УДК 528.3

S. PERIJ, I. POKOTYLO, T. KORLIATOVYCH

Department of Geodesy, of Lviv Polytechnic National University, 12, S. Bandera str., Lviv, Ukraine, 79013, e-mail: periy_ss@ukr.net, Ivan.Y.Pokotylo@lpnu.ua, tetiasek@gmail.com

# INVESTIGATION OF ACCURACY OF METHODS OF TRIGONOMETRIC LEVELLING DURING THE TRANSMISSION OF ELEVATIONS OVER WATER SURFACES

https://doi.org/10.23939/istcgcap2017.01.018

**Aims.** The investigations of the transmission of elevations over water surfaces remain topical, so the first aim is to develop the technique of increasing the accuracy of the trigonometric instead of geometrical one. The second aim is to investigate the methods of trigonometric which can be used for the transmission of elevations over water surfaces with the use of modern devices. **Methodology.** Simultaneous and non-simultaneous reciprocal observations over different water surfaces have been carried out. The results of the trigonometric leveling carried out by different methods have been processed. **Results.** On the territories of training geodetic site in Berezhany city (Ternopil region) and urban settlement Shatsk (Volyn' region) the activities directed to the creation of elevation geodetic base have taken place, including the investigations of the accuracy of trigonometric leveling during the transmission of elevation over the water surfaces. Vertical tie-in of geodetic datum has been done by the III class geometrical leveling. For determining heights on the investigated points, linear and angular measurements have been made with the use of automated tachymeters TPS 1201R Leica and TC 2003 Leica. According to the results of the observations on the geodetic site in urban settlement Shatsk, the comparison of the elevations obtained by the geometric leveling and trigonometric leveling by the methods "from inside", "across the point", "refraction basis" and non-simultaneous reciprocal leveling has been conducted. Balancing of the analytical network by a method of least squares has been done. According to the results obtained in Berezhany, the coefficients of vertical refraction along different directions have been computed and the comparison of elevations obtained from the III class geometric leveling and from the trigonometric leveling done by the method "from inside" have been conducted. The investigations of the accuracy of different methods of trigonometric leveling compared to III class geometric leveling have shown that they both generally meet the accuracy requirements of leveling on the flat grounds. Since the rough terrain has its peculiarities, some additional measured characteristics of the propagation of the axis of sight over different surfaces should be taken into account, for example, fluctuations of zenith distances determining atmosphere stratification or measured elevations. **Scientific novelty.** The application of non-simultaneous reciprocal trigonometric leveling over the water surfaces instead the III class geometric leveling on the paths under 1 km in length. **Practical applicability.** The possibility of replacing the cost-based geometric leveling with the trigonometric leveling without accuracy losses during the transmission of elevations over the water surfaces is shown.

*Key words:* trigonometric leveling, vertical refraction, geometric leveling, automated electronic tachymeter, refraction basis.

## Introduction

The creation and development of elevation sites on the territories with the significant amount of hydrological objects requires applying the combinations of different leveling methods providing given accuracy of the determination of elevations.

Geometric leveling is known to be one of the most accurate methods of determining elevations, but plotting level lines in the terrain with the developed hydrology has some difficulties, such as bypassing lakes and swamp lands, seeking for bridges over rivers etc. Avoidance of the water barriers causes the increase in length of the level line, which in turn increases errors of geometric leveling.

One of the most optimal solutions to this problem with the use of surface geodetic methods is the transmission of elevations over the water surface by the trigonometric leveling. This problem was investigated by such scientists as A. Ostrovskyi (Ostrovskyj, 2007, 1990), A. Celms (Celms, 2013), A. Brants, J. Walo (Walo, 2004), A. Pachuta, D. Maslitch (Maslitch, 1984), B. Tlustiak (Tlustiak, 1974, 1983), F. Brunner (Brunner, 1979) etc.

The method of trigonometric leveling is much more labor efficient than geometric leveling, but its accuracy is considered to be less than in geometric one. The biggest errors in trigonometric leveling are caused by vertical refraction (Brunner, 1979, Ostrovskyi, 1990, Dementiev, 2009). Therefore, the accuracy enhancement of the trigonometric leveling is possible only under the determining and taking into account the vertical refraction. However, this problem is complex and has not been completely resolved yet.

## Aim

The investigations of the transmission of elevations over the water surfaces are still topical nowadays, as well as the development of a technique for increasing accuracy of the trigonometric leveling instead of the geometric leveling. The aim of the article is to investigate the methods of trigonometric leveling, which can be used for the transmission of the elevations over the water surfaces with the use of modern devices.

## Methodology

According to the results of one-direction measurements of vertical angles and inclined distances, the elevation $h_{AB}$ without considering the deviation of plumb lines by the formula (Baran, 1996):

$$h_{AB} = D_{AB}\cos Z_{AB} + i_A - v_B + (1 - k_{AB})\frac{D_{AB}^2 \sin^2 Z_{AB}}{2R}, \quad (1)$$

where $D_{AB}$ is a slant distance measured between points of observation, which contains adjustment for meteorological conditions of propagation of light beam; $Z_{AB}$ is a measured zenith distance; $i_A$ is tachymeter height; $v_B$ is the height of sighting target; $R \cong 6380$ км is a radius of curvature of the Earth; $k_{AB}$ is a coefficient of vertical refraction over the line of sighting.

