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MODELING EVOLUTIONARY DYNAMICS OF COMPLEX ECOSYSTEMS USING COMBINED CHAOS THEORY AND NEURAL NETWORKS METHODS: I. FORMAL THEORETICAL BASIS FOR APPLICATION TO ENVIRONMENTAL RADIOACTIVITY DYNAMICS

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Abstract. We present elements of the formal mathematical approach to the analysis, modeling and further prediction of the nonlinear dynamics of chaotic systems based on the methods of nonlinear analysis and neural networks. As the object of studing is the environmental radioactivity dynamics. Using such a combined method is proposed for the first time in the environmental radioactivity dynamnics studying. Use of the information about the phase space in the simulation of the evolution of the physical process in time can be considered as a major innovation in the modeling of chaotic processes in the complex systems. This concept can be achieved by constructing a parameterized non-linear function $F(x, a)$, which transform $y(n)$ to $y(n+1) = F[y(n),a]$, and then use different criteria for determining the parameters a . Firstly to build the desired functions it is offered using the wavelet expansions. Further, since there is the notion of local neighborhoods, we can create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global non-linear model to describe most of the structure of the attractor.

Key words: environmental radioactivity dynamics, the ecological state, time series of concentrations, pollutants, analysis and prediction methods of the theory of chaos.

1. Introduction

One of the most actual and important problems of applied ecology and environment protection is associated with correct quantitative description of environmental radioactivity dynamics [1, 2]. In general, one should note the following actual problems: a long-term investigation of radionuclides behaviour in the environment; elucidation of the mechanism of radionuclides transfer in the environment by animals, through the food chain; elucidation of the mechanism of transformation and transportation of radioactive substances due to meteorological and hydrological phenomena and other factors; provision of a think-tank functioning for the recovery of the environment; conservation of research materials and samples and archiving of research methodologies and research objects and many others.

The problem of studying the dynamics of chaotic dynamical systems arises in many areas of science and technology. We are talking about classes of problems of identifying and estimating the

parameters of interaction between the sources of complex (chaotic) oscillations of the time series of experimentally observed values. Such problems arise in environmental sciences, such as geophysics, chemistry, biology, medicine, neuroscience, engineering, etc. [1–52]. The problem of analysis and forecasting of the impact of anthropogenic pressure on the state of atmosphere in an industrial city and development of consistent, adequate schemes for modeling the properties of concentration fields of air pollutions has been considered in details, for example, in Refs [12–22].

In this paper we present elements of the formal mathematical approach to the analysis, modeling and further prediction of the nonlinear dynamics of chaotic systems based on the methods of nonlinear analysis and neural networks. As the object of studing is the environmental radioactivity dynamics. The new element of our approach in comparison with the first version [2] is associated with the first application of the neural networks formalism [3] into problem of modeling of the environmental radioactivity.

Use of the information about the phase space in the simulation of the evolution of the physical process in time can be considered as a major innovation in the modeling of chaotic processes in the systems. Earlier it had been successfully realized in case of the atmosphere (air basin) of large industrial cities and some hydro-ecological systems (regions). Keeping in mind the listed problems and tasks here we present an improved generalized approach to the analysis and prediction of the nonlinear dynamics of chaotic systems based on the methods of nonlinear analysis and neural networks. As the object of study are the environmental radioactivity dynamics. Use of the information about the phase space in the simulation of the evolution of the physical process in time can be considered as a major innovation in the modeling of chaotic processes in the systems. This concept can be achieved by constructing a parameterized non-linear function $F(x, a)$, which transform $y(n)$ to $y(n+1) = F[y(n),a]$, and then use different criteria for determining the parameters a . Firstly to build the desired functions it is offered using the wavelet expansions. Further, since there is the notion of local neighborhoods, we can create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global non-linear model to describe most of the structure of the attractor.

