UDC 528.3

S. PERIY, T. KORLIATOVYCH, V. LITYNSKYY, I. POKOTYLO

Department of Geodesy, Lviv Polytechnic National University, 12, S. Bandera str., Lviv, 79013, Ukraine,
e-mail: periy_ss@ukr.net, tetiasek@gmail.com

INVESTIGATION OF REFRACTION FIELD OVER WATER SURFACES

https://doi.org/10.23939/istcgcap2017.02.011

Aim. To investigate the possibilities of applying non-simultaneous reciprocal trigonometrical levelling over water surfaces while partially taking into account vertical refraction. **Methodology**. Experimental investigations of vertical refraction over water surfaces during day time on a plain and hilly terrains in summer. Non-simultaneous reciprocal observations were made over different water surfaces with the use of an automated electronic tachymeter. For comparing the results, the elevations between the points of observation were determined by the method of geometric levelling. **Results**. Average integral quantities of vertical refraction have been computed. It has been revealed that either stable or unstable stratification of the terrestrial atmospheric layer can set in over the water surfaces. Average integral components of vertical refraction have been computed from the results of reciprocal observations and the extent of the fluctuations of zenith distances. The elevations obtained as a result of non-simultaneous reciprocal levelling with vertical refraction in the presence of fluctuations of zenith distances have been compared with those obtained from the equivalent heights elevations obtained by geometric levelling. **Scientific novelty.** The formulas for computing the abnormal component of vertical refraction according to reciprocal linear-angular observations, as well as for computing the extent of the fluctuation of zenith distances during reciprocal observation have been obtained. **Practical applicability.** The possibility of applying non-simultaneous reciprocal trigonometric levelling on the routs under 1 km in length instead of geometric levelling over water surfaces.

Key words. Trigonometric levelling, geometric levelling, vertical refraction, index of vertical refraction, fluctuations of zenith distances, automated electronic tachymeter.

Introduction

The creation and development of high-attitude sites on the territory having numerous hydrological objects requires the application of combined methods of levelling to accurately determine the elevations.

It is known that geometric levelling is one of the most accurate methods of determining elevations, however laying grade lines on the terrains with developed hydrology has some obstacles such as by-pass routes of lakes or wetlands and bridges. The by-pass routes of water obstacles causes the increase in material costs and errors in geometric levelling.

GNSS (Global Navigation Satellite System) levelling is a prospective method of determining elevations, but, as yet, it cannot be used with high-precision due to vegetation and relief of the terrain, especially at short distances.

The transmission of heights over water surfaces using trigonometric levelling is one of the optimal methods and enables the sufficient acceleration of levelling process in the areas with developed hydrology. The main obstacle in approaching high-precision results is the determination of the influence of vertical refraction on the over-water elevation.

This issue was considered by many scientists, such as K. Kazanskiy (1966), B. Tlustiak (1974), D. Maslitch (1984), A. Ostrovs`kyy (1990, 2007), P. Baran (1996), Walo and A. Pachuta (2004), V. Diemien`tiev (2009, 2011), J. O. Bjelotomić (2011), A. Celms, A. Brants (2013), K. Tretiak (2015).

According to O. Moroz (2003), vertical refraction causes significant errors in the results of trigonometric levelling. Therefore, the enhancement of its accuracy is possible when vertical refraction is taken into account. This problem is quite difficult and as of yet has not been solved.

Aim

The aim of this paper is to investigate the possibilities of applying non-simultaneous trigonometric levelling over water surfaces including partially taking into account the vertical refraction.

Methodology

According to one-sided measurements of vertical angles and inclined distances, elevation h_{AB} (without taking into account the deviation of plumb lines) is calculated by the formula (Periy, 2015)

$$h_{AB} = D_{AB}\cos Z_{AB} + i_A - v_B + (1 - k_{AB})\frac{D_{AB}^2 \sin^2 Z_{AB}}{2R}, \quad (1)$$

where D_{AB} is the inclined distance measured between fixed points; Z_{AB} is a measured zenith distance; i_A is tachymeter height; v_B is a height of sighting target; $R \cong 6380\ km$ is a mean radius of curvature of the Earth on the territory of Ukraine; k_{AB} is a coefficient of vertical refraction along the observation line determined at the observation point.