Reciprocal elevations $h_{AB}^1$ and $h_{BA}^1$ without taking into account vertical refraction are determined with formula (1) :

$$h_{AB}^1 = D_{AB}\cos Z_{AB} + i_A - v_B + \frac{D_{AB}^2 \sin^2 Z_{AB}}{2R}, \quad (2)$$

$$h_{BA}^1 = D_{BA}\cos Z_{BA} + i_B - v_A + \frac{D_{BA}^2 \sin^2 Z_{BA}}{2R}. \quad (3)$$

For reducing the effect of vertical refraction during the trigonometric leveling an observation method "from inside" is applied.

During the measurement of zenith distances and slant distances by the method "from inside" it can be assumed that if coefficients of vertical refraction on both arms of leveling are equal and the height of the device is stable, the elevation between the points (Fig. 1) is determined as a difference between formulae (2) and (3):

$$h_{AB} = h_2 - h_1 = D_2\cos Z_2 - D_1\cos Z_1 + \frac{1}{2R}\left(D_2^2\sin^2 Z_2 - D_1^2\sin^2 Z_1\right) - (v_2 - v_1) \quad (4)$$

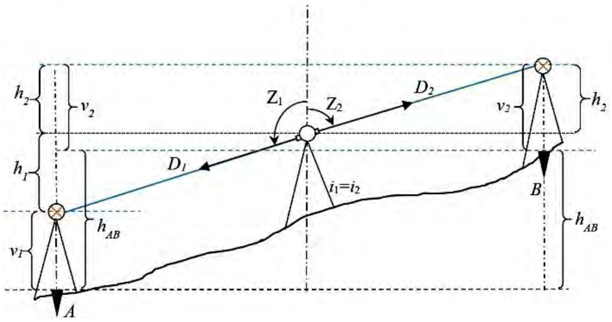


*Fig. 1. Trigonometric leveling "from inside"*

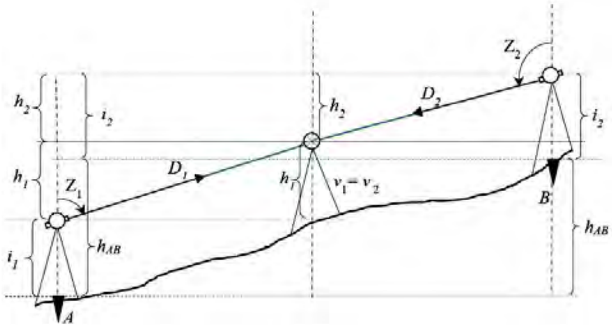


*Fig. 2. Trigonometric leveling "across the point"*

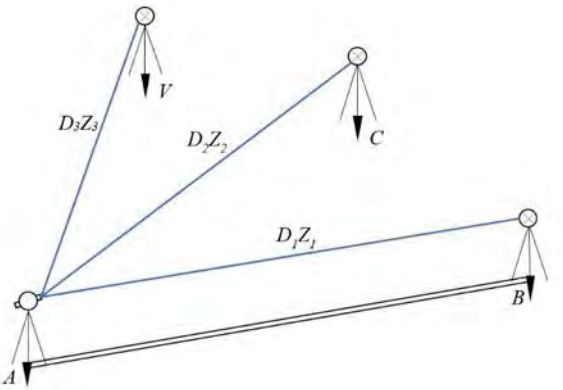


*Fig. 3. Trigonometric using the method of refraction basis*

Leveling according to the method "across the point" is based on measuring zenith angles $Z_1$, $Z_2$ and slant distances $D_1$ and $D_2$ towards the reflector installed approximately in the middle of the distance between the stations (see Fig. 2).

By analogy with the method "from inside", the elevation by the method "across the point" is determined by the formula:

$$h_{AB} = h_1 - h_2 = D_1\cos Z_1 - D_2\cos Z_2 + \frac{1}{2R}\left(D_1^2\sin^2 Z_1 - D_2^2\sin^2 Z_2\right) + (i_1 - i_2). \quad (5)$$

The difference between the two methods ("from inside" and "across the point") generally consists in the simultaneity of observations for the second one and measuring either instrument heights or sighting targets, which is not essential. Generally, the methods can be used for height transmission between the points for distance reduction and the choice of underlying surface between the points for the compensation of vertical refraction (Ostrovskyj, 1990; Baran, 1996).

The geodetic method of determining the constant of refraction (Ostrovskyj, 1990) is based on the so-called refraction basis (Fig. 3). The trigonometric leveling is done on the line of known heights between points determined by the high-precision (geometric) leveling. From formula (1), the constant of refraction $k$ is determined:

$$k_{12} = 1 - \frac{(H_2 - H_1) - (D_{12}\cos Z_{12} + i_1 - v_2)}{\Delta_R}, \quad (6)$$

where $H_1, H_2$ are the heights of terminal points of the line; $\Delta_R$ is the adjustment for the Earth curvature

$$\Delta_R = \frac{D_{12}^2 \sin Z_{12}^2}{2R}. \quad (7)$$

During the reciprocal trigonometric at known heights, the constants of refraction can be determined twice by formula (6), that is, $k_{12}$ and $k_{21}$ separately, and by their difference the quality of measurements and atmosphere stratification in two reciprocal directions can be estimated.

