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VIBRATION AND SOUND TRANSMISSION LOSS ACROSS A SANDWICH BEAM WITH THE DYNAMIC VIBRATION ABSORBERS

Метою дослідження є передбачення демпфування і акустичні властивості композитних багатошарових тонкостінних структур. У роботі було визначено ефективні константи жорсткості багатошарових пластин композитного типу і демпферні властивості, використовуючи процедуру, засновану на багаторівневих числових схемах. Наведений метод моделювання композитних ламінованих тонкостінних структур не залежить від строгих припущень щодо моделі структури.

Ключові слова – демпфування, багатошарова пластина, вібрація

The study aims to predict damping and acoustical properties of composite laminated thin-walled structures. Effective rigidity constants of sandwich type laminates and damping properties have been determined by using a procedure based on multi-level numerical schemes. The present method of laminated composite structures modeling does not rely on strong assumptions on the model of the structure.

Keywords – damping, sandwich type laminate, vibration

Inroduction

Structures composed of laminated materials are among the most important structures used in modern engineering and, especially, in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications. The rapid increase of the industrial use of these structures has necessitated the development of new analytical and numerical tools that are suitable for the mechanical properties identification and optimization of such structures [1, 2].

The transmission loss of a sound wave was investigated extensively about half a century ago. Many studies are limited to the acoustic field theory of isotropic materials. In [3] the wave transmission theory of elastic bodies is presented. In [4] a transmission matrix for the relationship between the velocity and pressure in an elastic solid body is obtained. In [5] is determined how the transmission matrix relates to different layered media, applying the elastic theory in many cases. To achieve effective damping over a wide frequency range, various methods are used. Active [4] and semiactive vibration control techniques [5] magnetic vibration dampers [6] and particle dampers [7] can achieve high damping over wide range of frequencies. However, active damping usually suffers from collateral effects [8,9]. Magnetic and particle vibration dampers have larger weight penalty. Passive damping using viscoelastic materials [10-13] is simpler to implement and more cost-effective than semiactive and active techniques.

High order asymptotic approaches

The present paper aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solution and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis.

Let us consider now such kinematical assumptions for symmetrical three layered plate (see Fig. 1):

$$U_e - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^e z^i \sin k\pi x / L, & \\ w = \sum\limits_{i,k} w_{ik}{}^e z^i \cos k\pi x / L, & 0 < z < H, \\ & 0 < x < L, \end{cases} \tag{1}$$

$$U_d - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^d z^i \sin k\pi x / L, & \\ w = \sum\limits_{i,k} w_{ik}{}^d z^i \cos k\pi x / L & H < z < H_p, \\ & 0 < x < L. \end{cases} \tag{2}$$

By substituting Eqs. (1), (2), into the following condition

$$\int\limits_{t_1}^{t_2}\int\limits_V (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz} - \rho\frac{\partial u}{\partial t}\delta\frac{\partial u}{\partial t} - \rho\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t})dVdt = 0, \tag{3}$$

and also assuming the unifrequency vibration

$$(u_{ik}^e = \overline{u}_{ik}^e e^{i\omega t},\ w_{ik}^e = \overline{w}_{ik}^e e^{i\omega t},\ u_{ik}^d = \overline{u}_{ik}^d e^{i\omega t},\ w_{ik}^d = \overline{w}_{ik}^d e^{i\omega t})$$

we obtain the set of linear algebraic equations for the amplitudes

$$[A]\overline{U} = \begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix}\begin{bmatrix} \overline{U}_e \\ \overline{U}_d \end{bmatrix} = 0 \quad . \tag{4}$$

For grater number of lamina it may be written in the form

$$U_d^n - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^n z_n{}^i x^{(2k-1)}, & H_p^{(n)} < z < H_p^{(n+1)}, \\ w = \sum\limits_{i,k} w_{ik}{}^n z_n{}^i x^{(2k-2)}, & 0 < x < L, \\ & n = 1, \dots, N, \end{cases} \tag{5}$$

Here $H_p^{(n+1)} - H_p^{(n)} = H_n$, $H_p^{(1)} = H$; H and H_n are one-half thicknesses of a core and the outer n-th layer, respectively. The corresponding frequency equation should be written such as in [3–7]. This is traditional unifrequency method of linear elastic system investigation.