The coefficient of vertical refraction is subdivided into two components; normal and abnormal (Izotov, 1955)

$$k_{AB} = k_{norm} + k_{abnorm}\ , \tag{2}$$

where k_{norm} is the coefficient of normal component of refraction; k_{abnorm} is the coefficient of abnormal component of refraction.

Coefficients of the normal component of vertical refraction are computed according to the values of atmospheric pressure *P,* and air temperature measured at the observation points when the dry-adiabatic temperature gradient in dry atmosphere is equal to $g_{adiab} = 0{,}0098\ K/m$ and with the use of the dependence (Izotov, 1955)

$$k_{norm} = 12{,}27\frac{P}{T^2}\ , \tag{3}$$

where P is atmospheric pressure in *mb*; T is air temperature in K^o, T = (273,15°+t°C).

The coefficient of the abnormal component of vertical refraction depending on meteorological values is represented by formula (4)

$$k_{\grave{a}bnorm} = 503\frac{P}{T^2}\frac{c}{h_e^b}, \tag{4}$$

where c is the abnormal temperature gradient at a height of 1 m; and h_e^b is the equivalent height of directional ray above the underlying surface.

The equivalent height of the directional ray above the underlying surface is calculated according to formula (5) (Izotov, 1955)

$$\frac{1}{h_e^b} = \frac{2}{D^2}\int_0^D \frac{l}{h^b}\,dl\ , \tag{5}$$

where b is the power roughly equal to 1, which depends on the stratification of the atmosphere.

A general formula for determining the coefficient of vertical refraction dependent on temperature and pressure is obtained on the basis of (2), (3) and (4) (Izotov, 1955).

$$k = 12{,}27\frac{P}{T^2} + 503\frac{P}{T^2}\frac{c}{h_e^b}\ . \tag{6}$$

The coefficients of the normal component of vertical refraction, as it can be derived from (3), is determined only according to the measurements of temperature and pressure on the observation points. Let us rewrite formula (1) as follows (Periy, 2015)

$$h^1_{AB} = D_{AB}\cos Z_{AB} + i_A - v_B + (1 - k_{norm})\frac{D^2_{AB}\sin^2 Z_{AB}}{2R}\ , \tag{7}$$

where h^1_{AB} is an elevation taking into account the Earth's curvature and the normal component of vertical refraction.

Let us write the system of equations for determining elevation on the basis of (1) for reciprocal observations taking into account (2) and (7):

$$\left.\begin{aligned} h_{AB} &= h^1_{AB} - k_{\grave{a}bnorm\ AB}\frac{D^2_{AB}\sin^2 Z_{AB}}{2R} \\ h_{BA} &= h^1_{BA} - k_{\grave{a}bnorm\ BA}\frac{D^2_{BA}\sin^2 Z_{BA}}{2R} \end{aligned}\right\} \tag{8}$$

The integral value of the coefficient of the normal component of vertical refraction is determined by formula (9) obtained from the system of equations (8) under the condition $k_{\text{з}nt.(\grave{a}bnorm.)} = k_{\grave{a}bnorm.AB} = k_{\grave{a}bnorm.AB}$:

$$k_{\text{з}nt(\grave{a}bnorm)} = \frac{(h^1_{AB} + h^1_{BA}) \times R}{\left(\frac{D_{AB} + D_{BA}}{2}\right)^2 \times \left[\sin\left(\frac{Z_{AB} + p - Z_{BA}}{2}\right)\right]^2}\ . \tag{9}$$

The coefficients of vertical refraction can be also determined by fluctuations of zenith distances with the use of the laws of statistical physics of atmosphere for different values of atmosphere stratification using the simplified formulae (Periy, 2013)

$$k_{(unstable)} = k_{\acute{I}} - 7{,}17\,D^{-1/2}h_e^{-2/3}m_Z^{''}\ , \tag{10}$$

$$k_{(indifferent)} = k_{\acute{I}}\ \mathbf{m}\ 8{,}35\ D^{-1/2}h_e^{-2/3}m_Z^{''}\ , \tag{11}$$

$$k_{(stable)} = k_{\acute{I}} + 13{,}3\ D^{-1/2}h_e^{-2/3}m_Z^{''}\ . \tag{12}$$

Numerical values of coefficients in formulae (10)–(12) are the aggregate values for different atmosphere stratification. Actually, the values of these coefficients change depending on turbulence, particularly for stable stratification, when dynamic turbulence exceeds the thermal one.