## Results

The investigations of the leveling accuracy over the water surface with the use of simultaneous observations of trigonometric leveling were conducted in summer in the town of Berezhany on the territory of training geodetic ground (see Fig. 4). For this purpose, eight benchmarks were established in advance: 4 benchmarks on the left and 4 on the right bank of the pond at a height of about 2 m–10 m–20 m relative to the water surface.

Tie-in of established points to vertical datums was conducted by the III class geometric leveling. Simultaneous daytime and evening linear-angular measurements were made with automatic tachymeters TPS 1201R Leica (with instrument accuracy of measuring angles 1″ and lines 1mm+1.5ppm) and TC 2003 Leica.

At two bottom points of right and left banks, the tachymeters were installed, whose sighting targets were reflectors mounted on the bottom, middle and top points on two opposite banks.

Measuring angles and line lengths was carried out in ten hourly cycles in groups of ten at two positions of the vertical circle of tachymeter. Pointing to the reflectors was done in the automatic mode with the use of a special tachymeter function consisting in observation of maximum of reflected signal. The measurements of such meteorological variables as water and air temperature, as well as pressure was taken hourly.

The plan of hourly observations consisted in the following activities:

- From face left the tachymeter was in turn pointed towards the top, middle and bottom points. The position of vertical circle was changed.
- From face right it was directed to the bottom, middle and top points correspondingly.

Having obtained the elevation values from one-direction trigonometric observations and from geometric leveling, we computed hourly constants of refraction from two banks by formula (6). Obtained values of the constant of refraction taken from observations from the right bank to left bank bottom (LB), left middle (LM) and left top (LT) are shown in Fig. 5. Values of the constant from observations made from left bank to right bottom (RB), right middle (RM) and right top (RT) are depicted in Fig. 6.

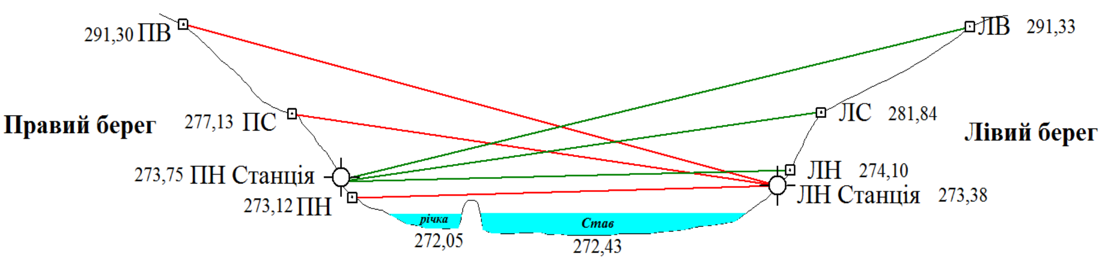


*Fig. 4. Layout of vertical section of the location of observation points on Berezhany pond*

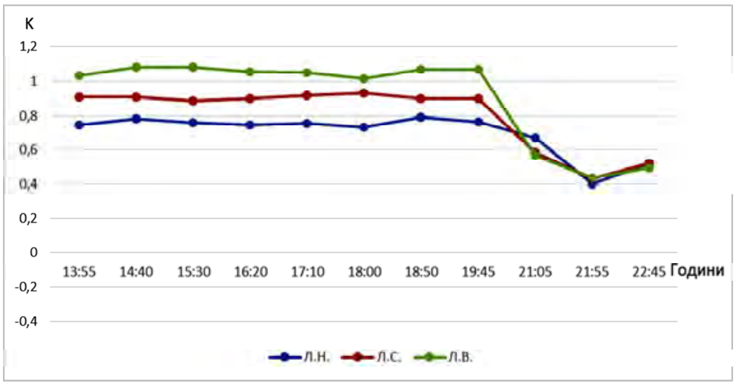


*Fig. 5. Daily variation of the vertical constant of refraction computed from observations from the right to the left bank*

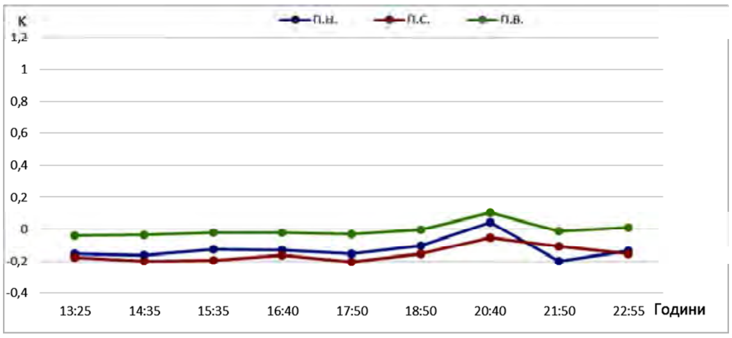


*Fig. 6. Daily variation of the vertical constant of refraction computed from observations from the left to the right bank*