Let us underline that earlier we have developed theoretical basis of studying the corresponding pollution dynamics in different ecological, hydrometeorological and hydroecological systems. Naturally, the problem concerns as different spatial as temporal scale levels [1–20]. As an example of problems whose solution lies in the problems considered in the article, it should be noted the analysis and prediction of the influence of anthropogenic impact on the water resources, river's systems and generally specking hydroecological systems. Earlier it has been considered in details (see [1, 14, 18–20]) a problem of using special mathematical technique for analysis and prediction of the influence of anthropogenic impact on the atmosphere of the industrial city , the development of adequate schemes modeling the properties of the fields of concentration of the air basin industrial city [10].

As the key blocks of the earlier developed approach remain the same in a case of environmental radioactivity dynamics below we are limited only by the key topics, focusing on features of environmental radioactivity.

2. Method of environmental analysis and prediction

2.1. Basic idea and construction of the model prediction for environmental radioactivity dynamics

The basic idea of the construction of our approach to prediction of chaotic properties of complex systems has been considered earlier (see, for example, Refs. [2, 13–24]) and following to these papers, it is in the use of the traditional concept of a compact geometric attractor in which evolves the measurement data, plus the implementation of neural network algorithms. Earlier this approach has been developed and used in problem of description of the dynamics of atmospheric systems such as air base in pollution pollution of industrial city. Here we consider the environmental radioactivity dynamics, more exactly, the corresponding radionuclide transfer. Shortly the analogous example has been considered in Ref. [21], namely, speech was about the dynamics of the air pollutants in an atmosphere of the industrial cities.

As the basis idea is remained the same, we shortly give it following to Ref. [2, 13, 18]. The meaning of the concept is in fact a study of the evolution of the attractor in the phase space of the system and, in a sense, modeling ("guessing") time-variable evolution.. From a mathematical point of

view, it is a fact that in the phase space of the system an orbit continuously rolled on itself due to the action of dissipative forces and the nonlinear part of the dynamics, so it is possible to stay in the neighborhood of any point of the orbit y (n) other points of the orbit y^r (n), r = 1, 2, ..., N_B, which come in the neighborhood y (n) in a completely different times than n. Of course, then one could try to build different types of interpolation functions that take into account all the neighborhoods of the phase space and at the same time explain how the neighborhood evolve from y (n) to a whole family of points about y (n+1).

Use of the information about the phase space in the simulation of the evolution of some geophysical (environmental, etc.) of the process in time can be regarded as a fundamental element in the simulation of random processes. In terms of the modern theory of neural systems, and neuro-informatics (e.g. [11]), the process of modeling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations) [3].

Imitating the further evolution of a complex system as the evolution of a neural network with the corresponding elements of the self-study, self-adaptation, etc., it becomes possible to significantly improve the prediction of evolutionary dynamics of a chaotic system. Considering the neural network (in this case, the appropriate term "geophysical" neural network) with a certain number of neurons, as usual, we can introduce the operators S_{ij} synaptic neuron to neuron u_i u_j, while the corresponding synaptic matrix is reduced to a numerical matrix strength of synaptic connections: $W = ||\ w_{ij}\ ||$. The operator is described by the standard activation neuro-equation determining the evolution of a neural network in time:

$$s_i^{'} = sign(\sum_{j=1}^{N} w_{ij} s_j - q_i), \qquad (1)$$

where $1 < i < N$. Here it is important for us another proven fact related to information behavior neuro-dynamical system. From the point of view of the theory of chaotic dynamical systems, the state of the neuron (the chaos-geometric interpretation of the forces of synaptic interactions, etc.) can be represented by currents in the phase space of the system and its the topological structure is obviously determined by the number and position of attractors. To determine the asymptotic behavior of the system it becomes crucial information aspect of the problem, namely, the fact of being the initial state to the basin of attraction of a particular attractor. Modeling each geophysical attractor by a record in memory, the process of the evolution of neural network, transition from the initial state to the (following) the final state is a model for the reconstruction of the full record of distorted information, or an associative model of pattern recognition is implemented. The domain of attraction of attractors are separated by separatrices or certain surfaces in the phase space. Their structure, of course, is quite complex, but mimics the chaotic properties of the studied object. Then, as usual, the next step is a natural construction parameterized nonlinear function $\mathbf{F}$ (x, $\boldsymbol{a}$), which transforms:

