Solution Refinement Effectiveness of Multi-Grid Accelerated, Cartesian Grid Based Navier Stokes Solver on Compressible and Laminar Flows

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***Abstract* – The developed Cartesian-grid based GeULER-NS code is validated for compressible and laminar flows on subsonic viscous flow over a NACA 0012 airfoil. Solution refinement effectiveness and Roe's flux calculation versus AUSM are tested. Drag coefficient results, skin friction coefficient distribution and symmetric behavior of the distribution are compared with the results from milestone computational studies found in literature.**

Key words – Cartesian grid generation, compressible flow, laminar flow, multi-grid method, Navier Stokes solver, solution refinement.

I. Introduction

Cartesian grid based flow solvers are rapidly increasing phenomena using which some problematic issues such as shock wave capturing capability, boundary layer separation point identification etc. are accomplished in a computationally inexpensive sense. In this study, Navier Stokes equations are implemented and solved on Cartesian grids. Resultingly, a multi-grid accelerated Navier Stokes solver, namely cartesian-Grid-generator-with-eULER-and-Navier-Stokes-flow-solver (GeULER-NS) is developed for compressible and viscous (laminar) flows. Shock wave capturing capability is tested in the previous study of the authors [1]. Same solver with the addition of solution refinement codes is used here for solving a subsonic test case on NACA 0012 airfoil with zero angle-of-attack at a Mach number of 0.5 and a Reynolds number of 5000. Results are compared with verification cases from literature.

II. NACA 0012: M = 0.5, Re = 5000, A Subsonic Test Case

The test case is a frequently studied [2] subsonic viscous flow over a NACA 0012 case with zero angle-of-attack at a Mach number of 0.5 and a Reynolds number of 5000. Solution refinement effectiveness and Roe's flux calculation versus AUSM are tested around symmetrical NACA 0012 airfoil. Drag coefficient (c_D) results and skin friction coefficient (c_F) distributions are compared with the results from the milestone computational studies found in literature. Convergence histories are shown in Figure 1 for both flux calculation techniques, i.e. Roe's approximate Riemann solver and AUSM. The first one enables faster convergence rates, especially for three solution refinement, also converges to ten times lower residual exponent with respect to AUSM. The solution accuracy of GeULER-NS [1] was tested by comparing c_D as shown in Table 1.

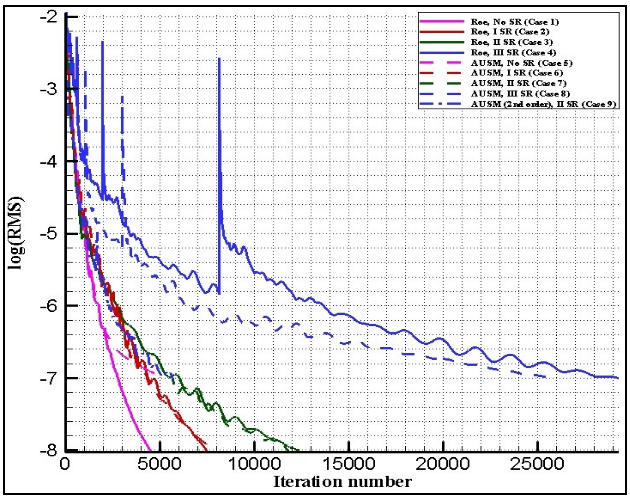

Fig. 1. Subsonic test case of NACA 0012: Convergence histories; $M_\infty = 0.5$, $\theta = 0.0°$, Re = 5000.

