

Fuzzy-Chance Constrained Economic Order Quantity Models

Olha Yegorova

Information Technology Design Department, Cherkasy State Technological University, UKRAINE, Cherkasy, T. Shevchenko Avenue, 460, E-mail: yegorovaov@gmail.com

Abstract – In this thesis fuzzy economic order quantity models with constraints on chances was offered. Hybrid intelligent algorithm based on fuzzy simulation, neural network and compositional technique overcoming uncertainty to solve hybrid programming models was considered. The proposed models and method form the methodological basis for decision-making processes optimization in forecasting stocking effects and preventing losses.

Key words – inventory, deteriorating items, misclassification errors, trade credit, directed optimization

I. Introduction

The development of the commodity market at the present level of the economy Ukraine constrained by low levels of implementation and use of modern methods of movement of material flow and inventory control systems in logistics.

One of the most effective ways to address these problems should consider improving mathematical models of inventory management.

II. Preliminaries

Definition 1 [1]. Let Θ be a nonempty set, and \mathcal{A} a σ -algebra of subsets (called events) of Θ . The set function \Pr is called a probability measure if

Axiom 1. (Normality) $\Pr\{\Theta\} = 1$;

Axiom 2. (Nonnegativity) $\Pr\{A\} \geq 0$ for any $A \in \mathcal{A}$;

Axiom 3. (Countable Additivity) For every countable sequence of mutually disjoint events $\{A_i\}$, we have

$$\Pr\left\{\bigcup_{i=1}^{\infty} A_i\right\} = \sum_{i=1}^{\infty} \Pr\{A_i\}. \quad (1)$$

Definition 2 [1]. Let Θ be a nonempty set, \mathcal{A} a σ -algebra of subsets of Θ , and \Pr a probability measure. Then the triplet $(\Theta, \mathcal{A}, \Pr)$ is called a probability space.

Definition 3. Let ξ be a fuzzy variable with membership function $\mu(x)$. Then the possibility measure of the fuzzy event $\xi \geq s$ can be represent by

$$\text{Pos}\{\xi \geq s\} = \sup_{x \geq s} \mu(x). \quad (2)$$

Definition 4 [1]. Let $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ be a fuzzy-probability vector on probability space $(\Theta, \mathcal{A}, \Pr)$ and $f_j: \mathfrak{R}^n \rightarrow \mathfrak{R}$ continuous functions, $j = 1, 2, \dots, m$.

Then elemental chances for fuzzy-probability event with function $f_j(\xi) \leq 0, j = 1, 2, \dots, m$, are a function on $[0, 1]$ to $[0, 1]$ defined by

$$\begin{aligned} \text{Ch}\{f_j(\xi) \leq 0, j = 1, 2, \dots, m\}(\alpha) &= \\ &= \sup\left\{\beta \mid \Pr\left\{\omega \in \Theta \mid \text{Pos}\left\{f_j(\xi(\omega)) \leq 0, \right. \right. \right. \\ &\quad \left. \left. \left. j = 1, 2, \dots, m\right\} \geq \beta\right\} \geq \alpha\right\}. \quad (3) \end{aligned}$$

III. Fuzzy-Chance Constrained Economic Order Quantity Models Formulation

In [2] were proposed three crisp economic order quantity (EOQ) models for deteriorating items with inspection errors, trade credit, time value of money and inflation. These models differ in the time resolution of supplier complaints.

The introduced mathematical models are based on the following assumptions:

- The inventory system under consideration deals with a single item.
- The rates D, λ, λ_r and the $\beta, a_1, a_2, \beta_r, a_{1r}, a_{2r}$ percentages are known constant, where D – demand rate, λ – supply rate, λ_r – supply rate of recovered imperfect quality items, β – percentage of defective items supplied, a_1 – percentage of non-defective items classifying as defective, a_2 – percentage of defective items classifying as non-defective, β_r – percentage of recovered imperfect quality items supplied, a_{1r} – percentage of non-defective recovered imperfect quality items classifying as defective, a_{2r} – percentage of defective recovered imperfect quality items classifying as non-defective.
- Only serviceable items deteriorate with constant rate θ .
- Shortages are allowed and are backlogged.
- Backlogged demands are made up at the beginning of the cycle.
- Deteriorated items and unrecoverable imperfect quality items are disposed.
- Recovered imperfect quality items are considered as good quality items.
- Replenishment rate is infinite.
- Unit selling price are greater than unit purchasing price.
- When $M \leq T$, the account is settled at $T = M$ and the supplier/retailer starts paying interest on the items in stock with the rate of interest i_p . When $M > T$, the account is settled at $T = M$ and the supplier/retailer does not need to pay any interest charge.
- When $M < N \leq T$, the account is settled at $T = N$ and the supplier/retailer starts paying interest on the items in stock with the rate of interest i_p between M to N and i_w between N to T . When $M < T < N$, the account is settled at $T = N$ and the supplier/retailer starts paying for the interest charges on the items in stock with rate i_p .