$$\mathbf{y}(n) \rightarrow \quad \mathbf{y}(n+1) = \mathbf{F}(\mathbf{y}(n), \mathbf{a}), \qquad (2)$$

and then to use the different (including neural network) criteria for determining the parameters a (see below). The easiest way to implement this program is in considering the original local neighborhood, enter the model(s) of the process occurring in the neighborhood, at the neighborhood and by combining together these local models, designing on a global nonlinear model. The latter describes most of the structure of the attractor.

Although, according to a classical theorem by Kolmogorov-Arnold -Moser, the dynamics evolves in a multidimensional space, the size and the structure of which is predetermined by the initial conditions, this, however, does not indicate a functional choice of model elements in full compliance with the source of random data. One of the most common forms of the local model is the model of the Schreiber type [3] (see also [10]).

Nonlinear modeling of chaotic processes is based on the concept of a compact geometric attractor, which evolve with measurements. Since the orbit is continually folded back on itself by the dissipative forces and the non-linear part of the dynamics, some orbit points $\mathbf{y}^r(k)$, r = 1, 2, …, N_B can be found in the neighbourhood of any orbit point $\mathbf{y}(k)$, at that the points $y^r(k)$ arrive in the neighbourhood of $\mathbf{y}(k)$ at quite different times than k. Then one could build the different types of interpolation functions that take into account all the neighborhoods of the phase space, and explain how these neighborhoods evolve from y (n) to a whole family of points about y (n + 1). Use of the information about the phase space in modeling the evolution of the physical process in time can be regarded as a major innovation in the modeling of

chaotic processes. This concept can be achieved by constructing a parameterized nonlinear function $F(x, a)$, which transform $y(n)$ to $y(n + 1) = F(y(n), a)$, and then using different criteria for determining the parameters a. Further, since there is the notion of local neighborhoods, one could create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global nonlinear model that describes most of the structure of the attractor.

As shown Schreiber [3], the most common form of the local model is very simple

$$s(n+\Delta n) = a_0^{(n)} + \sum_{j=1}^{d_A} a_j^{(n)} s(n-(j-1)\tau) \qquad (3)$$

where Δn – the time period for which a forecast has to be done. The coefficients $a_j^{(k)}$, may be determined by a least-squares procedure, involving only points $s(k)$ within a small neighbourhood around the reference point. Thus, the coefficients will vary throughout phase space. The fit procedure amounts to solving $(d_A + 1)$ linear equations for the $(d_A + 1)$ unknowns.

When fitting the parameters a, several problems are encountered that seem purely technical in the first place but are related to the nonlinear properties of the system. If the system is low-dimensional, the data that can be used for fitting will locally not span all the available dimensions but only a subspace, typically. Therefore, the linear system of equations to be solved for the fit will be ill conditioned.

However, in the presence of noise the equations are not formally ill-conditioned, but still the part of the solution (that relates the noise directions to the future point) is meaningless . Note that the method presented here is not only because, as noted above, the choice of fitting requires no knowledge of physics of the process itself. Other modeling techniques are described, for example, in [3, 10].

2.2. Wavelets for construction of model prediction for the environmental radioactivity dynamics

It is well known that the wavelets are fundamental building block functions, analogous to the sine and cosine functions [22]. Fourier transform extracts details from the signal frequency, but all information about the location of a particular frequency within the signal is lost. At the expense of their locality the wavelets have advantages over Fourier transform when non-stationary signals are analyzed [22–26]. Here, we use non-decimated wavelet transform that has temporal resolution at coarser scales.

