Development of Mathematical Model of Controlled Plant Using the Obtained Experimental Data

Roman Fedoryshyn, Sviatoslav Klos, Volodymyr Savytskyi

Department of Automation of Heat and Chemical Processes, Lviv Polytechnic National University, UKRAINE, Lviv, S. Bandery street 12, E-mail: romanfedoryshyn@yahoo.com

***Abstract* – The results of the experimental study of the dynamical and static properties of an electric oven as a controlled plant are presented in the paper. Up-to-date microprocessor instruments were applied during the experiments. Mathematical model of the electric oven was developed on the basis of the experimental data. The adequacy of the model was verified.**

Key words – controlled plant, electric oven, mathematical model, step response curve, simulation.

I. Introduction

Development of a mathematical model is an important step on the way of a controlled plant investigation. The model provides the possibility of studying the behavior of the plant at disturbances of various types and of various values by modeling without making the additional experiments. The mathematical model enables also the tuning parameters of an automatic controller to be designed in order to provide the needed quality of the transient processes in the closed loop control system.

The goal of this paper is to present the results of experimental study of an electric oven as a controlled plant and the developed mathematical model of the oven on the basis of the obtained experimental data.

II. Experimental Study

The experimental facility consisted of a laboratory electric oven (СУОЛ-0,15.2/12М), air blower (ВК-2) for blowing the air through the oven and a resistance thermometer (Pt100 of Techprylad company) installed inside the oven. The thermometer was connected according to a three-wire diagram to a programmable logic controller МІК-51Н of Microl company. The output signal of the thermometer was recorded by means of a personal computer with application of specialized software for data logging (МІК-registrator 1.1.14). A variable autotransformer (РНО-250-2) was used to change the electric power at the input of the oven. The secondary connection of the transformer was rotated by means of an electric actuator with a position meter (КДУ-1). A position indicator (ДУП-К) was used for defining the position of the electric actuator.

Fifteen step response curves were recorded during the experimental study. Thirteen curves were obtained for the input control channel by making a step change of the electric power at the input of the oven. Two curves were obtained for the disturbance channel by making a step change of the air flowrate through the oven (by changing the position of the butterfly valve at the output of the air blower).

III. Mathematical Model

The mathematical model of the electric oven was developed in the form of static characteristics equations, transfer functions and a nonlinear differential equation.

The static characteristics equations were developed on the basis of the experimental data for the input control channel (1) and for the disturbance channel (2):

$$\theta = 0.5446 \cdot \mu_{VATR} + 32.5359, \quad (1)$$

$$\theta = -0.2320 \cdot \mu_{valve} + 68.7000, \quad (2)$$

where θ is the output signal of the thermometer (°C); μ_{VATR} is the position of the actuator by means of which the secondary connection of the variable autotransformer was rotated (%); μ_{valve} is the position of the butterfly valve at the output of the air blower (%).

The maximum relative error of the calculated values of the static characteristics according to the equation (1) is 5 % in the range of μ_{VATR} from 5 to 80 % at μ_{valve} = 100 %. The maximum relative error for the equation (2) is 0.5 % in the range of μ_{valve} from 50 to 100 % at μ_{VATR} = 24 %.

Based on the obtained experimental step response curves for the input control channel an averaged normalized step response curve was built. And it was approximated by a theoretical curve. The dynamics of the transient processes is defined by the inertia of the electric oven itself and by the inertia of the thermometer. Since the controlled plant is a dual-capacity object, its behavior can be described by the second order lag element. The following transfer function was accepted for building the theoretical step response curve

$$W_{CP}^{x}(p) = \frac{k_x}{T_{2x}^2 p^2 + T_{1x} p + 1}, \quad (3)$$

where $W^{x}_{CP}(p)$ is the transfer function of the controlled plant; p is the Laplace operator; k_x is the proportional gain of the controlled plant; T_{1x}, T_{2x} are the time constants of the controlled plant.

The approximation of the experimental step response curve was made by the following three methods: graphic-analytical, numerical and combined method. According to the graphic-analytical method an inflection point should be found on the curve and a tangent line should be built through this point. The values of the time constants are calculated using the length of some segments and the area between the curve and the asymptote. The graphic-analytical method of defining the time constants on the basis of the step response curves is described in [1].

The numerical method of defining the time constants consists in application of "fminsearch" function of Matlab software. "Fminsearch" finds the minimum of a multi-parameter goal function using the Nelder-Mead simplex method. In our case the goal function was the sum of squares of deviations of the theoretical curve from the experimental one. And the time constants T_{1x} and T_{2x} were the parameters defined by "fminsearch". The results of the time constants definition by various methods are presented in Table 1. There are also the values of maximum reduced errors for each method in the table.

As we can see from Table 1 the smallest error is provided by the numerical method. However, the dual-capacity plant is approximated by a first-order lag element since the time constant T_{2x} is approximately equal to zero in this method. The biggest error is given by the graphic-analytical method. But this method provides a good description of the dual-capacity plant behavior. There is an inflection point on the theoretical curve at the same time moment as on the experimental curve (see Fig.1, *a*).

