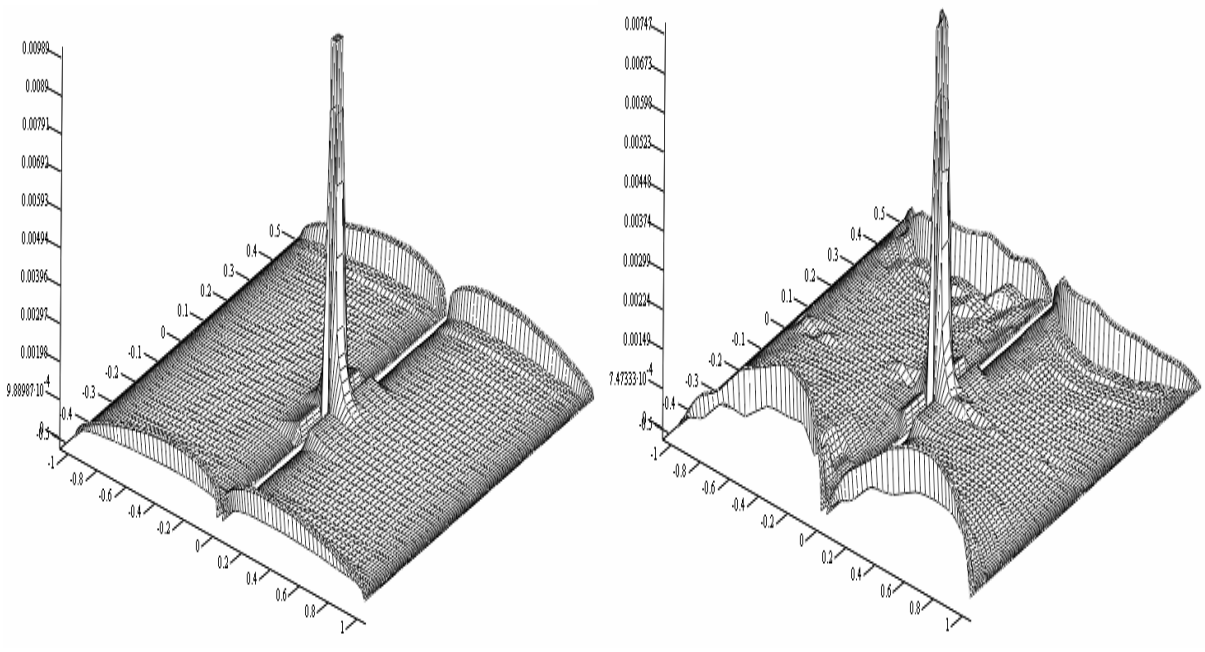
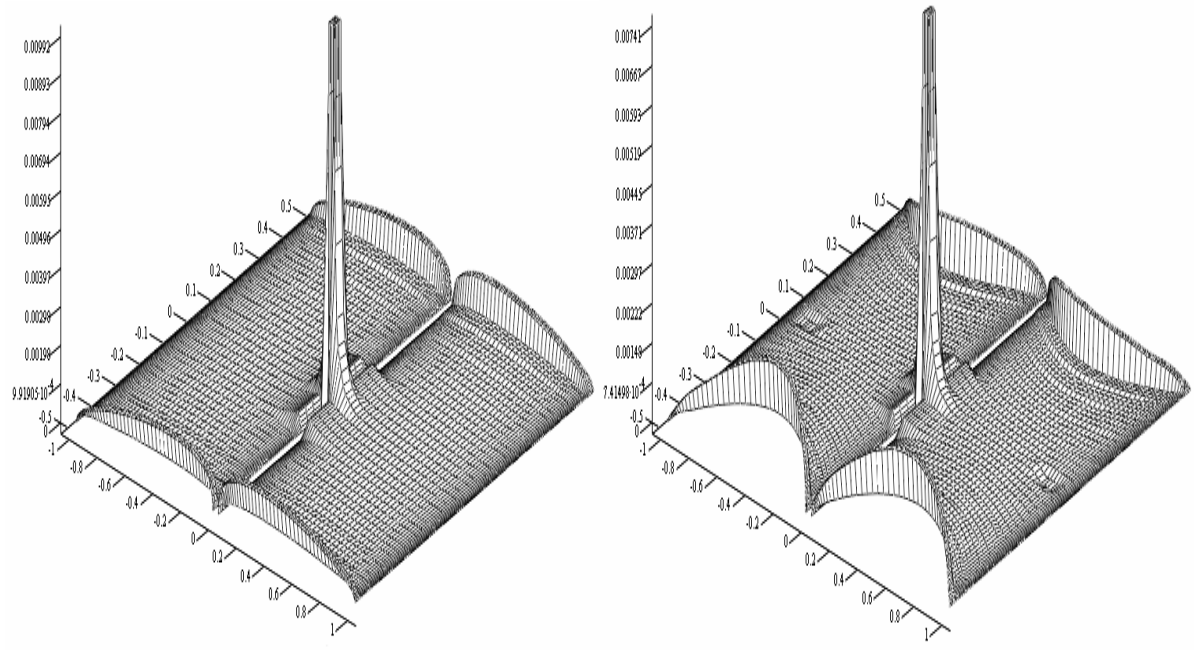