*Table 1*

**Comparison of the results of third class geometric leveling and trigonometric leveling using the "from inside" method performed from the right to the left bank**

| Time of observation | Elevation h3-h2 (m) | Elevation h3-h1 (m) | Elevation h2-h1 (m) | $h^{geom}$ -$h^{trig}$ (h3-h2) (m) | $h^{geom}$ -$h^{trig}$ (h3-h1) (m) | $h^{geom}$-$h^{trig}$ (h2-h1) (m) |
|---|---|---|---|---|---|---|
| | $\mathbf{h^{geom}}$=-7.739 | $\mathbf{h^{geom}}$ =-17.229 | $\mathbf{h^{geom}}$ =-9.490 | | | |
| | $\mathbf{h^{trig}}$ | $\mathbf{h^{trig}}$ | $\mathbf{h^{trig}}$ | | | |
| 13:29-14:15 | -7.757 | -17.270 | -9.513 | -0.019 | -0.041 | -0.022 |
| 14:16- 15:05 | -7.754 | -17.272 | -9.518 | -0.015 | -0.043 | -0.028 |
| 15:06-15:55 | -7.754 | -17.274 | -9.520 | -0.015 | -0.045 | -0.030 |
| 15:56-16:45 | -7.756 | -17.272 | -9.516 | -0.017 | -0.043 | -0.026 |
| 16:46-17:35 | -7.757 | -17.271 | -9.514 | -0.019 | -0.042 | -0.024 |
| 17:36-18:25 | -7.761 | -17.270 | -9.508 | -0.022 | -0.041 | -0.018 |
| 18:26-19:15 | -7.752 | -17.269 | -9.517 | -0.013 | -0.040 | -0.027 |
| 19:16-20:10 | -7.755 | -17.272 | -9.517 | -0,016 | -0.043 | -0.027 |
| 20:41- 21:28 | -7.733 | -17.227 | -9.494 | 0.006 | 0.002 | -0.004 |
| 21:29-22:18 | -7.744 | -17.238 | -9.495 | -0.005 | -0.009 | -0.005 |
| 22:19- 23:08 | -7.740 | -17.233 | -9.493 | -0.002 | -0.004 | -0.002 |
| | | | $Fh_{abs.\ mean}$ | **-0.013** | **-0.032** | **-0.019** |
| | | | MSE | **0.009** | **0.018** | **0.011** |

From Fig. 5 it can be seen that in observations from the right bank to the left one the constant of refraction in daylight hours increases with the increase in height of the target point and equals 0.75 (LB), 0.9 (LM) and 1.05 (LT). During nighttime, the constant of refraction is identical for all directions and is equal to about +0.45. Such a variation in vertical refraction can be explained by the existing complex refraction field formed above the river and in general described by the rules of the dynamic turbulence of the atmosphere.

From Fig. 6 it can be seen that in observations from the left bank to the right one the constant of refraction in daylight hours and in the nighttime also increases with the increase in height. The values of the constant of vertical refraction are small and negative, which show the normal structure of the atmosphere above the water surface. Daily variation of the vertical constant of refraction is smooth, which characterizes the beam propagation over the water surface with stable temperature.

The comparison of the results of the III class geometric leveling and trigonometric leveling with the use of "from inside" method from the right bank to the left bank and vice versa has been conducted. The results of this comparison are shown in Tables 1 and 2. The value **h3-h2** is the elevation between LB and LM; **h3-h1** is the elevation between LB and LT; **h2-h1** is the elevation between LM and LT. The difference between geometric leveling and the trigonometric leveling by the "from inside" method in the direction **h3-h2** on average equals -0.013 m, in the direction **h3-h1** it is equal to -0.032 m, in the direction **h2-h1** it equals 0.019 m. At the increase in the height of the observation beam, the mean square error (MSE) and absolute values of errors increase. It indicates changing the constant of refraction with the height and the necessity of taking into account the height difference of observed objects.

The values ***h3-h2*** are elevations between RB and RM; ***h3-h1*** are elevations between RB and RT; ***h2-h1*** are elevations between RM and RT. The difference between geometric leveling and the trigonometric leveling by the "from inside" method in the direction **h3-h2** on average equals -0.002 m, in the direction **h3-h1** it equals -0.011 m; in the direction ***h2-h1*** it equals -0.013 m. The mean square error (MSE) does not exceed 5 mm, which indicates the regularity of refraction change in the directions over the big water surfaces with the stable temperature.

The investigations of reciprocal non-simultaneous leveling was conducted on the territory of urban settlement Shatsk along the lakeside of Pisochne. For the experiment, four points T1, T2, T3, T4 were arranged on the lakeside in a way allowing the formation of two triangles (ΔT1T2T3, ΔT1T2T4), Fig.7, with reciprocal visibility between the points. One of the factors considered in establishing the points was their nearby location to the traverse net points, which were used for III class leveling with a digital level Dini 22 Trimble (tie-in was done using one leveling station).

The measurements were made with a tachymeter Leica TC 2003.

The sequence of the observations was as follows:

1. A tripod with supports was mounted and centered on the points T1, T2, T3. Above the point T1 the device was mounted, and on T2 and T3 reflectors were installed.