TABLE 1

RESULTS OF TIME CONSTANTS DEFINITION

Method	T_{1x}, s	T_{2x}, s	δ_{max}, %
Graphic-analytical	352.827	112.238	14
Numerical	524.77	$3.2 \cdot 10^{-5}$	5
Combined	524.77	112.238	7

It is proposed to apply the combined method of the time constants definition for the controlled plant under consideration. According to this method the time constant T_{1x} is taken from the numerical method and T_{2x} from the graphic-analytical one. The combined method provides an adequate description of the dual-capacity plant behavior (see Fig. 1, *b*) with an acceptable error.

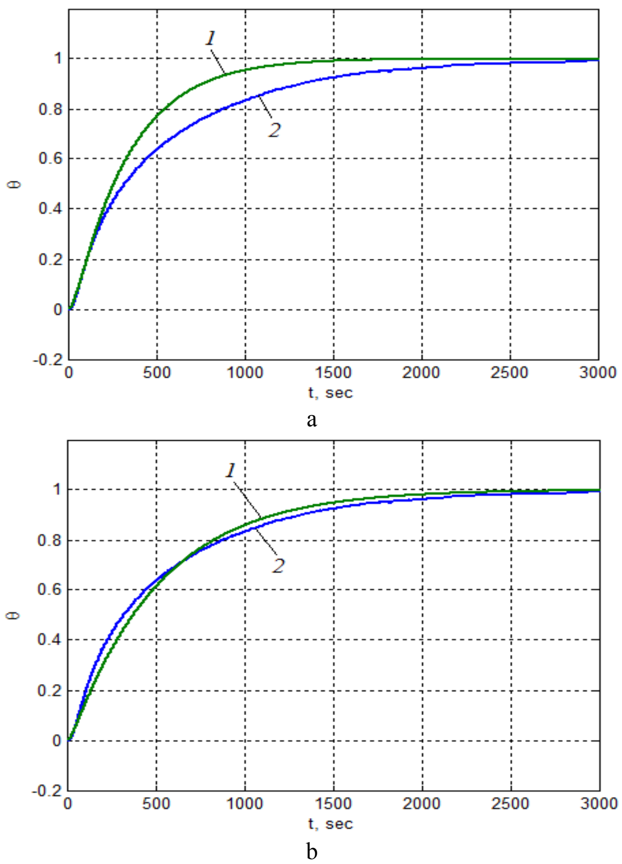

Fig. 1. Comparison of the theoretical (*1*) and the experimental (*2*) normalized step response curves: *a* – graphic-analytical method; *b* – combined method.

Trasfer function (3) can also be presented in the following way

$$W_{CP}^{x}(p) = \frac{k_x}{(a_x p + 1)} \cdot \frac{1}{(b_x p + 1)}. \quad (4)$$

In this case the time constants are as follows: $a_x = 499.55$ s $b_x = 25.22$ s.

Based on the obtained experimental step response curves for the disturbance channel it was defined that the rate of temperature signal change during heaing is different for that during cooling. In order to take into account this kind of nonlinearity the mathematical model of the controlled plant for the disturbance channel was developed in two stages.

At the first stage an averaged normalized step response curve was built. And it was approximated by a theoretical curve using the same methodology as for the input control channel (graphic-analytical, numerical and combined method). However, none of the methods provided an acceptable error of approximation. That is why the structure of the transfer function was modified and a correction element was introduced

$$W_{CP}^{z}(p) = \frac{k_z}{(a_z p + 1)} \cdot \frac{1}{(b_z p + 1)} - \frac{k_k p}{(a_k p + 1)} \cdot \frac{e^{-\tau p}}{(b_k p + 1)}. \quad (5)$$

At the second stage the transfer function of the first term's first multiplier in (5) was replaced by the following non-linear differential equation

$$a_z \cdot \frac{dy_1}{dt} + (y_1)^n = c \cdot k_z \cdot z. \quad (6)$$

By means of this non-linear equation it was provided that the rate of temperature signal change during heaing and cooling is different. The obtained parameters of the model for the disturbance channel are as follows: $a_z = 90.46$ s $b_z = 10.91$ s, $k_k = -37.8464$ °C/%, $\tau = 35$ s, $a_k = b_k = 282.66$ s, $n = 0.8494$, $c = 0.6913$. Maximum reduced error of the model for heating is 9 % and for cooling it is 6 %.

The developed mathematical model was implemented in a PLC (MIK-51H) in order to simulate the controlled plant (electric oven under investigation). The simulated transient processes were compared to the experimental ones and the adequacy of the model was proved.

Conclusion

The mathematical model of the electric oven was developed on the basis of the experimental and theoretical research. The model consists of the static characteristics equations, transfer functions and a nonlinear differential equation. The relative error of the calculated points of the static characteristics based on the developed equation does not exceed 5 %. The maximum reduced error of the theoretical step response curve for the input control channel (change of the electric power at the input of the oven) is 7 %. For the disturbance channel (change of the air flowrate through the oven) this error is 9 % for heating and 6 % for cooling.

Thus we conclude that the developed mathematical model of the electric oven is adequate and can be applied at practice for studying the transient processes and for designing the tuning parameters of an automatic controller.

References

[1] Plaskowski A. Experimental definition of the controlled plant dynamical properties. Science and Engineering Publishing House, Warsaw, 1965, 177 p. (published in Polish).