a) b)

Fig. 4. Modulus of current distribution $\dot{J}xt$ calculated by crystal_har program.

a) model1

b) model2

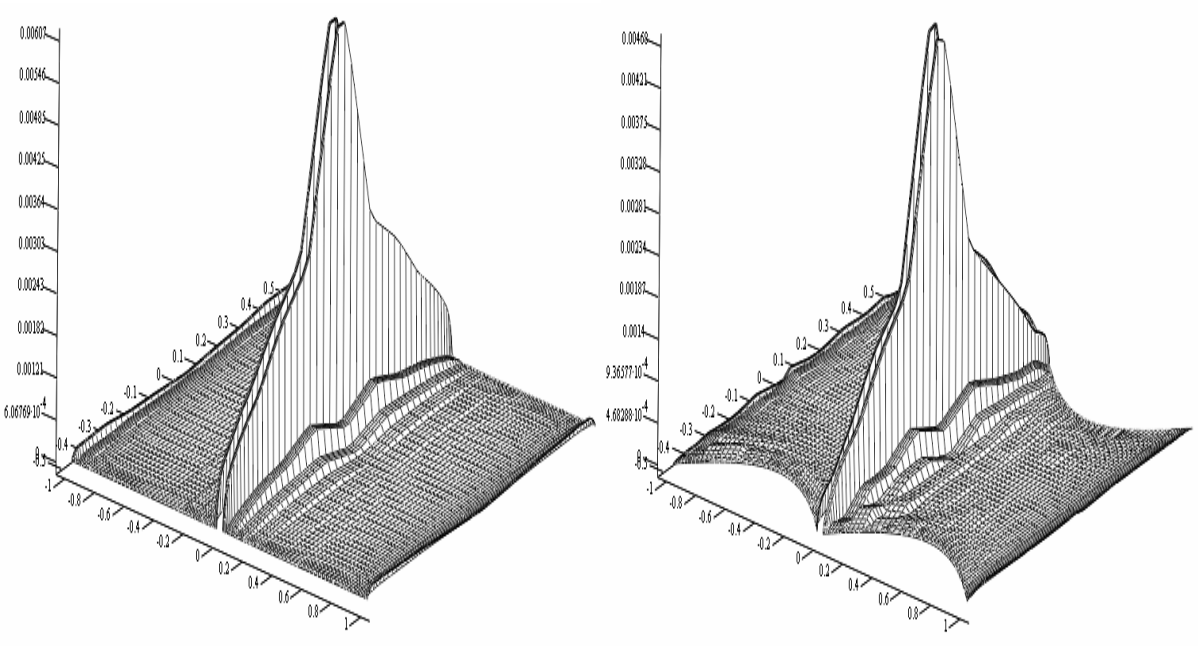


a) b)