Comparing (6) to (10)–(12), we can see the functional relationship between the abnormal component of the coefficient of vertical refraction and the fluctuations of zenith distances. But, fluctuations of zenith distances m_Z'' are determined as root-mean-square deviations from average values of observations received on certain targets. They do not contain the information about atmosphere stratification, that is, do not take into account the sign of temperature gradient *c*.

The atmosphere stratification can be determined using the reciprocal observations according to the sign and value of the integral coefficient of vertical refraction (9) and under the condition of its equality in mutually opposite directions. In the periods of indifferent stratification (close to the periods of steady images), the accurate determination of the type of atmosphere stratification is problematic due to the possibility of transient periods.

Corrected elevations, taking into account the vertical refraction and measurement errors in zenith distances, are obtained by a technique for processing the results of reciprocal trigonometric levelling using the fluctuations of zenith distances (Periy, 2015)

$$h_{AB}^{rec.trig.} \cong \frac{h_{AB}^1 - h_{BA}^1}{2} - \left(\frac{m_{Z_{AB}}^b - m_{Z_{BA}}^b}{m_{Z_{AB}}^b + m_{Z_{BA}}^b}\right)\left(\frac{h_{AB}^1 + h_{BA}^1}{2}\right), \quad (13)$$

where $m_{Z_{AB}}$ and $m_{Z_{BA}}$ are the fluctuations of zenith directions, determined from received observations; and *b* is the power depending on atmosphere stratification (for unstable stratification *b*≈1).

The corrected elevation can also be obtained by the equivalent heights if formula (13) is to be represented as follows:

$$h_{AB}^{rec.trig.} \cong \frac{h_{AB}^1 - h_{BA}^1}{2} + \left(\frac{h_{e\,AB}^b - h_{e\,BA}^b}{h_{e\,AB}^b + h_{e\,BA}^b}\right)\left(\frac{h_{AB}^1 + h_{BA}^1}{2}\right), \quad (14)$$

The power *b* can be determined from formula (13) by substituting the value of elevation of reciprocal trigonometric levelling $h_{AB}^{rec.trig.}$ with the elevation from high-precision geometrical levelling h^{geom}. For simplifying the formula, let us substitute its part for a coefficient A:

$$A = \frac{\left[\left(\dfrac{h_{AB}^1 - h_{BA}^1}{2}\right) - h^{geom}\right]}{\left(\dfrac{h_{AB}^1 + h_{BA}^1}{2}\right)}, \quad (15)$$

where h^{geom} is the elevation obtained from geometrical levelling.

Therefore, the final formula for determining the power will look like

$$b = \frac{\ln\left(\dfrac{1+A}{1-A}\right)}{\ln\left(\dfrac{m_{Z_{AB}}}{m_{Z_{BA}}}\right)}. \quad (16)$$

Results

For conducting the experiment, four points of observation (T1, T2, T3, T4) were set at the shore of Pisochne Lake creating 2 triangles (ΔT1T2T3, ΔT1T2T4) with mutual visibility between the points (Periy, 2017).

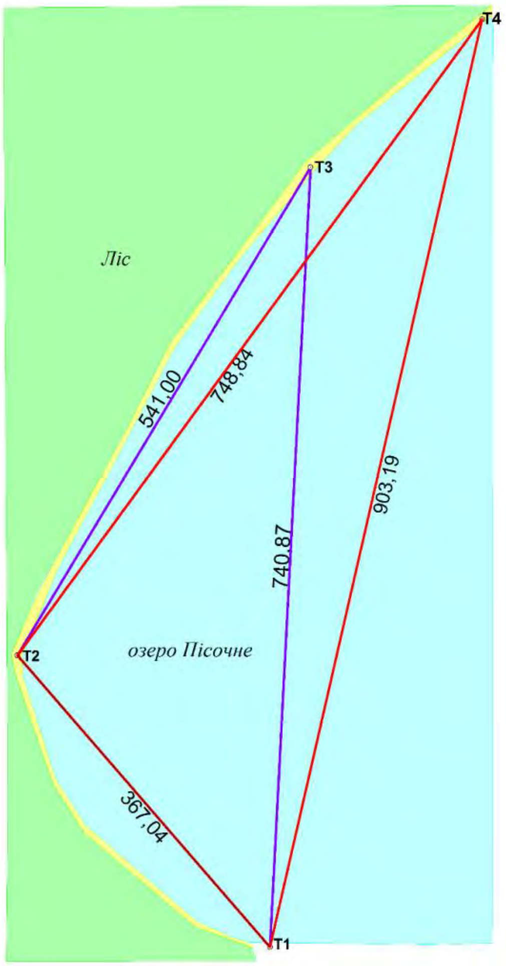

Fig. 1. The plan of network on Pisochne Lake

The measurements on the points are carried out with the use of tachymeter Leica TC 2003 (the accuracy of measuring angles 0.5″, lines 1mm + 1ppm).