*Table 2*

**Comparison of the results of III class geometric leveling and trigonometric leveling using the "from inside" method made from the left to the right bank**

| Time of observation | Elevation h3-h2 (m) | Elevation h3-h1 (m) | Elevation h2-h1 (m) | $h^{geom}$-$h^{trig}$ (h3-h2) (m) | $h^{geom}$-$h^{trig}$ (h3-h1) (m) | $h^{geom}$-$h^{trig}$ (h2-h1) (m) |
|---|---|---|---|---|---|---|
| | **$h^{geom}$**=-4.009 | **$h^{geom}$**=-18.180 | **$h^{геом}$**=-14.171 | | | |
| | **$h^{trig}$** | **$h^{trig}$** | **$h^{trig}$** | | | |
| 12:53-13:52 | -4.008 | -18.190 | -14.182 | 0.001 | -0.010 | -0.011 |
| 14:00-15:05 | -4.006 | -18.191 | -14.185 | 0.003 | -0.011 | -0.014 |
| 15:0916:04 | -4.003 | -18.189 | -14.186 | 0.006 | -0.010 | -0.015 |
| 16:09-17:14 | -4.006 | -18.190 | -14.183 | 0.003 | -0.010 | -0.013 |
| 17:24-18:19 | -4.005 | -18.191 | -14.186 | 0.004 | -0.011 | -0.015 |
| 18:24-19:19 | -4.005 | -18.189 | -14.184 | 0.004 | -0.009 | -0.013 |
| 20:05-21:15 | -4.000 | -18.187 | -14.187 | 0.009 | -0.008 | -0.017 |
| 21:16-22:21 | -4.018 | -18.197 | -14.179 | -0.009 | -0.018 | -0.009 |
| 22:26-23:25 | -4.008 | -18.193 | -14.186 | 0.001 | -0.014 | -0.015 |
| | | | $Fh_{abs.mean}$ | **0.002** | **-0.011** | **-0.013** |
| | | | MSE | **0.005** | **0.003** | **0.002** |

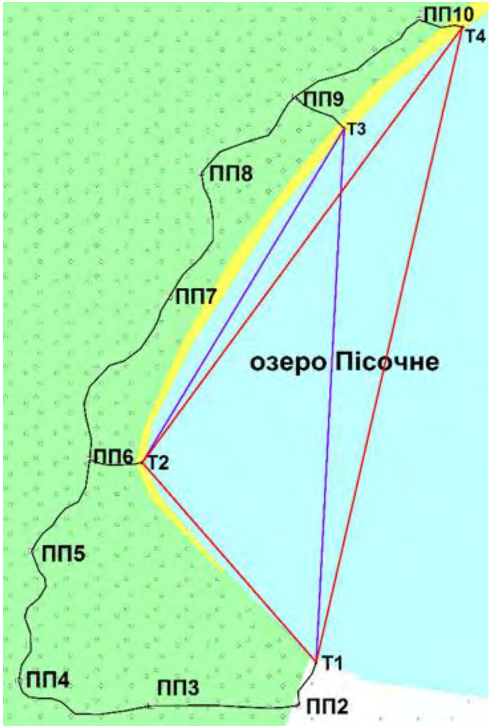


*Fig. 7. Location of the observation points and their elevation referencing to polygon measurement points on the territory of Pisochne lake field*

2. The heights of the device and reflectors were measured with special equipment, readings being taken with the use of the method of photofixation. At the point with the device, the air and water temperature, as well as atmospheric pressure were measured.
3. From the point T1 towards two points of imaginary triangle T2 and T3, vertical and horizontal circuits were read in turn and the slope distance was measured. The measurements were taken in 10 rounds.
4. In turn, the device was mounted in other points, being replaced by the reflector, and observations were done in the same way.

The same operations were performed in the triangle T1 T2 T4.

The results obtained were processed by the method "from inside" according to formula (4), "across the point" with formula (5) and the "refraction basis" method.

The results of the comparison of the elevations from geometric leveling with those from the method "from inside" are shown in Table 3.

The results of comparison of the elevations from geometric leveling with the elevations obtained from the "across the point" method are given in Table 4.

The results of comparison of the elevations from geometric leveling with the elevations obtained from the "refraction basis" method are given in Table 5.

The mean absolute error between the geometric leveling and trigonometric leveling by the "from inside" method in Δ 123 equals -0.005 m and MSE is equal to 0.004 m.

The mean absolute error between the geometric leveling and trigonometric leveling by the "from inside" method in Δ 124 equals -0.008 m and MSE is equal to 0.009 m.

The mean absolute error between the geometric levelling and trigonomentric leveling by the "across the point" method in Δ 123 equals 0.01 m and MSE is equal to 0.002 m.

The mean absolute error between the geometric levelling and trigonomentric leveling by the "across the point" method in Δ 124 equals 0.017 m and MSE is equal to 0.007 m.

From Table 5 it can be seen that the least difference between the geometric elevation and the elevation computed by the method of refraction basis equals 0.008 m, and the biggest one is -0.039 m. The thorough analysis of the results obtained by the method of refraction basis shows that the longest direction should be chosen as the base line, so that it might be possible to predict the value of the constant of refraction along the other directions.

As Tables 3, 4, 5 show, the least difference occurs between the geometric leveling and the trigonometric one obtained by the method "from inside", and the least accurate prediction results are obtained by the method of refraction basis. For increasing the accuracy of refraction basis, it is desirable to apply the additional characteristics of the observed line associated with atmosphere turbulence.