In the fuzzy environment, we assume that the demand rate, selling prices, interest paid rate, interest earned rate are fuzzy numbers and denoted by \tilde{D} , \tilde{p}_v , \tilde{p}_b , \tilde{i}_p , \tilde{i}_w , \tilde{i}_e respectively. Here, we assume that

$$\begin{aligned} \tilde{D} &= [D_1, D_2, D_3, D_4], \quad \tilde{p}_v = [p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}] \\ \tilde{p}_b &= [p_{b_1}, p_{b_2}, p_{b_3}, p_{b_4}], \quad \tilde{i}_p = [i_{p_1}, i_{p_2}, i_{p_3}] \\ \tilde{i}_w &= [i_{w_1}, i_{w_2}, i_{w_3}] \text{ and } \tilde{i}_e = [i_{e_1}, i_{e_2}, i_{e_3}] \end{aligned}$$

are non-negative trapezoidal and triangular fuzzy numbers respectively.

Then, the fuzzy EOQ models with constraints on chances on the probability space (Θ, A, Pr) can be constructed as follows.

Model 1. EOQ for deteriorating items with inspection errors, trade credit, time value of money and inflation in case when settlement of claims will take place immediately after the entrance control basic party supplies.

$$Arg \max_q \max_{I \in \Omega} \tilde{H}^q(I), \quad (4)$$

$$\begin{aligned} I &= (I_b, I_m, I_s, I_r), \quad I_b \in \Omega_b, \quad I_m \in \Omega_m, \quad I_s \in \Omega_s, \\ &\quad I_r \in \Omega_r, \\ \Omega_i &= [a_i, b_i], \quad i \in \{b, m, s, r\} = J, \quad \Omega = \times \Omega_i, \quad I \in \Omega, \\ &\quad q = \overline{1, 20}, \end{aligned}$$

subject to the constraints:

$$Ch\{H^q(I) \geq \tilde{H}^q, q = 1, 2, \dots, 20\}, (\xi) \geq \delta,$$

investment amount on total purchase cost have an upper and lower limits

$$Ch\{V_{\min} - \tilde{P}^q \leq V_{\max}\}, (\xi_1) \geq \tau_1;$$

warehouse space where the items are to be stored is limitation

$$Ch\{y \cdot z \cdot d \cdot \lambda \cdot (\tilde{t}_1 + \tilde{t}_2) \leq W\}, (\xi_2) \geq \tau_2;$$

holding cost cannot be more than total purchase cost

$$Ch\{\tilde{C}_h < \tilde{P}^q\}, (\xi_3) \geq \tau_3;$$

providing good service of customers

$$Ch\{((1-\beta)a_1 + \beta(1-a_2)) \cdot \lambda \cdot \tilde{t}_2 \leq u\}, (\xi_4) \geq \tau_4,$$

where

$$\tilde{H}^q(I) = \frac{\begin{pmatrix} \tilde{R}^q(I) - K - \tilde{C}_{si}(I) - \tilde{C}_{adi}(I) - \\ - \tilde{C}_{rnd}(I) - \tilde{C}_{rg}(I) - \tilde{C}_s(I) - \\ - \tilde{C}_{un}(I) - \tilde{C}_d(I) - \tilde{C}_h(I) - \\ - \tilde{P}^q(I) - \tilde{I}P^q(I) + \tilde{I}E^q(I) \end{pmatrix}}{\tilde{R}^q(I)},$$