Linear-angular observations on the points were conducted 10 times. Pointing to the reflectors was done in the automatic mode using the special function of the tachymeter consisting of maximum observation of the reflected signal.

Non-simultaneous reciprocal observations were conducted in sunny weather.

From the results of reciprocal observations, the integral abnormal components of the coefficients of vertical refraction were calculated for each line using formulae (9) (Table 1).

Analyzing signs and values of the integral abnormal components of coefficients of vertical refraction are shown in Table 1, where we can observe the unstable atmosphere stratification over the lake during the period of observation. It means that air temperature decreases when height increases.

Table 1

Values of integral abnormal coefficients of vertical refraction as a result of observation on Pisochne Lake

Name of line	D (m)	$k_{anom.refr.}$
T1-T3	740	-0.675
T2-T3	541	-1.389
T1-T2	349	-1.995
T1-T4	903	-0.491
T2-T4	748	-0.979

For the same lines, the elevations are calculated using formula (12) with different values of power *b* 1/2, 1/3,-1/6,-1/3,-1/2,-2/3. These elevations were compared to elevations obtained from the results of repeated III class geometric levelling (Moroz, 2015). Differences between elevations obtained by trigonometric and geometric levelling are given in Table 2.

Analyzing Table 2, we can see the ambiguity in determining the power along the directions of observation. The least mean value of errors is located between values obtained by geometrical and trigonometric levelling taking into account the fluctuations of zenith distances in powers -2/3 and equals to 1.6 mm, and the least mean square deviation from the mean square error (MSE) with the use of the power 1/2 equals 0.9 mm.

The power *b* of calculations using formula (16) is given in Table 3.

As we can derive from Table 3, the peak-to-peak value of the power is considerable and changes from 2 to -2. It points to errors, which are connected with non-simultaneity of observations along the lines, and to device errors contained in the values of fluctuation.

The experiment was repeated for the experimental geodetic ground in Berezhany city in the summer of 2017.

Investigations were conducted over the water surface of a pond.

For the experiment, 4 points were chosen, 2 on the left and 2 on the right shore (see Fig. 2). Elevations between these points were determined with a digital level Dini03 with the use of invar rods according to the program of class II geometric leveling.

Linear – angular measurements were conducted with the tachymeter Leica TC 2003 according to the special program of observations.

Table 2

Differences of elevations between geometric and trigonometric levelling taking into account the fluctuations of zenith distances with different powers *b* (Lake Pisochne)

Line	*b*^1 (mm)	*b*^1/2 (mm)	*b*^1/3 (mm)	*b*^0 (mm)	*b*^-1/6 (mm)	*b*^-1/3 (mm)	*b*^-1/2 (mm)	*b*^-2/3 (mm)
T1-T3	5.4	4.6	4.3	3.7	3.4	3.1	2.8	**2.5**
T2-T3	7.9	3.3	1.7	**-1.5**	-3.1	-4.7	-6.3	-7.9
T1-T2	7.3	3.9	2.7	**0.2**	-1.0	-2.3	-3.5	-4.6
T1-T4	**3.6**	5.0	5.4	6.3	6.8	7.3	7.7	8.2
T2-T4	**-0.3**	2.8	3.8	5.9	6.9	7.9	9.0	10.0
Average	4.8	3.9	3.6	2.9	2.6	2.3	2.0	**1.6**
MSE	3.3	**0.9**	1.4	3.5	4.6	5.6	6.7	7.8

Table 3

Calculated values of power *b* (Pisochne Lake)

Line	*b*
T1-T3	-2.15
T3-T2	0.16
T2-T1	-0.03
T1-T4	2.31
T2-T4	0.96
Average	**0.25**

Table 4

Values of integral abnormal coefficients of vertical refraction by the results of observation over Berezhany pond

Line	*D* (m)	**k.**$_{\mathbf{abnorm.refr.}}$
1-3	1111	0.1556
1-4	1092	0.1556
2-3	1107	0.1520
2-4	1089	0.1626

The sequence of observations is as follows:

1. On the points 1, 2, 3, 4, supports with tripods were installed. Over point 1 the device was installed, on points 2, 3, 4 reflectors were mounted.