The results of the comparison of the geometric and non-simultaneous reciprocal trigonometric leveling are given in Table 6.

As this Table shows, the elevation differences between geometric leveling and the reciprocal non-simultaneous leveling are not very significant: they correspond with the accuracy of the III class geometric leveling.

The balancing of analytical network was performed by the least-squares method with the use of weights $P$ computed according to the fluctuations of measured elevations $m_h$ from 10 observations by the formula

$$P = \frac{1}{m_h^{2/3}} . \qquad (7)$$

*Table 3*

**Comparison of the results of third class geometric and trigonometric using the "from inside" method at Δ 123, Δ 124**

| Triangle Δ123 | | | | | Triangle Δ124 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Station | Direction | $H^{trig}$(m) | $H^{geom}$(m) | Δ(m) | Station | Direction | $H^{trig}$(m) | $H^{geom}$(m) | Δ(m) |
| 1 | 2<br>3 | -0.158 | -0.157 | -0.001 | 1 | 2<br>4 | 0.583 | 0.582 | 0.001 |
| 2 | 1<br>3 | -1.013 | -1.004 | -0.010 | 2 | 1<br>4 | -0.278 | -0.265 | -0.013 |
| 3 | 1<br>2 | -0.85 | -0.847 | -0.003 | 4 | 1<br>2 | -0.86 | -0.847 | -0.013 |
| | | | **$Fh_{abs\ m.}$** | **-0.005** | | | | **$Fh_{abs\ m.}$** | **-0.008** |
| | | | **MSE** | **0.004** | | | | **MSE** | **0.009** |

*Table 4*

**Comparison of the results of III class geometric and trigonometric using the "across the point" method at Δ 123, Δ 124**

| Triangle Δ123 | | | | | Triangle Δ124 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Station | Direction | $H^{trig}$(m) | $H^{geom}$(m) | Δ(m) | Station | Direction | $H^{trig}$(m) | $H^{geom}$(m) | Δ(m) |
| 1<br>2 | 3 | -0.839 | -0.847 | 0.008 | 1<br>2 | 4 | -0.832 | -0.847 | 0.015 |
| 1<br>3 | 2 | -0.991 | -1.004 | 0.012 | 1<br>4 | 2 | -0.24 | -0,265 | 0.025 |
| 2<br>3 | 1 | -0.148 | -0.157 | 0.009 | 2<br>4 | 1 | 0.593 | 0.582 | 0.011 |
| | | | **$Fh_{abs\ m.}$** | **0.01** | | | | **$Fh_{abs\ m.}$** | **0.017** |
| | | | **MSE** | **0.002** | | | | **MSE** | **0.007** |

*Table 5*

**Comparison of the results of III class geometric and trigonometric using the refraction basis method**

| Base line | K | $K^{avr}$ | Observation directions | $H^{geom}$ (m) | $H^{trig}$(m) | $H^{geom}$-$h^{trig}$(m) |
|---|---|---|---|---|---|---|
| **1-2**<br>**2-1** | -1.88<br>-1.83 | -1.85 | **1-4** | -0.265 | -0.226 | -0.039 |
| **1-4**<br>**4-1** | -0.25<br>-0.45 | -0.35 | **1-2** | -0.846 | -0.861 | 0.015 |
| **2-3**<br>**3-2** | -1.18<br>-1.32 | -1.25 | **3-1** | 1.004 | 1.027 | -0.024 |
| **3-1**<br>**1-3** | -0.62<br>-0.45 | -0.54 | **3-2** | 0.157 | 0.139 | 0.018 |
| **2-3**<br>**3-2** | -1.18<br>-1.32 | -1.25 | **2-4** | 0.581 | 0.606 | -0.024 |
| **2-4**<br>**4-2** | -0.70<br>-0.97 | -0.83 | **2-3** | -0.157 | -0.165 | 0.008 |

*Table 6*

**Results of the comparison of geometric and reciprocal non-simultaneous trigonometric**

| Name of the station | Name of the point | $h_{geometr.}$ (m) | $h_{recipr.geometr.}$ (m) | Δh1 (mm) |
|---|---|---|---|---|
| 1 | 3 | -1.0035 | -0.9998 | 3.7 |
| | 2 | -0.8465 | -0.8467 | -0.2 |
| | 4 | -0.265 | -0.2586 | 6.4 |
| 2 | 4 | 0.5815 | 0.5874 | 5.9 |
| | 3 | -0.157 | -0.1554 | 1.6 |

The mean value of differences between the elevations obtained from the geometric leveling and the trigonometric leveling according to the results of balancing by the least squares method at the accepted weights in Δ123 equals 0.002 m and in Δ124 is equal to 0.003 m.

The order 2/3 of $m_h$ gives the best results of balancing; it can be explained by the systematic effect of the vertical refraction in the observation triangles (Tretyak, 2015).

The results of the comparison of elevations of geometric leveling and balanced elevations over Δ123 and Δ124 are shown in Table 7.