q – set of timing of payments on the loan, I_b – shortage level, I_m – maximum inventory level, I_s – inventory level at the time of delivery of goods to settle claims, I_r – inventory level after the delivery of goods to settle claims, R – sale revenue, K – ordering cost per order, C_{si} – screening cost, C_{adi} – cost of accepting a defective items, C_{rnd} – cost of rejection a non-defective items, C_{rg} – cost for return the rejection items to supplier, C_s –

shortage cost due to backlog, C_{un} – opportunity cost due to lost sale, C_d – deterioration cost, C_h – inventory holding cost, P – purchase cost, IP – interest paid cost, IE – interest earned, V_{\min} – lower limit of total purchase sum, V_{\max} – upper lower limit of total purchase sum, y, z, d – overall dimension of production unit, W – capacity of warehouse, u – maximum allowable number of defective items in the lot, $Ch\{\}$ – chance measure, (ζ, δ) – optimistic function value, $\xi_1, \tau_1, \xi_2, \tau_2, \xi_3, \tau_3, \xi_4, \tau_4$ – advance defined trust levels.

Model 2. EOQ for deteriorating items with inspection errors, trade credit, time value of money and inflation in case when settlement of claims will take place at the end of the term exhaustion of inventories created from the start of supply.

$$Arg \max_q \max_{I \in \Omega} \tilde{H}^q(I), \quad (5)$$

$$\begin{aligned} I &= (I_b, I_m, I_s), \quad I_b \in \Omega_b, \quad I_m \in \Omega_m, \quad I_s \in \Omega_s, \\ \Omega_i &= [a_i, b_i], \quad i \in \{b, m, s\} = J, \quad \Omega = \times \Omega_i, \quad I \in \Omega, \\ &\quad q = \overline{1, 14}, \end{aligned}$$

subject to the constraints:

$$Ch\{H^q(I) \geq \tilde{H}^q, q = 1, 2, \dots, 14\}, (\zeta) \geq \delta,$$

investment amount on total purchase cost have an upper and lower limits

$$Ch\{V_{\min} - \tilde{P}^q \leq V_{\max}\}, (\xi_1) \geq \tau_1;$$

warehouse space where the items are to be stored is limitation

$$Ch\{y \cdot z \cdot d \cdot \lambda \cdot (\tilde{t}_1 + \tilde{t}_2) \leq W\}, (\xi_2) \geq \tau_2;$$

holding cost cannot be more than total purchase cost

$$Ch\{\tilde{C}_h < \tilde{P}^q\}, (\xi_3) \geq \tau_3;$$

providing good service of customers

$$Ch\{((1-\beta)a_1 + \beta(1-a_2)) \cdot \lambda \cdot \tilde{t}_2 \leq u\}, (\xi_4) \geq \tau_4,$$

where

$$\tilde{H}^q(I) = \frac{\begin{pmatrix} \tilde{R}^q(I) - K - \tilde{C}_{si}(I) - \tilde{C}_{adi}(I) - \\ - \tilde{C}_{rnd}(I) - \tilde{C}_{rg}(I) - \tilde{C}_s(I) - \\ - \tilde{C}_{un}(I) - \tilde{C}_d(I) - \tilde{C}_h(I) - \\ - \tilde{P}^q(I) - \tilde{I}P^q(I) + \tilde{I}E^q(I) \end{pmatrix}}{\tilde{R}^q(I)},$$

I_b – shortage level, I_m – maximum inventory level, I_s – inventory level at the time of delivery of goods to settle claims.

Model 3. EOQ for deteriorating items with inspection errors, trade credit, time value of money and inflation in case when settlement of claims will take place after the input control basic party supplies, but before exhaustion created stock.

