

# Mathematical Modeling of Linear Integration Based on Reverse Formulas

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**Abstract** – In the research work the accuracy of calculation of transient processes based on mathematical models generated using two multi-implicit methods, formulas of back-differentiation and back-integration. Object of study - line 750 kV "Zachidnoukrainsk - Vinnytsia" for which were formed mathematical models based on analytical solution of equations of the transition process, the method of back-differentiation formulas and back-integration method formulas that have been implemented in the software complex MathCad. Research transients when switched on unloaded line system (source EDS) showed significantly higher accuracy of results obtained using the method of back-integration formulas ago, compared with the method of back-differentiation formulas.

Key words – line, modeling, mathematical model, analytical method, a method of differentiation formulas back, method formulas integrating back, power system.

## I. Introduction

Power System (EPS) is composed of a set of power plants that are interconnected including via lines. The system can be the transition from one power system to another steady mode. This process is called transients. In order to minimize the impact thereof on the electric power system installed appropriate security devices. This requires constructing mathematical models of electric power system elements and bringing them to automatic calculation of the minimum amount of time, therefore, for the investigated object build mathematical models based on analytical method (AM), differentiation formulas back (FDN) and methods formulas of integrating back (FIN) and examine the adequacy of the mathematical model and methods FDN and FIN by comparison with results obtained analytical method.

FDN method refers to numerous multi-implicit methods for solving differential equations and described in [1].

For numerical solution of the equations of transients was developed implicit multistep method FIN, described in [2].

In the method, FIN, after the integration equations, integral changes in expression

$$\int_{t_k}^{t_{k+1}} x dt = h \cdot \sum_{i=0}^p \beta_i \cdot x_{k+1-i}$$

where  $\beta_i$  – are constant coefficients.

## II. Characteristics of the research object

Object is a 750 kV line between substation (SS) "Western Ukrainian" (C1 system) and substation (SS) "Vinnytsia" (C2 system) circuits which are shown in Fig. 1.

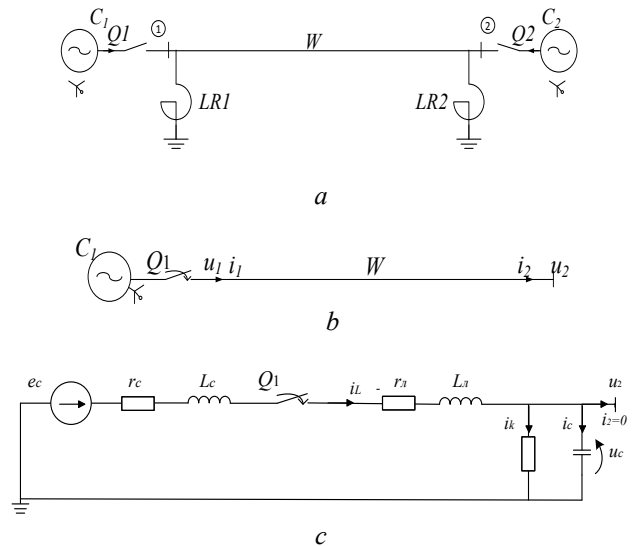


Fig. 1. Scheme of the investigated lines:  
a) principle; b) investigated the principal;  
c) equivalent circuit

Consider mode switch on switch Q1 unloaded line C1 of the system, while the switch Q2 – off, that is, off course lines. Shunt reactors LR1 and LR2 of the systems C1 and C2 respectively off.

Based on these analytical formulas and methods for given real parameters of line integration step  $h = 0.4$  ms that 50 points for the period of the fundamental frequency, calculate the current line at the beginning and end of line voltage.

To avoid errors of different values for both methods FIN and FDN and early settlement, current  $i_L(t)$  and voltage  $u_c(t)$  values in the first two points for  $t_0$  and  $t_1$  precise and calculated by analytical formulas.

## III. Results

Results are given for phase voltages in [V] the currents in [A].

### Experiment 1 ( $h = 0.4$ ms)

Results calculate currents and voltages are shown in Figs. 2, 3.

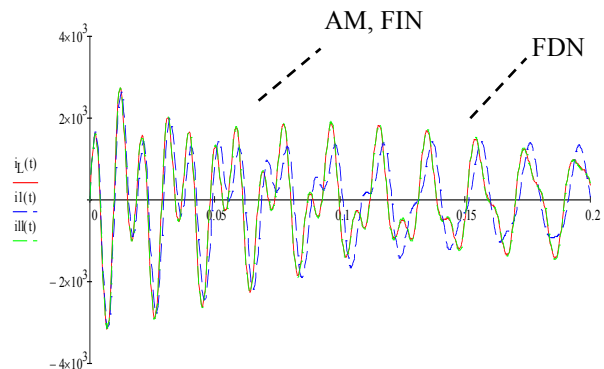


Fig. 2. The current  $i_L$  at the beginning of the line:  
 $i_L(t)$  – analytical method (AM);  
 $i_L(t)$  – method FDN;  $i_L(t)$  – method FIN