Timoshenko beam

Timoshenko beam is a particular case of presented above occasions (5) given by the kinematical hypothesis. By

$$U(x,z,t) = z\gamma(x,t), \quad W(x,z,t) = w(x,t) \tag{6}$$

and (3) we obtain the Timoshenko beam equations [T]

$$-EI\frac{\partial^2\gamma}{\partial x^2} + SG\left(\frac{\partial w}{\partial x} + \gamma\right) + \rho I\frac{\partial^2\gamma}{\partial t^2} = 0, \quad SG\left(\frac{\partial\gamma}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \rho S\frac{\partial^2 w}{\partial t^2} = q \quad . \tag{7}$$

For the case of steady state vibrations

$$\gamma = \gamma_0 e^{i\omega t} e^{ikx\sin\varphi}, \quad w = w_0 e^{i\omega t} e^{ikx\sin\varphi}, \quad q = q_0 e^{i\omega t} e^{ikx\sin\varphi} \tag{8}$$

we obtain

$$\left(\frac{(SGk_s)^2}{EJk_s^2 + SG - \rho I \omega^2} - SGk_s^2 + S\rho\omega^2 \right) w_0 = q_0 \quad , \quad k_s = k \sin\varphi \quad . (9)$$

Here E, G are rigidity modules. They are in the general case the complex frequency dependent functions.

Transition to the Timoshenko beam

The rapid increase of the industrial use of these structures has necessitated the development of new analytical and numerical tools that are suitable for the mechanical properties identification and optimization of such structures [14, 15]. The numerical method developed follows a semi-analytical approach with analytical field applied in longitudinal direction and layer-wise displacement field employed in transverse direction [16-18]. Identification of elastic properties of laminated plates from the measured eigen-frequencies has been performed. Elastic constants of laminates have been determined by using an identification procedure based on experiment design, and multilevel theoretical approach. The variety of NS and its interaction with testing methods and experimental modeling conditions are presented in [19, 20].

Let us consider three-layered symmetrical beam (Fig. 1). Its mechanical properties are: core material (honeycomb polymer filled structure): compressive modulus – 1.076 GPa; tensile modulus – 3.96 GPa; flexural modulus – 1.020 GPa; shear modulus – 0.638 GPa. Face material (fiber composite material): tensile modulus – 26 GPa; flexural modulus ≈ 6 GPa; shear modulus ≈ 0.6 GPa.

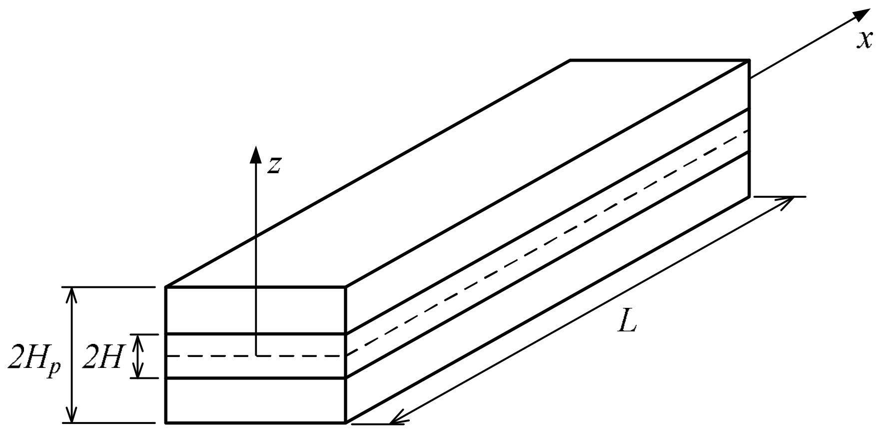
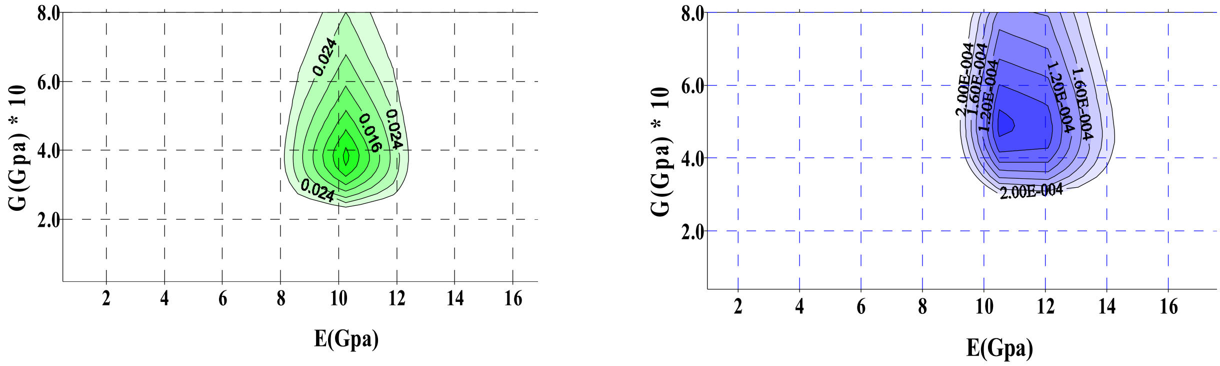


