Models of representation of selenopotentials by spherical functions and using point masses

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Abstract – there are many approaches to studying the gravitational field of the moon, and depending on the method chosen and the input information, each of them has certain advantages.

Key words – gravitational potential, gravitation field, selenopotential, Moon, modelling.

I. Introduction

The complicated gravitational field of the Moon is the result of the heterogeneity of the density of the internal structure of the moon and its figures. In addition to the expansion of selenopotentials in a series of spherical functions, other methods of representing the gravitational field of the moon are widely used. One of these methods is representation using discrete models – point masses or disks.

II. Presenting main material

The application of the selenopotential to a schedule in a series of spherical functions has become widespread. In this case, the form of the entry will look like this [1]:

$$\Delta g = \left(\frac{GM_{(}}{R^2}\right)\sum_{n=3}^{n}(n-1)\left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n}\left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\cos\vartheta) \qquad (1)$$

r, θ, λ – spherical coordinates; GM – Moon gravitational constant; R(– mean Moon radius; r – distance from the origin to an arbitrary point of space; n – model degree,

Cnm and Snm- fully normalized spherical harmonics,

Pnm(cosϑ) – fully normalized associated Legendre functions.

Along with the expansion of the selenium potential in a series of spherical functions, it is convenient to use a model of point masses to display local gravitational anomalies of short lengths but large in magnitude. We define the capacity definition in the following way

$$V = GM\sum_{1}^{n}\frac{m_i}{r_i} \qquad (2)$$

m_i – mass in units M(, r_i – distance to point mass.

From formula (2) we find a formula for determining gravitational anomalies

$$\Delta g = \frac{\partial V}{\partial r} \qquad (3)$$

Receiving partial derivatives, we get

$$\Delta g_R = \frac{GM}{R}\sum_{1}^{n}\left(\frac{m_i(r-d_i\cos\psi)}{(d_i^2+r^2-2d_i r\cos\psi)^{3/2}}\right) \qquad (4)$$

$$\cos\psi = \cos\vartheta\cos\vartheta_i\cos(\lambda-\lambda_i) \qquad (5)$$

It should be remembered that the total mass of the moon remains unchanged, the sum of the quantities of abnormal masses must be equal to zero. [2]

When determining the magnitudes of gravitational anomalies, we will accept the mean radius of the moon R=1737,6 km. [3]

TABLE 1

PARAMETERS OF MODEL

	μi	di	ϑi	λi
1	1.622399	0.8	51.89999	342
2	0.7166	0.8	46.39999	77.7
3	-1.5189	0.7654	59.89999	2.7
4	-0.5405	0.8087	39.09999	70.89999
5	0.3831	0.8153	93.09999	118.6
6	0.2139	0.8954	75.09999	226.6
7	0.9928	0.7366	95.89999	173.9
8	-0.1889	0.8459	119.2	78.5
9	-1.6884	0.6457	45	337.3999
10	0.29971	0.77	109.7	26
11	-0.38289	0.7724	78.79999	35.7
12	0.28328	0.8076	4.299999	150
13	-0.07552	0.8735	77	96.5
14	1.626699	0.8	63.39999	13.7
15	-0.4453	0.8	73.2	17
16	0.5914	0.7757	130.8	275.8999
17	-0.4093	0.8211	109.1	261.3999
18	0.6445	0.7995	103.2	232
19	-0.8094	0.7686	123.6	196.8
20	-0.4516	0.8181	84.5	147.9
21	-0.6546	0.7164	80.59999	195.7
22	-0.0846	0.9253	127.9	331.2998
23	-0.15868	0.8583	28.7	229
24	0.22094	0.8026	170.9	323.2
25	-0.18684	0.8263	102	350

di – distance to point mass in units R(,

μi – mass in units M(,

ϑi – polar distance,

λi – selenographic longitude.

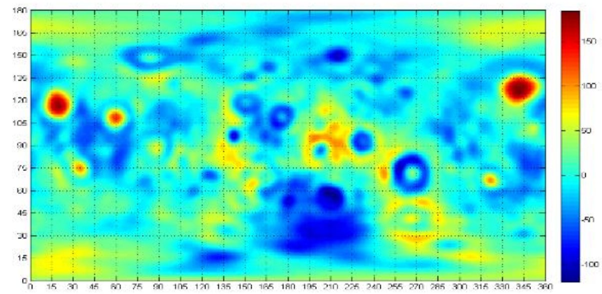

Fig.1 Anomalies of gravity Δg at an altitude of 100 km, calculated by the expansion in a series of spherical functions using model values Cnm , Snm

Spherical functions are their own functions for the sphere, and their widespread use is due to the slight deviation of the surface of the planets of the spherical. This form of presentation is quite convenient in constructing theories of the motion of artificial satellites of the Moon and for studying the planetary features of the gravitational field structure.

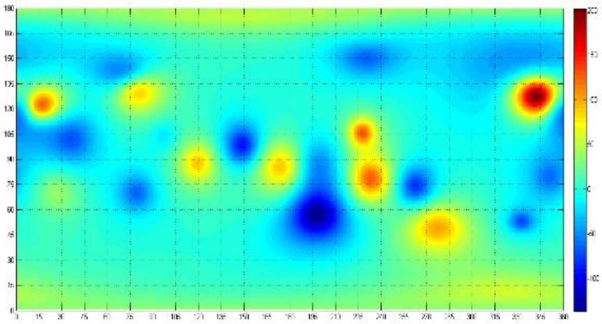

Fig.2 Anomalies of gravity Δg at an altitude of 100 km, calculated using a model of point masses

When solving certain problems due to the features of the gravitational field of the moon, the representation of the potential by a set of point masses may be more efficient and convenient. This method is especially convenient when describing the gravitational field of separate regions of the moon, and the ones where maskons are located.

Conclusion

In this paper we describe the gravitational field of the Moon by means of determining the gravitational anomalies by a decomposition in a row with spherical functions, as well as constructing a model of point masses. The analysis of feasibility of using one or another method of representation of selenopotentials depending on the tasks set.

References

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