2. The heights of device and reflectors were measured with the help of special equipment with the use of reading by the method of photofixation. On the point of device location, the temperature of air and water, as well as atmospheric pressure was measured.

3. From point 1 towards the other three points of quadrangle 2, 3, 4, horizontal and vertical circuits were in turn read and inclined distances were measured. Measurements were taken in 10 steps.

4. Observations on other points were conducted in the same way. The device was turn-by-turn mounted on consequent points, and reflectors were mounted and oriented instead of it on vacant places.

According to results obtained from reciprocal observations, integral abnormal components of the coefficients of vertical refraction for every line were calculated using formula (9) (Table 4).

As we can derive from Table 4, the values of the abnormal component of the integral coefficient of vertical refraction are positive, which corresponds with stable atmosphere stratification over Berezhany pond. It is caused by the increasing temperature with height (temperature inversion). For these lines, the elevations were calculated by formula (13) with different power values: - 1, -2/3, -1/3, 0, 1/3, 1.

5. Moreover, elevations were calculated taking into account equivalent heights (14) instead of the fluctuations of zenith distances. Equivalent ray heights were determined by (5): along the lines 1-3, 1-4, 2-3 i 2-4 with height equals to 10 m, while along the lines 3-1, 3-2, 4-1, 4-2 heights were equal to 5 m. These elevations were compared to the values of elevations obtained from class II geometric levelling, and the differences are shown in Table 5.

Fig. 2. Layout of the experiment over Berezhany pond

Table 5

Differences between elevations from geometric and trigonometric levelling taking into account the fluctuations of zenith distances (with different powers), and taking into account equivalent heights (Berezhany pond)

Line	Δ mh^-1	Δ mh^-2/3	Δ mh^-1/3	Δ mh^0	Δ mh^1/3	Δ mh^1	Δ he
1-3	**0.3**	2.0	3.8	5.6	7.5	10.9	**0.6**
1-4	3.6	3.1	2.5	2.0	1.5	**0.4**	-2.8
2-3	4.1	4.7	5.3	5.9	6.5	7.7	**1.0**
2-4	**0.7**	1.9	3.2	4.5	5.8	8.3	**-0.6**
Average error	**2.2**	2.9	3.7	4.5	5.3	6.8	**-0.4**
MSE	1.9	1.3	**1.2**	1.8	2.6	4.5	1.7

As we can derive from Table 5, the least average error occurs between the values of geometric and trigonometric levelling taking into account the equivalent heights and equal to 0.4 mm, and MSE equal to 1.7 mm. The least value of MSE is that with the use of the fluctuations of zenith distances raised to the power -1/3 and equal to 1.2 mm. Both methods meet the accuracy of class II geometric leveling. It shows the inverse relationship between the fluctuations and the value of vertical refraction and can be explained by the presence of dynamic turbulence on bigger heights of propagation of a directional ray over the water surface in the investigated area.

Powers calculated by (16) and shown in Table 6 confirm the suggested hypothesis of backward linkage. Air flows over Berezhany pond move along the Zolota Lypa Valley with high river banks, which form a dynamic wind tunnel.

Table 6

Calculated values of power *b* (Berezhany pond)

Line	***b***
1-3	-1.07
1-4	1.27
2-3	-3.39
2-4	-1.18
average	**-1.09**

For increasing the accuracy of levelling, the authors plan to create the technique for determining the powers at the fluctuations of zenith distances and equivalent heights during the performance of simultaneous reciprocal trigonometric observations.

Conclusions

1. The possibility of non-simultaneous reciprocal trigonometric levelling for height transmission over water surfaces while partially taking into account the vertical refraction and accuracy of class II geometric levelling is shown.

2. The occurrence of stable, as well as unstable stratifications of surface air is revealed.

3. Fluctuations of zenith heights for determining elevations for stable atmosphere stratification have stronger relationship with dynamic turbulence. Therefore, their powers are negative while determining partial values of vertical refraction in the range from -1 to -0.3.

4. During unstable stratification for reciprocal trigonometric levelling, powers of fluctuations of zenith distances are in the range from +1 to +0.3.

5. At stable stratification under the conditions of strong dynamic turbulence, it is better to use equivalent heights for determining partial values of vertical refraction.