Fig. 1. The sandwich beam with inner viscoelastic layer (core) and face layers.

In Fig. 2 the values of elastic constants E_T, G_T of equivalent Timoshenko's beam are presented. For the elastic modules identification the procedure was applied by comparing the elastic energy of the two beams: one of them – non-uniform, and the – uniform. The isolines in Fig.4 present the constant values of difference of Timoshenko beam elastic energy and the energy of analytically modelling beam $\int_V (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz})dV$. Two methods were applied: analytical and approximation [18-20].

Fig. 2 presents the theoretically found elasticity constants E_T, G_T for the Timoshenko's beam analogue of above beam sample (corresponding to minimum difference of energies). A small variance in values is caused by the different elastic energy components taken into account in both cases. In the approximation method the transverse energy component $\int_V (\sigma_{zz}\delta\varepsilon_{zz})dV$) is very small and not taken into account. For symmetrical structure with identical face layers by use of Eqs. (1-9), the equivalent rigidity Timoshenko's beam theory may be found [18-20].



(a) **(b)**

Fig.2. Equivalent module E,G: (a) – analytical approach; (b) – approximation approach

Estimation of frequency dependent coefficients

Loss factors in layered beams (plates in cylindrical bending) is found by deformations energy comparison [17–20]:

$$\eta_\Sigma = \frac{1}{\eta_1} \frac{\frac{L^3}{3}\sum_{i=1}^{N}\int_{H_p^{(i)}}^{H_p^{(i+1)}} \eta_i \sigma_{xx}^{(i)} \varepsilon_{xx}^{(i)} dz + L\sum_{i=1}^{N}\int_{H_p^{(i)}}^{H_p^{(i+1)}} \eta_i \frac{\tau_{xz}^{(i)2}}{G_i} dz}{\frac{L^3}{3}\sum_{i=1}^{N}\int_{H_p^{(i)}}^{H_p^{(i+1)}} \sigma_{xx}^{(i)} \varepsilon_{xx}^{(i)} dz + L\sum_{i=1}^{N}\int_{H_p^{(i)}}^{H_p^{(i+1)}} \frac{\tau_{xz}^{(i)2}}{G_i} dz}, \qquad (10)$$

where η_i is the loss factor of i-th layer. Here the small transverse stresses are neglected. This result may be achieved by direct computation by use of stiffness matrix if damping matrix is congruent to the stiffness matrix.

$$\eta = \frac{\eta_1 [q]^T |A_1| |q| + \eta_2 [q]^T |A_2| |q| + \ldots + \eta_N [q]^T |A_N| |q|}{[q]^T |A| |q|} \qquad (11)$$

Here: $|A|$ – stiffness matrix, $|q|$ – vector of displacement component, $|A_i|$ – stiffness matrix component corresponding to i-th layer ($|A| = \sum_i |A_i|$). In Fig.3 damping coefficients for fife-layered beam with damping interlayers for various approximation levels are presented.