TABLE 1

COMPARISON OF THE GEULER-NS RESULTS WITH FIRST AND SECOND REFERENCES FOR SUBSONIC FLOW AROUND NACA 0012

Case no - (Scheme)	Ref. no	C_D	Cell no	Time
1 (Roe)	0	0.0524	4304	2 min. 43 sec.
2 (Roe)	1	0.0418	11684	16 min. 20 sec.
3 (Roe)	2	0.0345	34698	1 h. 9 min. 2 sec.
4 (Roe)	3	0.0303	129981	7 h. 15 min. 28 sec.
5 (AUSM)	0	0.0523	4304	3 min. 29 sec.
6 (AUSM)	1	0.0426	11684	10 min. 9 sec.
7 (AUSM)	2	0.0333	53143	1 h. 34 min. 32 sec.
8 (AUSM)	3	0.0302	140268	7 h. 24 min. 52 sec.
9 (AUSM)	2 (2nd order)	0.0324	53143	NA
- (SVS)	-	0.0321 [2]	40960	25 h. 25 min. 16 sec.
- (AUSMV)	6	0.0311 [3]	335606	2 min. 43 sec.

The aim is to obtain the closest c_D compared to the reference results of Gooch [2] and Şahin [3]. The reference drag coefficient results of Gooch [2] involve a structured grid generation based solver with finite volume, cell-centered method employing State-Vector-Splitting (SVS) scheme. The results of Şahin [3] involve a Cartesian grid generator based solver with finite volume, cell-centered method employing AUSMV. From now on, the reference results of Gooch [2] will be called shortly as first reference and the reference results of Şahin [3] will be called shortly as second reference. In cases where solution adaptation is not used, both methods overestimate c_D by as much as 69 % in comparison with first reference and 68.4 % in comparison with second reference. But, as the solution refinement numbers are increased, both methods give closer results to the reference data. Among first order solutions, the minimum overestimating percent is obtained from case 7, which is 3.7 % compared to first reference and 7.1 % compared to second reference. Second order solution scheme gives better results but takes more time in comparison with first order schemes. The reason for high computational time is the use of second order gradients and limiters for each cell in every iteration step. The computational time doubles but more accurate c_D output, 0.9 % in comparison with first reference, is accomplished which encourages the use of high order schemes in convenient cases.

Comparing Figure 2.a) and 2.b), it can be seen that case 4 gives the most accurate pressure coefficient (c_P) distribution results with respect to Gooch [2], though in the trailing edge of NACA 0012 case 4 slightly overestimates c_P distribution.

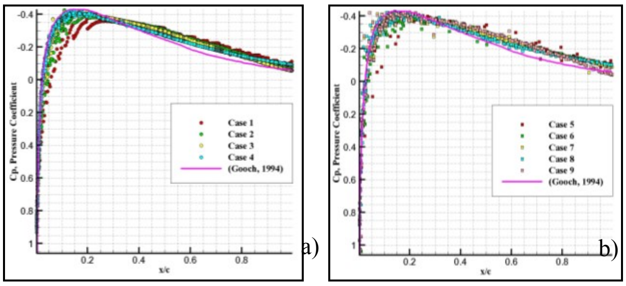

Fig. 2. GeULER-NS results of c_P along the surface of NACA0012 airfoil by using (a) Roe's approximate Riemann solver and (b) AUSM; $M_\infty = 0.5$, $\theta = 0.0°$, $Re = 5000$

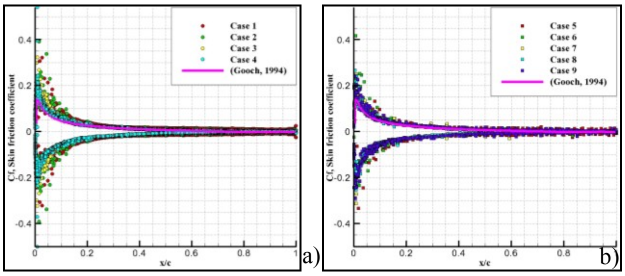

Fig. 3. GeULER-NS results of c_F distributions along the surface of NACA0012 airfoil by using (a) Roe's approximate Riemann solver and (b) AUSM; $M_\infty = 0.5$, $\theta = 0.0°$, $Re = 5000$

In Figure 3, the symmetric behavior of skin friction coefficient, c_F, distribution about the camber line of NACA 0012 airfoil is presented for all cases. The GeULER-NS results are compared with Gooch's studies [2]. The lowest peak of c_F near the leading edge is predicted by AUSM results. Among them, as seen in Figure 3.b), the closest scatter plot with respect to Gooch [2] solid line representation is case 9, second order scheme with three solution refinement using AUSM. In Figure 4, pressure and Mach number contours around the airfoil are compared with Gooch [2]. Similarities between these figures can be easily seen. The close-up view near the trailing edge of the airfoil shows the symmetric separation region with wake vortices in Figure 4.c).