REFERENCES

Baran P. I., Solovjov F. F., Chornokon' V. Ja. *Trygonometrychne niveljuvannja v inzhenerno-geodezychnyh robotah* [Trigonometric leveling in engineering geodetic operations]. Kyiv: *Ukrgeodezkartografija*, 1996, p. 130.

Bjelotomić O., Grgić I, Bašić T. Prefactory accuracy of levelling figure closure by combined measurements of trigonometric and geometric levelling. Proceeding of the 5th international conference on engineering surveying: «Ingeo 2011» Brijuni, Croatia, 22–24 September 2011, pp. 185–192;

Celms A., Ratkevičs A., Brants A., Kauranens E. Accuracy of height measurements for levelling across wide water bodies. *Suchasni dosjagnennja* *geodezichnoї nauki ta virobnictva* [Modern achievements in geodetic science and industry]. 2013 issue II (26), pp. 97–101.

Dement'ev V. E. *Refrakcija i mirazhi* [Refraction and mirages]. Moscow: OOO “Galleja-Print”, 2009, p. 391.

Dement'ev V. E. *Refraktsiia v turbulentnii atmosferi* [Refraction in the turbulent atmosphere]. Moscow: OOO “Galleja-Print”, 2011, p. 398.

Yzotov A. A., Pellynen L. P. *Yssledovanye zemnoy refraktsyy y metodov heodezycheskoho nyvelyrovanyya* [Research on terrestrial refraction and methods of geodetic levelling]. *Trudyi tsentralnogo nauchno-issledovatelskogo instituta geodezii, aerosemki i kartografii* [Works of the Central Research Institute of Geodesy, Aerospace and Cartography], issue. 102, Moscow: 1955, p. 176;

Kazanskiy K. V. *Zemnaya refraktsiya nad obshirnyimi vodnyimi poverhnostyami* [Terrestrial refraction over vast water surfaces. L: Hydrometeoyzdat, 1966, p. 192.

Maslich D. I., Perij S. S. *Osobennosti refrakcionnogo polja na morskih trassah v pribrezhnom subtropicheskom rajone* [Peculiarities of refraction field on marine routs in the offshore subtropical area]. *Heodeziia, kartohrafiia i aerofotoznimannia* [Geodesy, cartography and aerial photography]. Lviv Polytechnic Publishing House: 1984, issue 39, pp. 38–46.

Moroz O. I. *Vyznachennia ta vrakhuvannia vertykalnoi refraktsii pid chas heodezychnykh vymiriuvan* [Determining and taking into account vertical refraction during geodetic measurements: monograph]. Lviv Polytechnic Publishing House, 2003, p. 224.

Moroz O. I., Korlyatovych T. Yu., Pokotylo I. Ya., Yamelynets' S. P. *Stvorennya heodezychnoho polihona navkolo ozera Pisochne Shats'koho natsional'noho pryrodnoho parku* [Creation of geodetic ground around Pisochne lake in Shatsk national nature park]. *Visnyk heodeziyi ta kartohrafiyi*. [Herald of geodesy and cartography]. 2015, issue 5–6, pp. 21–23.

Ostrovskij A. L., Dzhuman B. M, Zablockij F. D., Kravcov N. I. *Uchet atmosfernyh vlijanij na astronomo-geodezicheskie izmerenija* [Taking into account weather impacts on astrogeodetic measurements.]. Moscow: *Nedra,* 1990, p. 235.

Ostrovs'kij A., Kravchuk O. *Doslidzhennja zakonomirnostej diї vertikal'noї refrakciї ta tochnosti trigonometrichnogo niveljuvannja nad vodnimi poverhnjami* [Investigations of the laws of vertical refraction and accuracy of trigonometric leveling over water surfaces]. *Geodesy, cartography and aerial photography*. 2007, issue 69, pp. 83–90.

Periy S .S., Moroz O. I. *Vyznachennya vertykal'noyi refraktsiyi za fluktuatsiyamy vertykal'nykh kutiv* [Determination of vertical refraction according to fluctuations of vertical angles] *Naukovi pratsi Donets'koho natsional'noho tekhnichnoho universytetu*, [Scientific works of Donetsk National Technical University] Donetsk. Series: Mining and Geological. 2013, issue 1(18), pp. 49-55.

