Presentation of the Selenoid Mass Distribution Based on the Impact of the Ellipsoidal Shape and Rotating Component

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Abstract – This research is intended to demonstrate influence of ellipsoidal shape and rotating component for gravitational potential and mass distribution of the Moon. As are well known, gravitational potential of the planets formed by two components – gravitational potential and rotation. For ellipsoidal-shaped body effect of these components is different. Obviously, expansion ellipticity of the planet will raise radius vector for second component. Issues of the contribution of each of the components.

Key words – ellipticity, gravitational potential, three layered planet, gravitation field

I. Introduction

The problem of studying the internal structure of planets is a fundamental and crucial for understanding creation and development of the solar system bodies. Undoubtedly, the main physical characteristics of heterogeneity bowels of the planet are density, since it generates a gravitational field and has a significant impact on other fields [2]. Over the past twenty years there has been significant progress in obtaining new results suitable for studying gravitational field and internal structure of the Moon. The Moon is a key to deciphering the evolutionary history of the terries-trial planets because it is the most accessible planetary body that pre-serves a surface record spanning most of solar system history [3].

II. Potential of the triaxial ellipsoid

Potential of the homogeneous triaxial ellipsoid $\tau\left\{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1\right\}$ is defined as:

$$V=\frac{3}{4}fV_{\tau}\delta\int_{0,\xi}^{\infty}\left(1-\frac{x^2}{a^2+u}-\frac{y^2}{b^2+u}-\frac{z^2}{c^2u}\right)\frac{du}{Q(u)} \qquad (1)$$

where:

V_{τ} - volume of ellipsoid τ,

δ - density of mass distribution($\delta = const$),

ξ - ellipsoidal coordinate (the largest root of cubic equation [1]):

$$\frac{x^2}{a^2+\xi}+\frac{y^2}{b^2+\xi}+\frac{z^2}{c^2+\xi}=1 \qquad (2)$$

To determine the gravitational potential consider various cases:

- Internal potential $W_{\tau_i}^{b}\left(\delta_j\right)$ for ellipsoid defines as[1]:

$$W_{\tau_i}^{b}(\delta_j)=\frac{3fV_{\tau}}{4}\delta_i-\left[\rho_i^2M_{00}(0)-\rho^2\left[M_{10}(0)\sin^2\theta+M_{01}(0)\cos^2\theta\right]\right]$$

$$P\in\tau_i \qquad (3)$$

- External potential for ellipsoid:

$$W_{\tau_i}^{z}=\frac{3}{4}fV_{\tau}\delta_i\rho_i^3\left\{M_{00}\left(\frac{\varsigma}{\rho_i^2}\right)-\rho^2\left[M_{10}\left(\frac{\varsigma}{\rho_i^2}\right)\sin^2\theta+M_{01}\left(\frac{\varsigma}{\rho_i^2}\right)\right]\right\}$$

$$\tau_i\left(P\notin\tau_i\right) \qquad (4)$$

- Potential $W_{G_i}^{P}$ in ellipsoidal cavity segment $G_i\left(\rho_{i-1}\le\rho\le\rho_i\right)$

$$W_{G_i}^{P}=\frac{3fV_{\tau}}{4}\delta_i\left[\rho_i^2M_{00}(0)-\rho^2[M_{10}(0)\sin^2\theta+M_{01}(0)\cos^2\theta]\right] \qquad (5)$$

Formulas (3)-(5) allow determining the gravitational potential and its radial derivatives at any point in space

TABLE 1

BASIC PARAMETERS A THREE-LAYERED MOON

Mean density δ_c g/cm^3		3.3464
Volumetric mean radius $R\cdot 10^3$ *м*		1738
Core radius $R_я\cdot 10^3$ *м*		340
Mantle radius $R_м\cdot 10^3$ *м*		1678
Density value g/cm^3	In core	5.22
	In mantle	3.38
	In crust	2.92

As we can see in TABLE 1 density value of core, mantle and crust of the Moon considering depth of occurrence changes quite uniformly. But transitions core-mantle and mantle-crust changes quite sharply.

TABLE 2

THE PARAMETERS THAT CHARACTERIZE THE ELLIPTICITY OF THE MOON

Gravitational constant *GM* $\times 10^{14}\ \frac{м^2}{c^2}$	0.049028 [4]
Equatorial radius $a_e\times 10^3$ *м*	1738.1 [4]
Moment of inertia $\alpha=\frac{A-C}{A}$	0.00039 [4]
Rotation speed $\omega \times 10^{-5}\ \frac{rad}{s}$	0.26611 [4]

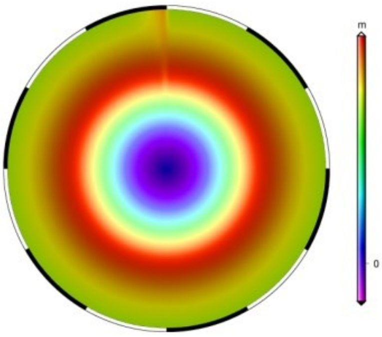

Fig. 1 The potential difference ellipsoidal and spherical Moon

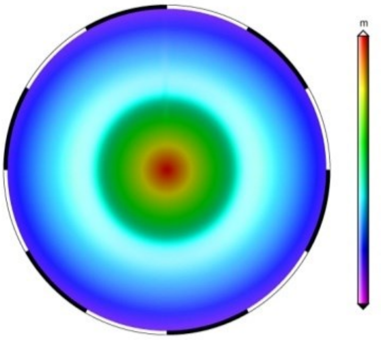

Fig.2 The difference between derivative potential of the ellipsoidal and spherical Moon

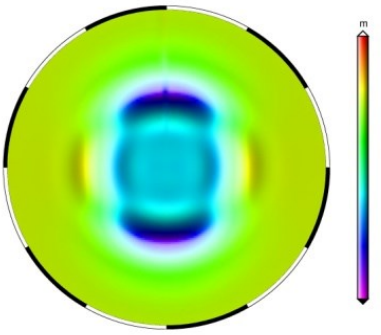

Fig. 3 The difference between the second derivative of potential of the ellipsoidal and spherical Moon

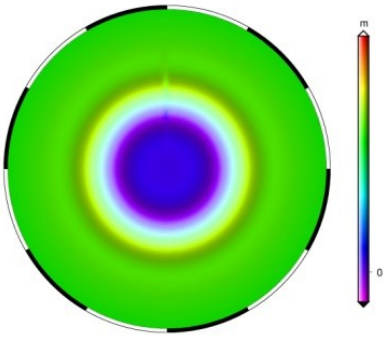

Fig. 4 The difference between derivative potential of the ellipsoidal and spherical Moon and the rotation of the Moon

Conclusion

This figure clearly shows characteristics of the Moons potential and its derivatives. Transitions core-mantle and crust-mantle are distinctly traceable. The difference between the values of the potential in spherical and ellipsoidal cases Moon is quite small, but it is necessary to be considered for precise modeling.

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