$$Arg \max_q \max_{I \in \Omega} \tilde{H}^q(I), \quad (6)$$

$$\begin{aligned} I &= (I_b, I_m, I_r), \quad I_b \in \Omega_b, \quad I_m \in \Omega_m, \quad I_r \in \Omega_r, \\ \Omega_i &= [a_i, b_i], \quad i \in \{b, m, r\} = J, \quad \Omega = \times \Omega_i, \quad I \in \Omega, \\ &\quad q = \overline{1, 14}, \end{aligned}$$

subject to the constraints:

$$\text{Ch}\{H^q(I) \geq \tilde{H}^q, q=1,2,\dots,14\}, (\xi_1) \geq \delta,$$

investment amount on total purchase cost have an upper and lower limits

$$\text{Ch}\{V_{\min} - \tilde{P}^q \leq V_{\max}\}, (\xi_2) \geq \tau_1;$$

warehouse space where the items are to be stored is limitation

$$\text{Ch}\{y \cdot z \cdot d \cdot \lambda \cdot (\tilde{t}_1 + \tilde{t}_2) \leq W\}, (\xi_3) \geq \tau_2;$$

holding cost cannot be more than total purchase cost

$$\text{Ch}\{\tilde{C}_h < \tilde{P}^q\}, (\xi_3) \geq \tau_3;$$

providing good service of customers

$$\text{Ch}\{((1-\beta)a_1 + \beta(1-a_2)) \cdot \lambda \cdot \tilde{t}_2 \leq u\}, (\xi_4) \geq \tau_4,$$

where

$$\tilde{H}^q(I) = \frac{\begin{pmatrix} \tilde{R}^q(I) - K - \tilde{C}_{si}(I) - \tilde{C}_{adi}(I) - \\ - \tilde{C}_{rnd}(I) - \tilde{C}_{rg}(I) - \tilde{C}_s(I) - \\ - \tilde{C}_{un}(I) - \tilde{C}_d(I) - \tilde{C}_h(I) - \\ - \tilde{P}^q(I) - \tilde{I}P^q(I) + \tilde{I}E^q(I) \end{pmatrix}}{\tilde{R}^q(I)},$$

I_b – shortage level, I_m – maximum inventory level, I_r – inventory level at the time of delivery of goods to settle claims.

In the next section, a hybrid intelligent algorithm is introduced to find near optimum solutions of the formulated problems in (4), (5) and (6).

IV. A Hybrid Intelligent Algorithm

In order to solve hybrid programming models, we may integrate hybrid simulation, neural networks and directed optimization [3] to produce a hybrid intelligent algorithm as follows.

Step 1. Generate training input-output data for uncertain functions like

$$\begin{aligned} U_1 : I &\rightarrow E[H^q(I)] \\ U_2 : I &\rightarrow \text{Ch}\{g_j(I) \leq 0, j=1,2,\dots,p\} \\ U_3 : I &\rightarrow \max\{f\} \text{Ch}\{H_q(I) \geq \tilde{H}^q\} \geq \alpha \end{aligned}$$

by the hybrid simulation.

Step 2. Train a neural network to approximate the uncertain functions according to the generated training input-output data.

Step 3. Initialize population of potential solutions whose feasibility may be checked by the trained neural network.

Step 4. Update the potential solutions by operator of mutation in which the feasibility of offspring may be checked by the trained neural network.

Step 5. Calculate the objective values for all potential solutions by the trained neural network.

Step 6. Compute the fitness of each potential solution according to the objective values.

Step 7. Select the potential solutions by computation optimal measure.

Step 8. Repeat the fourth to seventh steps for a given number of cycles.

Step 9. Report the best potential solution as the optimal solution.

Step 10. Repeat the first to ninth steps for a given number of subfunctions.

V. Compositional Directed Optimization Method

In order to solve the models (4), (5) and (6), we design evolutionary technologies of directed optimization [3] based on the fuzzy simulation to obtain the approximate optimal solution, in which simulation algorithms are employed to check the feasibility of solutions and compute the objective values if analytic methods are invalid.

Step 1. Set iteration counter $t = 0$.

Step 2. Define the structure I of potential solution \mathbf{x} .

Step 3. Define the population size ζ , $\zeta = \overline{1, \rho}$, and generate uniformly distributed on Ω representative population of potential solutions $X = \mathbf{x}_1^t, \mathbf{x}_2^t, \dots, \mathbf{x}_\rho^t$, so $\Omega_i = [a_i, b_i]$, $\Omega = \times \Omega_i$, $I \in \Omega$.

Step 4. Choose from the given set H objective function H_q , $q = \overline{1, u}$, calculate the value of this function f in $\mathbf{x}_1^t, \mathbf{x}_2^t, \dots, \mathbf{x}_\rho^t$: $f_\zeta^t = H_q(\mathbf{x}_\zeta^t)$.