The dilation and translation of the mother wavelet $\psi(t)$ generates the wavelet as follows

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k). \qquad (4)$$

The dilation parameter j controls how large the wavelet is, and the translation parameter k controls how the wavelet is shifted along the t-axis. For a suitably chosen mother wavelet $\psi(t)$, the set $\{\psi_{j,k}\}_{j,k}$ provides an orthogonal basis, and the function f which is defined on the whole real line can be expanded as

$$f(t) = \sum_{k=-\infty}^{\infty} c_{0k}\varphi_{0,k}(t) + \sum_{j=1}^{J}\sum_{k=-\infty}^{\infty} d_{jk}\psi_{j,k}(t), \qquad (5)$$

where the maximum scale J is determined by the number of data, the coefficients c_{0k} represent the lowest frequency smooth components, and the coefficients d_{jk} deliver information about the behavior of the function f concentrating on effects of scale around 2^{-j} near time $k \times 2^{-j}$. This wavelet expansion of a function is closely related to the discrete wavelet transform (DWT) of a signal observed at discrete points in time.

In practice, the length of the signal, say n, is finite and, for our study, the data are available monthly, i.e. the function $s(t)$ in Eq. (3) is now a vector $f = (f(t_1),\ldots, f(t_n))$ with $t_i = i/n$ and $i = 1,\ldots,n$. With these notations, the DWT of a vector f is simply a matrix product $d = Wf$, where d is an $n \times 1$ vector of discrete wavelet coefficients indexed by 2 integers, d_{jk}, and W is an orthogonal $n \times n$ matrix associated with the wavelet basis.

For computational reasons, it is simpler to perform the wavelet transform on time series of dyadic (power of 2) length. One particular problem with DWT is that, unlike the discrete Fourier transform, it is not translation invariant. This can lead to Gibbs-type phenomena and other artefacts in the reconstruction of a function. The non-decimated wavelet transform (NWT) of the data $(f(t_1), \ldots, f(t_n))$ at equally spaced points $t_i = i/n$ is defined as the set of all DWT's formed from the n possible shifts of the data by amounts i/n; $i = 1, \ldots, n$.

Thus, unlike the DWT, there are 2^j coefficients on the jth resolution level, there are n equally spaced wavelet coefficients in the NWT

$$d_{jk} = n^{-1}\sum_{i=1}^{n} 2^{j/2}\psi\left[2^j(i/n - k/n)\right]y_i, \qquad (6)$$
$$k = 0, \ldots, n-1,$$

on each resolution level j. This results in $\log_2(n)$ coefficients at each location. As an immediate consequence, the NWT becomes translation

invariant. Due to its structure, the NWT implies a finer sampling rate at all levels and thus provides a better exploratory tool for analyzing changes in the scale (frequency) behavior of the underlying signal in time. These advantages of the NWT over the DWT in time series analysis are demonstrated in [21]. As in the Fourier domain, it is important to assess the power of a signal at a given resolution. An evolutionary wavelet spectrum (EWS) quantifies the contribution to process variance at the scale j and time k. From the above paragraphs, it is easy to plot any time series into the wavelet domain. Another way of viewing the result of a NWT is to represent the temporal evolution of the data at a given scale. This type of representation is very useful to compare the temporal variation between different time series at given scale. To obtain the results, smooth signal S_0 and the detail signals D_j (j =1, …, J) are

$$S_0(t) = \sum_{k=-\infty}^{\infty} c_{0k}\varphi_{0,k}(t)$$

$$D_j(t) = \sum_{k=-\infty}^{\infty} d_{jk}\psi_{j,k}(t)\cdot \qquad (7)$$

The fine scale features (high frequency oscillations) are captured mainly by the fine scale detail components D_J and D_{J-1}. The coarse scale components S_0, D_1, and D_2 correspond to lower frequency oscillations of the signal. Note that each band is equivalent to a band-pass filter. Further one could use the Daubechies wavelet as mother wavelet. This wavelet is bi-orthogonal and supports discrete wavelet transform. Furthermore, formally the neural network algorithm is launched, in particular, in order to make training the neural network system equivalent to the reconstruction and interim forecast the state of the neural network (respectively, adjusting the values of the coefficients).