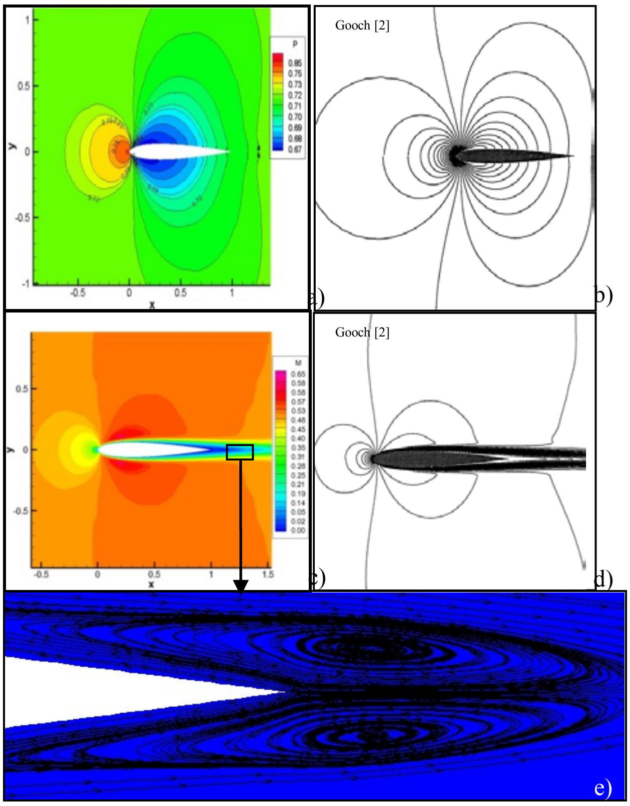

Fig. 4. Pressure contours of (a) case 9, (b) Gooch [2]; Mach contours of (c) case 9, (d) Gooch [2]; (e) close-up view of separation region streamlines around trailing edge of NACA 0012; $M_\infty = 0.5$, $\theta = 0.0°$, $Re = 5000$

Conclusion

In this study, a developed Navier Stokes solver is tested for its solution refinement effectiveness on a subsonic, laminar flow test case using Cartesian grid techniques and multi-grid approach. Performance of the GeULER-NS solver fulfilled the expections by accurately eliciting the drag coefficient results, pressure coefficient and skin friction coefficient distributions. At the end of the study, the symmetric behavior of friction distribution about the camber line of the airfoil is presented and a close-up view of separation region streamlines around the trailing edge is successfully captured. As the future work, solution refinement effectiveness of the GeULER-NS solver will be enhanced for three-dimensional studies.

References

[1] E. Kara, A. İ. Kutlar and M. H. Aksel, "Shock Wave Capturing with Multi-grid Accelerated, Solution Adaptive, Cartesian Grid Based Navier Stokes Solver," *Journal of Aeronautics and Space Technologies*, vol. 9, no. 2, July 2016, pp. 63-73.

[2] C. F. Gooch, "Solution of the Navier-Stokes equations on Locally Refined Cartesian Meshes using State-vector Splitting," Ph.D. dissertation, Department of Aeronautics and Astronautics, Stanford University, Palo Alto, CA, USA, 1994.

[3] M. S. Sahin, "Development of a Two-dimensional Navier-Stokes Solver for Laminar Flows using Cartesian Grids," M.S. Thesis, Mechanical Engineering Department, Middle East Technical University, Ankara, TURKEY, 2011.