Perij S. S *Aprobacija zastosuvannja neodnochasnogo dvostoronn'ogo trigonometrichnogo niveljuvannja z urahuvannjam vertikal'noї refrakciї* [Approbation of application of non-simultaneous bilateral trigonometric levelling taking into account vertical refraction]. *Heodeziia, kartohrafiia i aerofotoznimannia* [Geodesy, cartography and aerial photography]. Lviv Polytechnic Publishing House, 2015, issue 82, pp. 19–28.

Periy S. S., Moroz O. I., Korlyatovych T. Yu., Pokotylo I. Ya. *Zastosuvannya tryhonometrychnoho nivelyuvannya dlya peredavannya vysot nad vodnymy poverkhnyamy na terytoriyi Shats'koho natsional'noho pryrodnoho parku* [Application of trigonometric levelling for height transmission over water surfaces in Shatsk national nature park]. *Suchasni dosjagnennja geodezichnoї nauki ta virobnictva* [Modern achievements in geodetic science and industry]. 2017, issue I (33), pp. 65–68.

Tlustjak B. T. *Issledovanie zakonomernostej izmenenija zemnoj refrakcii v pribrezhnoj zone bol'shih vodnyh poverhnostej* [Investigation of laws of changing terrestrial refraction in offshore of vast water surfaces]. *Heodeziia, kartohrafiia i aerofotoznimannia* [Geodesy, cartography and aerial photography]. Lviv: 1974, issue 20, pp. 86–93.

Tretyak K., Perij S. S., Sidorov I. S., Babiy L. Complex High Accuracy Satellite and Field Measurements of Horizontal and Vertical Displacements of Control Geodetic Network on Dniester Hydroelectric Pumped Power Station (HPPS). Geomatics and Environmental Engineering, vol. 9, no. 1, 2015, pp. 83–96. Available at: http://dx.doi.org/10.7494/geom.2015.9.1.83

Walo J., Pachuta A. Olszak T. The Height transfer over the Czorsztyn artificial lake applying trigometric levelling. Reports on Geodesy. 2004. Tom 2/69, pp. 31–32.

С. ПЕРІЙ, Т. КОРЛЯТОВИЧ, В. ЛІТИНСЬКИЙ, І. ПОКОТИЛО

Кафедра геодезії, Національний університет “Львівська політехніка”, вул. С. Бандери, 12, Львів, 79013, Україна,
ел. пошта: periy_ss@ukr.net, tetiasek@gmail.com

ДОСЛІДЖЕННЯ РЕФРАКЦІЙНОГО ПОЛЯ НАД ВОДНИМИ ПОВЕРХНЯМИ

Мета. Дослідити можливості застосування двостороннього неодночасного тригонометричного нівелювання над водними поверхнями з частковим урахуванням вертикальної рефракції. **Методика.** Виконані експериментальні дослідження вертикальної рефракції над водними поверхнями в денний період у рівнинній та горбкуватій місцевості у літню пору року. Виконані неодночасні взаємні лінійно-кутові спостереження над різними водними поверхнями із застосуванням роботизованого тахеометра. Для порівняння, визначено перевищення між спостережуваними пунктами геометричним нівелюванням. **Результати.** Обчислені середньоінтегральні коефіцієнти вертикальної рефракції. Виявлено, що над водними поверхнями може встановлюватись як стійка, так і нестійка стратифікація приземного атмосферного прошарку. Обчислені середньоінтегральні складові вертикальної рефракції із двосторонніх спостережень та степені флуктуацій зенітних відстаней. Порівняно перевищення отримані із двостороннього неодночасного тригонометричного нівелювання із урахуванням вертикальної рефракції за флуктуаціями зенітних відстаней, а також отриманих із урахуванням еквівалентних висот із перевищеннями отриманими геометричним нівелюванням. **Наукова новизна.** Отримана формула розрахунку аномальної складової вертикальної рефракції за двосторонніми лінійно-кутовими спостереженнями. Отримана формула обчислення степеня флуктуацій зенітних відстаней під час двосторонніх спостережень. **Практична значущість.** Показана можливість застосування неодночасного двостороннього тригонометричного нівелювання на трасах до 1 км взамін геометричного нівелювання над водними поверхнями.

Ключові слова: тригонометричне нівелювання; геометричне нівелювання; вертикальна рефракція; коефіцієнт вертикальної рефракції; флуктуації зенітних відстаней; роботизований електронний тахеометр

Received 04.09.2017