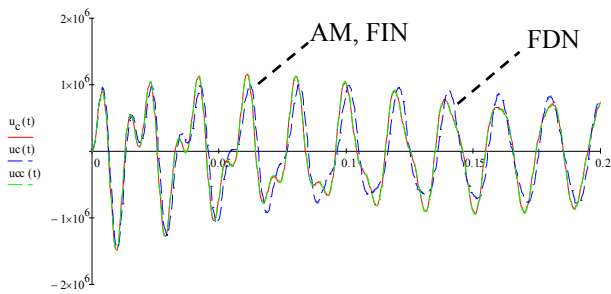


Fig. 3. The voltage  $u_2$  at the end of the line:  
 $u_c(t)$  – analytical method (AM);  
 $uc(t)$  – method FDN;  $ucc(t)$  – method FIN

Results are alternating character. From Fig. 2 and 3 shows that the method is relatively FDN analytical solution has a significant error of. To assess the accuracy of the results calculated absolute error methods FDN and FIN as the difference between these methods and the values obtained analytical method.

**Experiment 2** ( $h' = 0.02$  ms)

To the method FDN get approximately the same accuracy as the FIN ( $h = 0.4$  ms) needed to reduce integration step 20 times, that is to  $h' = 0.02$  ms.

Results calculate currents and voltages in increments  $h'$  shown in Figs 4, 5.

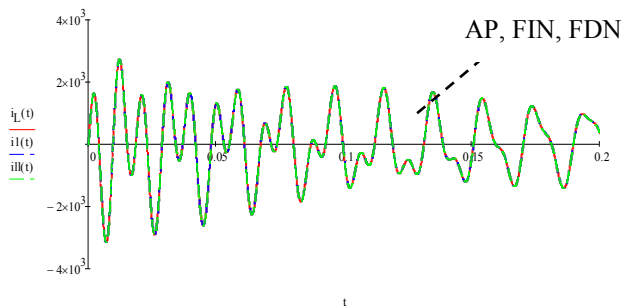


Fig. 4. The current  $i_L$  at the beginning of the line:  
 $i_L(t)$  – analytical method (AM);  
 $i1(t)$  – method FDN;  $i11(t)$  – method FIN

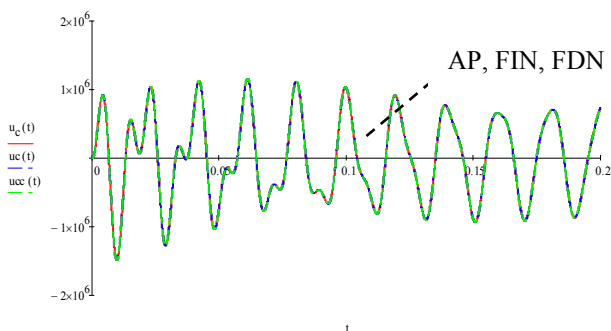


Fig. 5. The voltage  $u_2$  at the end of the line:  
 $u_c(t)$  – analytical method (AM);  
 $uc(t)$  – method FDN;  $ucc(t)$  – method FIN

From Figs 4, 5 shows the results for the examined three graphs visually practically coincided. To quantify the accuracy error calculated similarly as in the first experiment.

The analysis shows that under these conditions error method FDN ( $h' = 0.02$  ms) had decreased and the same order as the method of FIN  $h = 0.4$  ms.

This error of method of FIN step  $h' = 0.02$  ms decreased approximately three orders of magnitude, compared with an error of step  $h = 0.4$  ms.

That same steps for integration FIN method provides results with higher accuracy than the method of FDN.

## Conclusion

Comparing the results of computer simulation of mathematical models developed formulas based on methods of integrating back (FIN) and back differentiation formulas (FDN), carried out by determining appropriate values of absolute coordinates mode errors regarding the results obtained by accurate analytical formulas and showed a:

- absolute error results in both methods have sustained oscillation;
- value absolute error method FDN has the same order as the coordinates of the regime that caused significant phase shift with respect to the exact;
- values of absolute errors for FIN by two orders of magnitude smaller than the coordinates and visually mode phase shift is observed with respect to accurate results, because practically no phase shift to the exact results;
- value absolute error method FDN exceed the errors in methods FIN over procedure;
- to obtain results by FDN with the same accuracy as the method of FIN necessary step in the method of integrating FDN take 20 times less than in the method of FIN;
- if the method also adopt FIN step 20 times smaller, the accuracy of the results increases as the absolute error decreases by more than two orders of magnitude.

The research confirmed that the application of FIN method for developing mathematical models to significantly increase the accuracy and adequacy of the results of computer simulation method compared with the use FDN.

## References

- [1] Perhach V. S., “Matematicheskie zadachi e'lektro-energetiki”, [“Mathematical problems of electroenergy”], High School Publ., 1982, 380 pages.
- [2] UDC 621.3: 517,513: 518.5. AHM. Lysyak. Implicit multistep method formulas backward integration for the study of transients of electrical systems. – Technical Electrical №5, 2002.