Fig. 5. Modulus of current distribution $\dot{J}xt$ calculated by crystal_tol1 program.

a) model1

b) model2

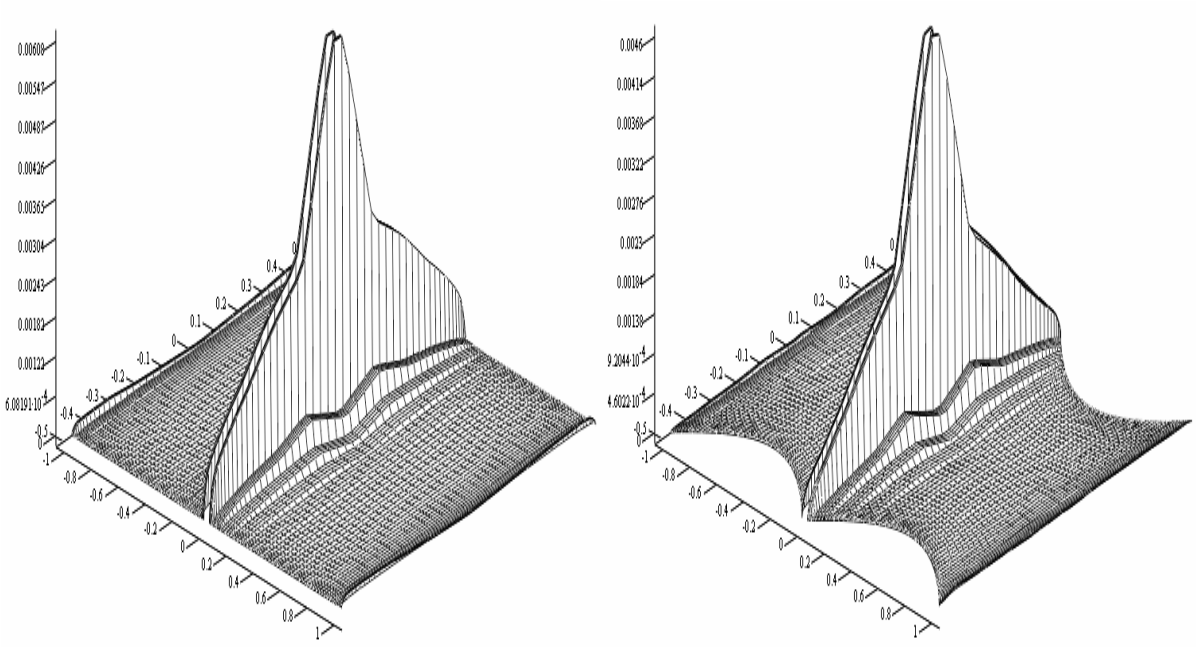


a) b)

Fig. 6. Modulus of current distribution $\dot{J}yt$ calculated by crystal_har program.