*Table 7*

**Results of the comparison of the elevations of geometric and adjusted using the least squares method with weights in Δ 123 and Δ 124**

| Triangle Δ123 | | | | Triangle Δ124 | | | |
|---|---|---|---|---|---|---|---|
| Directions | $H^{geom}$ (m) | $H^{trig}$ (m) | Δ (м) | Directions | $H^{geom}$ (m) | $H^{trig.}$ (m) | Δ(m) |
| **1-2** | -0.846 | -0.849 | -0.002 | **1-2** | -0.846 | -0.850 | -0.003 |
| **2-3** | -0.157 | -0.155 | 0.002 | **2-4** | 0.581 | 0.586 | 0.005 |
| **3-1** | 1.004 | 1.004 | -0.0005 | **4-1** | 0.265 | 0.263 | -0.002 |
| | | **M=** | **0.002** | | | **M=** | **0.003** |

The analysis of the processed results of the observations made on the Pisochne lake shows that the best results from the comparison of the geometric and trigonometric leveling were obtained from the balancing analytical networks by the least squares method with the use of weights. Quite satisfying results were obtained from reciprocal non-simultaneous leveling and the trigonometric leveling by the "from inside" method.

The results only half as good as previous were obtained according to the same data from the trigonometric leveling by the "across the point" method. It can be explained in the first place by the non-simultaneity of the observations performed by this method.

The determination of the elevations with the use of the method of refraction basis showed much worse results. The hypothesis about the equality of the constants of refraction in different directions is not confirmed in all cases, especially over the water surfaces.

## Scientific novelty and practical value

The possibility of applying the non-simultaneous reciprocal trigonometric leveling, leveling with the use of "from inside" method and balancing of analytical network by the least squares method with the use of weights for elevation transmission over the water surfaces has been proposed and approved for applying instead of the geometric leveling in distances up to 1 km.

The prospects of further investigations must consist in introducing clarity into the technique and choosing optimal lines and networks for replacing the geometric leveling.

## Conclusions

1. Conducted investigations proved the possibility of applying the non-simultaneous reciprocal trigonometric leveling instead of the III class geometric leveling for elevation transmission over the water surfaces.

2. For the reliability and accuracy evaluation, it is recommended that observations should be made within the specially created local analytical networks.

3. Refraction bases should be chosen along the long lines over the homogeneous surfaces with the directions, which could be observed.

REFERENCES

Baran P. I., Solovjov F. F., Chornokon' V. Ja. *Trygonometrychne niveljuvannja v inzhenerno-geodezychnyh robotah* [Trigonometric in geodetic works]. Kyiv: *Ukrgeodezkartografija*, 1996, 130 p.

Brunner F. K.: Vertical Refraction Angle Derived from the Variance of the Angle-Of-Arrival Fluctuations. In: Refractional Influences in Astronomy and Geodesy, Tengström E., Teleki G. (Eds.), 1979, pp. 227–238.

Celms A., Ratkevičs A., Brants A., Kauranens E. Accuracy of height measurements for levelling across wide water bodies. *Suchasni dosjagnennja geodezichnoï nauki ta virobnictva* [Modern achievements in geodetic science and industry]. 2013 issue II (26), pp. 97–101.

Dement'ev V. E., Pellinen L. P. *Refrakcija i mirazhi* [Refraction and mirages]. Moscow: OOO "Galleja-Print", 2009, 391 p.

Dement'ev V. E. *Refraktsiia v turbulentnii atmosferi* [Refraction in a turbulent atmosphere]. Moscow: OOO "Galleja-Print", 2011, 398 p.

Maslich D. I., Perij S. S. *Osobennosti refrakcionnogo polja na morskih trassah v pribrezhnom subtropicheskom rajone* [Features of the refractive field on the sea routes in the coastal subtropical region]. *Geodezija, kartografija i ajerofotos#emk*a [Geodesy, cartography and aerial photography]. Lviv: 1984, issue 39, pp. 38–46.

Moroz O. I. *Vyznachennia ta vrakhuvannia vertykalnoi refraktsii pid chas heodezychnykh vymiriuvan: Monohrafiia*. [Definition and calculation of vertical refraction during geodetic measurements]. Lviv: Lviv Polytechnic Publishing House, 2003, 224 p.

Ostrovskij A. L., Dzhuman B. M, Zablockij F. D., Kravcov N. I. *Uchet atmosfernyh vlijanij na astronomo-geodezicheskie izmerenija* [Accounting of atmospheric effects on astronomical and geodetic measurements.]. Moscow: *Nedra*, 1990, 235 p.

Ostrovs'kij A., Kravchuk O. *Doslidzhennja zakonomirnostej diï vertikal'noï refrakciï ta tochnosti trigonometrichnogo niveljuvannja nad vodnimi poverhnjami* [The study of the regularities of the action of vertical refraction and the accuracy of trigonometric over water surfaces]. *Geodezija, kartografija i aeroznimannja.* [Geodesy, cartography and aerial photography]. 2007, issue 69, pp. 83–90.

Perij S. S. *Vikoristannja dvostoronn'ogo trigonometrichnogo niveljuvannja u svitloviddalemirnij poligonometrii̇̈* [Using of bilateral trigonometric levelling in light range-finder polygonometry]. *Geodezija, kartografija i aerofotoznimannja* [Geodesy, cartography and aerial photography]. 2001, issue 61, pp.112–117.