Step 5. Normalize value f_ζ^t so $f_\zeta^{ht} \in [0;1]$, $\sum_{\zeta=1}^{\rho} f_\zeta^{ht} = 1$.

Step 6. Form a matrix of pairwise comparisons Saaty S as follows. Among normalized values function to find the minimum f_ζ^{ht} , split segment $[0;1]$ on 10 intervals: $[0;0,1)$, $[0,1;0,2)$, ..., $[0,9;1]$. Then all $h \in \{1,2,\dots,p\}$, if $f_\zeta^{ht} \in [0,1k;0,1+0,1k)$ and $f_\zeta^{ht} \in [0,1l;0,1+0,1l)$, where $k, l \in \{0,1,\dots,9\}$, then $s_{ch} = l - k + 1$. Other elements of the matrix S calculated as: $s_{rq} = s_{cq} / s_{cr}$.

Step 7. Calculate the eigenvalues of matrix S and for the maximum number of own a_{\max} find the corresponding eigenvector w . Values w_ζ indicate as optimal (quasi optimality) measure of potential solution \mathbf{x}_ζ^t .

Step 8. Generate offspring and create a new population of potential solutions, based on the optimal measure w_ζ of potential solution \mathbf{x}_k^t .

For this find the minimum w_{\min} and the maximum w_{\max} elements of the eigenvector. Sort descending optimal measure of potential solutions on interval $[w_{\max}; w_{\min}]$ and divide them into three groups: close to optimal (large) A_L , quasi optimal (medium) A_M and suboptimal (small) A_S .

For potential solutions from A_L group, $k \in \{1, \dots, \lambda\} = 4 \cdot \zeta$ offspring generate accordingly [4]

if w_ζ is A_L then $\mathbf{x}_k^{A_L,t+1} = \mathbf{x}_\zeta^{A_L,t} + \boldsymbol{\sigma}^{A_L,t+1} \cdot \mathbf{z}_k^t$,
where

$$\boldsymbol{\sigma}^{A_L,t+1} = \boldsymbol{\sigma}_\zeta^t \cdot \exp^{\frac{1}{d_i} \left(\|\mathbf{s}_\zeta^t\| \sqrt{2\pi} / 2 - 1 \right)} \times \exp^{\frac{c_\sigma}{d} \left(\|\mathbf{s}_\zeta^t\| \Gamma\left(\frac{n}{2}\right) / \sqrt{2} \Gamma\left(\frac{n+1}{2}\right) - 1 \right)},$$

$$\mathbf{s}_\zeta = (1 - c_\sigma) \mathbf{s}_\zeta + \sqrt{c_\sigma(2 - c_\sigma)} \frac{\sqrt{\zeta}}{\zeta} \sum_{z_k \in A_L} \mathbf{z}_k, \mathbf{z}_k = N(0, \mathbf{I}),$$

$$\mathbf{x} = \frac{1}{\zeta} \sum_{\mathbf{x}_k \in A_L} \mathbf{x}_k, \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \dots, \sigma_n) - \text{vector of global}$$

mutation step-size, $\mathbf{z}_k \in R^n$ – independent realization of a random number with zero mean, $\mathbf{I} \in R^{n \times n}$ – the identity matrix, $d_i \approx 3 \cdot n$, $d \approx 1 + \sqrt{\zeta/n}$, $\|\mathbf{s}_\zeta\|$ – length of the search path, $N(0,1)$ – independent realization of a random number with zero mean and standard deviation 1, $c_\sigma \approx \sqrt{\zeta/(n+\zeta)}$, $i = \overline{1, n}$, n – search space dimension.