a) model1

b) model2

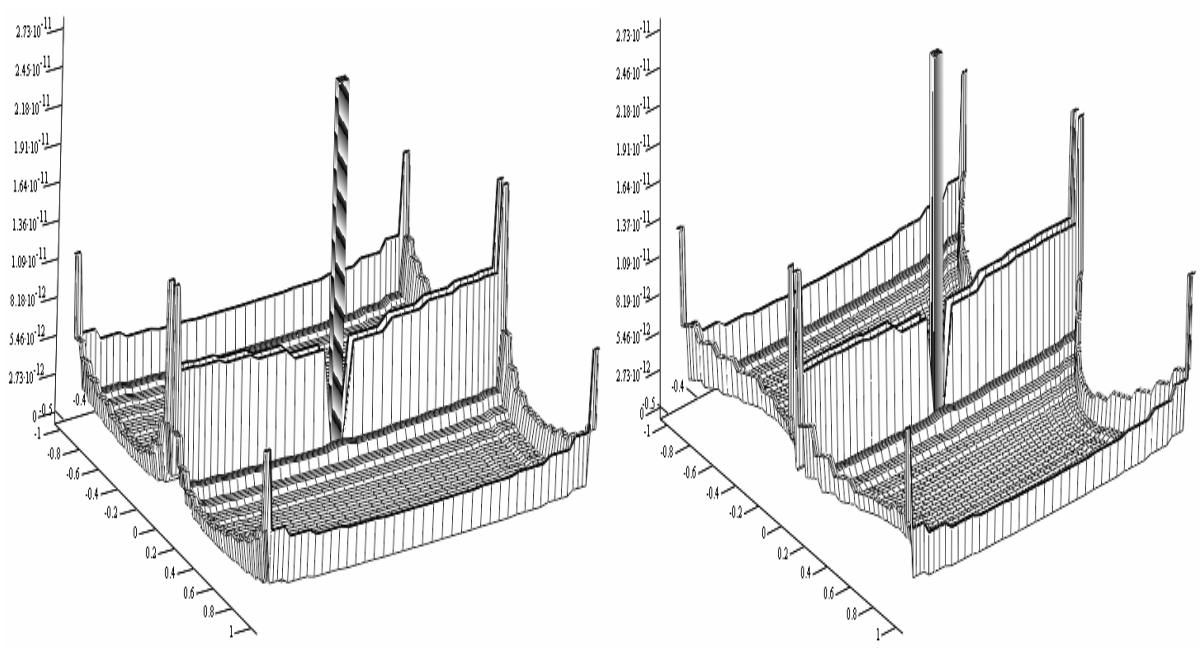


a) b)

Fig. 7. Modulus of current distribution $\dot{J}yt$ calculated by crystal_tol1 program.

a) model1

b) model2

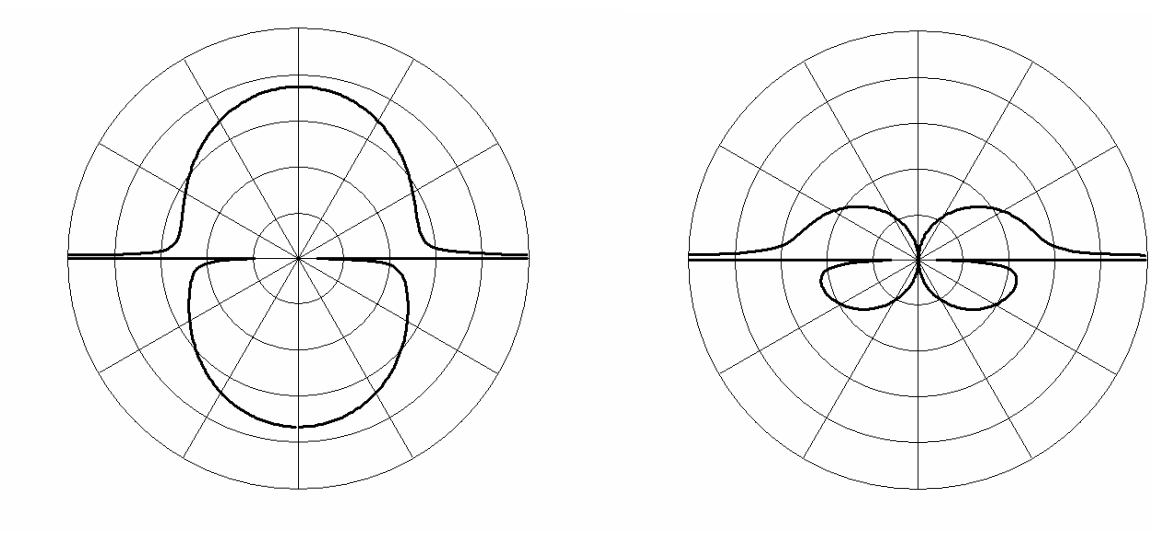


a) b)

Fig. 8. Modulus of charge distribution ρ calculated by crystal_tol1 program.

a) model1

b) model2



a) b)

Fig. 9. Modulus of axial ratio calculated by crystal_tol1 program.

a) model2 in range 0 – 0.1

b) model3 in range 0 – 0.01