Perij S. S *Aprobacija zastosuvannja neodnochasnogo dvostoronn'ogo trigonometrichnogo niveljuvannja z urahuvannjam vertikal'noї refrakciї* [Approbation of application of non-simultaneous bilateral trigonometric levelling taking into account vertical refraction]. *Geodezija, kartografija i aerofotoznimannja* [Geodesy, cartography and aerial photography]. 2015, issue 82, pp. 19–28.

Tlustjak B. T. *Issledovanie zakonomernostej izmenenija zemnoj refrakcii v pribrezhnoj zone bol'shih vodnyh poverhnostej* [Investigation of the regularities of changes in terrestrial refraction in the coastal zone of large water surfaces]. *Geodezija, kartografija i ajerofotos#emka* [Geodesy, cartography and aerial photography]. Lviv: 1974, issue 20, pp. 86–93.

Tlustjak B. T., Perij S. S. *Statisticheskaja harakteristika refrakcii svetovyh luchej na morskih trassah.* [Statistical characteristic of refraction of light rays on sea routes]. *Tez. dokl. Vsesojuz. soveshh. po refrakcii jelektromagnitnyh voln v atmosfere* [Abstracts of the All-Union Conference on the refraction of electromagnetic waves in the atmosphere]. Tomsk, 1983, pp. 281–284.

Tretyak K., Perij S. S., Sidorov I. S., Babiy L. Complex High Accuracy Satellite and Field Measurements of Horizontal and Vertical Displacements of Control Geodetic Network on Dniester Hydroelectric Pumped Power Station (HPPS). Geomatics and Environmental Engineering, vol. 9, no. 1, 2015, pp. 83–96. Available at: http://dx.doi.org/10.7494/geom.2015.9.1.83

Walo J., Pachuta A., Olszak T. The Height transfer over the Czorsztyn artificial lake applying trigometric levelling. Reports on Geodesy. 2004, tom 2/69, pp. 315–321.

С. ПЕРІЙ, І. ПОКОТИЛО, Т. КОРЛЯТОВИЧ

Кафедра геодезії, Національний університет “Львівська політехніка”, вул. С. Бандери, 12, Львів, Україна, 79013, ел. пошта: periy_ss@ukr.net, Ivan.Y.Pokotylo@lpnu.ua, tetiasek@gmail.com

## ДОСЛІДЖЕННЯ ТОЧНОСТІ МЕТОДІВ ТРИГОНОМЕТРИЧНОГО НІВЕЛЮВАННЯ ПІД ЧАС ПЕРЕДАЧІ ВИСОТ ЧЕРЕЗ ВОДНІ ПОВЕРХНІ

**Мета.** Актуальними залишаються дослідження передачі висот через водні поверхні, опрацювання методики підвищення точності тригонометричного нівелювання замість геометричного. Дослідити методи тригонометричного нівелювання, які можна застосовувати для передачі висот над водними поверхнями із використанням сучасних приладів. **Методика.** Виконані одночасні та неодночасні взаємні спостереження над різними водними поверхнями. Виконано опрацювання результатів тригонометричного нівелювання різними методами. **Результати.** На територіях навчально-геодезичних полігонів у місті Бережани Тернопільської області та смт Шацьк Волинської області виконані роботи зі створення висотної геодезичної основи, зокрема і для дослідження точності тригонометричного нівелювання під час передачі висот над водними поверхнями. Виконана висотна прив’язка геодезичної основи геометричним нівелюванням ІІІ класу. Для визначення висот на досліджуваних пунктах проведено лінійно-кутові вимірювання із використанням автоматизованих тахеометрів TPS 1201R Leica та TC 2003 Leica. За результатами отриманими із спостережень на геодезичному полігоні у смт Шацьк виконано порівняння перевищень геометричного нівелювання та перевищень отриманих із тригонометричного нівелювання методами “із середини”, “через точку”, “рефракційного базису” та неодночасним двостороннім нівелюванням. Виконано зрівноваження аналітичної мережі методом найменших квадратів. За результатами спостережень виконаними в Бережанах визначено коефіцієнти вертикальної рефракції за різними напрямками та порівняння перевищень отриманих із геометричного нівелювання ІІІ класу із перевищенням з тригонометричного нівелювання методом “із середини”. Дослідження точності різних методів тригонометричного нівелювання порівняно із геометричним нівелюванням ІІІ класу показало, що вони переважно задовольняють вимоги точності нівелювання на рівнинній місцевості. Для пересіченої місцевості необхідно враховувати її особливості та враховувати виміряні додаткові характеристики розповсюдження променя візування над підстильними поверхнями, що характеризують стратифікацію атмосфери такі, як флуктуації зенітних віддалей або виміряних перевищень. **Наукова новизна.** Запропоновано застосування неодночасного двостороннього тригонометричного нівелювання над водними поверхнями взамін геометричного нівелювання ІІІ класу на трасах до 1 км. **Практична значущість.** Показано можливість заміни затратного геометричного нівелювання тригонометричним нівелюванням без втрати точності під час передачі висот через водну поверхню.

*Ключові слова*: тригонометричне нівелювання; вертикальна рефракція; геометричне нівелювання роботизований електронний тахеометр; рефракційний базис.

Received 25.04.2017