3. Illustrative example and conclussions

In order to present an illustrative example of application of the presented approach, we use our data of studying dynamics of the nitrates concentrations in the Small Carpathians watersheds in the Earthen Slovakia during 1969–1996 years. The physical and chemical features of any radionuclide isotopes transfer and temporal and spatial distribution are in many aspects analogous to considered example with nirates pollutans in the water systems. It explaines using such an example here as illustration. All calculations below are performed with using "Geomath" and "Quantum Chaos" computational codes [2–21, 52–86]. In Refs. [3, 5, 19] the chaotic behaviour in the nitrates and sulphates concentration time series in the watersheds of the Small Carpathians was investigated. In this refs. it can be found all detailed information about problem. To reconstruct the corresponding attractor, the time delay and embedding dimension are calculated by the methods of autocorrelation function and average mutual information, and the latter is calculated by means of correlation dimension method and algorithm of false nearest neighbours. It is shown that low-dimensional chaos exists in the time series under investigation. In figure 1 we present the original data (solid lines) and 8-month forecasts (dashed lines) for the nitrates concentrations in the watershed Ondava (Stropkov; Slovakia) concerning the time 1969–1996 years.

These results can be considered as the example of quite successful short-range forecast for the concentrations of watershed pollutants. Obviously, one should wait for the principally same situation in a case of transformation and transportation of radioactive substances due to meteorological and hydrological phenomena. The presented non-linear prediction method provides quite satisfactory results even in the case, when the concentrations are sharply rising; at least, all the tendencies to the rising were revealed by the method. In addition, we have used the simplest scheme for the approximation of local model and more complicated methodology can provide hopefully the better forecasts.

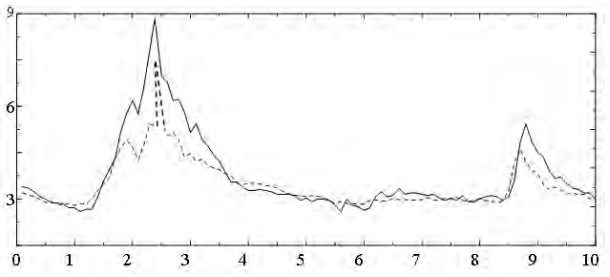

Fig. 1. Original data (solid lines) and 8-month forecasts (dashed lines) for the nitrates concentrations in the watershed Ondava (Stropkov; Slovakia) for period 1969-1996 (Axe X – the serial number of the term) (see text and ref. [3])

Therefore, we have presented the elements of a new approach to nonlinear modeling and prediction of chaotic processes with orientation on the environmental radioactivity dynamics. The approach is based on two key functional elements. Besides using other elements of starting chaos theory method

the proposed approach includes the application of the concept of a compact geometric attractor, and one of the neural network algorithms, or, in a more general definition of a model of artificial intelligence. The starting point is a formal knowledge of the time series of the main dynamic parameters of a chaotic system, and then to identify the state vector of the matrix of synaptic interactions $||w_{ij}||$ etc. The main difficulty here lies in the implementation of the process of learning neural network to simulate the complete process of change in the topological structure of the phase space of the system and use the output results of the neural network to adjust the coefficients of the function display. The meaning of the latter is precisely the application of neural network to simulate the evolution of the attractor in phase space, and training most neural network to predict (or rather, correct) the necessary coefficients of the parametric form of functional display. As alternative and addition simultaneously, one should use our proposal at first to use the wavelet expansion for construction of the parametrized model prediction functions. In any case these alternative should be checked at concrete modelling examples, namely, reproducing time environmental radioactivity dynamics in the concrete geospheres. The concrete such example of application of the presented approach will be listed in the next paper.

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