For potential solutions from A_M group, $k \in \{1, \dots, \lambda\} = 2 \cdot \zeta$ offspring generate accordingly:

if w_ζ is A_M then $\mathbf{x}_k^{A_M,t+1} = \mathbf{x}_\zeta^{A_M,t} + N(0, \boldsymbol{\sigma}_\zeta^{A_M,t+1})$,
where $N(0, \boldsymbol{\sigma}_\zeta^t)$ – independent realization of a random number with zero mean and standard deviation of

$$\text{potential solution } \zeta, \quad \boldsymbol{\sigma}_\zeta^{A_M,t+1} = \frac{1}{3} \max \left\{ \begin{array}{l} d(\mathbf{x}_\zeta^t, \mathbf{x}_L^t) \\ d(\mathbf{x}_\zeta^t, \mathbf{x}_R^t) \end{array} \right\},$$

$d(\mathbf{x}_\zeta^t, \mathbf{x}_L^t)$ – distance between a potential solution ζ and left next (or point a) neighbor interchange from *medium* group, $d(\mathbf{x}_\zeta^t, \mathbf{x}_R^t)$ – distance between a potential solution ζ and right next (or point b) neighbor interchange from *medium* group.

For potential solutions from A_S group, $k \in \{1, \dots, \lambda\} = \zeta$ offspring generate accordingly

if w_ζ is A_S then

$$\mathbf{x}_k^{A_S,t+1} = \mathbf{x}_\zeta^{A_S,t} + \boldsymbol{\sigma}_\zeta^{A_S,t} \cdot e^{(\tau \cdot N(0,1) + \tau_\zeta \cdot N_\zeta(0,1))} \cdot N(0,1),$$

where $\boldsymbol{\sigma}_\zeta^{A_S,t}$ – vector of standard deviations of potential solution ζ from A_S group on t -th iteration, τ – global mutation step for all suboptimal solutions, τ_ζ – mutation step for ζ -th potential solutions according to $N_\zeta(0,1)$.

Step 9. Find the corresponding value of the function f_k^t for offspring of potential solutions. According to these

values, and the values $f_1^t, f_2^t, \dots, f_\rho^t$ determine λ best solutions and create a new population of potential solutions $X^{t+1} = \mathbf{x}_1^{t+1}, \mathbf{x}_2^{t+1}, \dots, \mathbf{x}_\rho^{t+1}$.

Step 10. Check the convergence condition of the iterative process

$$\begin{aligned} & \varpi_1 \cdot |f_{H_q \max}^t - f_{H_q \max}^{t-1}| + \varpi_2 \cdot |f_{H_q \text{avg}}^t - f_{H_q \text{avg}}^{t-1}| + \\ & \varpi_3 \cdot |f_{H_q \min}^t - f_{H_q \min}^{t-1}| \leq \varepsilon, \end{aligned}$$

where $\varpi_1, \varpi_2, \varpi_3$ – weights, $f_{H_q \max}$ – the maximum value of the objective function, $f_{H_q \text{avg}}$ – average value of the objective function, $f_{H_q \min}$ – the minimum value of the objective function.

Weight coefficients calculated based on the analytic hierarchy process T. Saaty considering competence of experts by decision-maker.

Step 11. If the convergence condition is not met, then increase the number of the current iteration $t = t + 1$ and go to step 5.

Step 12. Steps 4-11 to repeat the entire set of objective functions H^q , $q = \overline{1, u}$.

Step 13. Determine the best value of the objective function and the corresponding solution.

Step 14. End.

Conclusion

In this paper fuzzy economic order quantity models with constraints on chances is given. The compositional technique overcoming uncertainty to solve such complex problems has many structural implementations. The offered models and method aiming at near optimum solutions in a reasonable time.

References

- [1] Лю Б. Теория и практика неопределенного программирования [Текст] / Б. Лю; пер с англ. – М.: БИНОМ. Лаборатория знаний, 2014. – 416 с.
- [2] Єгорова О. В. Композиційний метод розв'язання задачі формування запасів товарів, які втрачають природні властивості [Текст] / О. В. Єгорова // Технологічний аудит і резерви виробництва. – 2015. – Вип. 4, № 2(24). – С. 29-34.
- [3] Єгорова О. В. Нечіткі штрафні функції в задачі управління запасами з урахуванням товарних втрат [Текст] / О.В. Єгорова, В.Є. Снитюк // Молодий вчений. – 2016. – № 5(32). – С. 228-234.
- [4] Hansen, N. Evolution strategies [Text] / Nikolaus Hansen, Dirk V. Arnold, Anne Auger // Springer Handbook of Computational Intelligence / Janusz Kacprzyk, Witold Pedrycz (Editors). – Berlin: Springer Berlin Heidelberg, 2015. – Pp. 871-898.