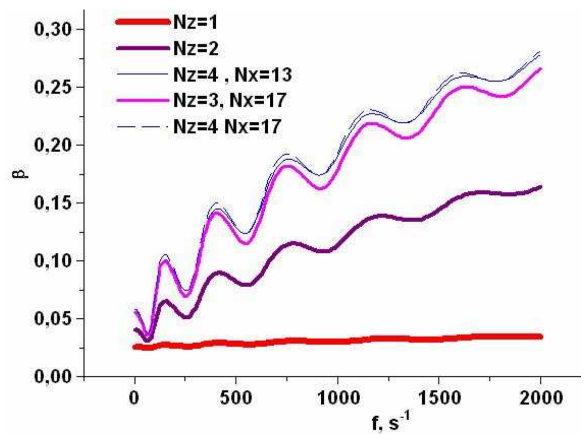


Fig. 3. Theory dependent damping prediction accuracy for the five-layered beam

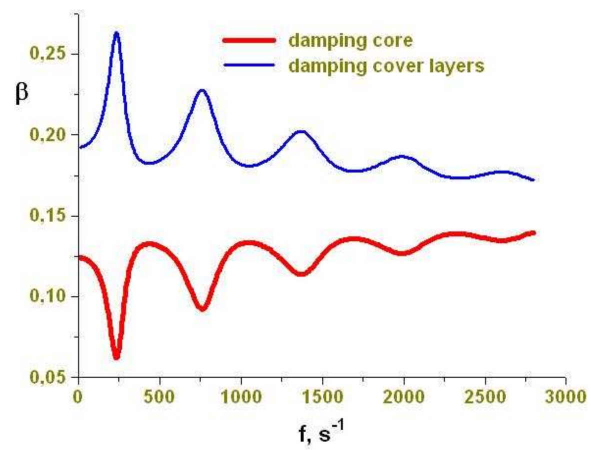


Fig.4. Damping properties for various thicknesses of damping properties distribution (L=0.6m)

Here N_x – is a number of terms in longitudinal direction and N_z – in normal to the beam direction for every layer. The strong model depending quantities may be seen also for middle frequency range.

In previous chapter the frequency influence on the damping properties of sandwich panels may be seen. For this purpose a few investigations are made for damping properties investigation in frequency domain. In Fig.4 the frequency depended damping is presented for various thicknesses of damping properties distribution. Two cases are under investigation: 1) a damping core; 2) a damping outer layer (properties of layers are such as in Fig.4).

In Fig.4 monotonic damping decreasing for outer damping layer and damping increasing for inner damping layer may be seen. Damping frequency fluctuations also may be seen. These fluctuations are correlated with appropriate FRF. The local damping minimums are appropriately coincided with FRF resonance picks.

Frequency dependent rigidity

Let us consider now the frequency dependent characteristics of laminated beam. In Fig.5 the frequency dependent rigidity ratios E/Es of three-layered symmetrical beam and Euler beam with bending stiffness $EI_S = \int_{-H}^{H} E(z)z^2 dz$ for various layers rigidity are presented.

The ratio E/E_S was found by equation $E/E_S = f_i^2 / f_{si}^2$. (Here f_i is defined by eigenfrequencies and by $f_{iS=H/2\pi} \sqrt{E_S g/(12\rho)}\alpha_i^2/L^2, \alpha_1 = 1.875, \alpha_2 = 4.694,...$ known console Euler beam eigenfrequencies). Cases were discussed, when cover layer are ten miles more rigid than inner layer, uniform beam and beam with ten miles more rigid than core.

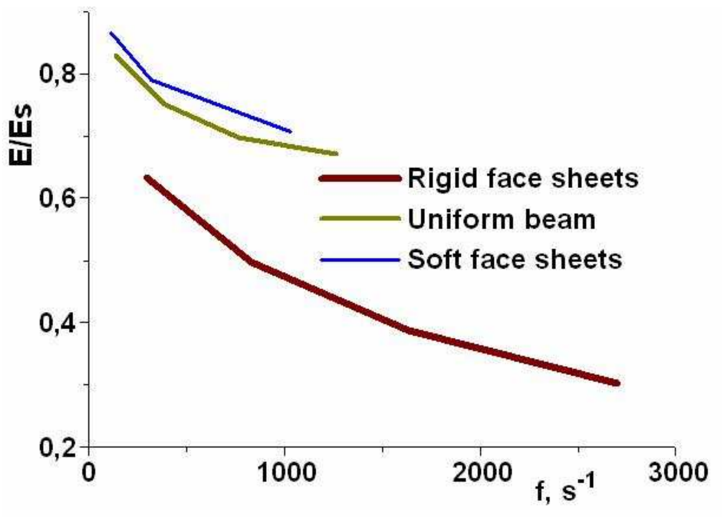
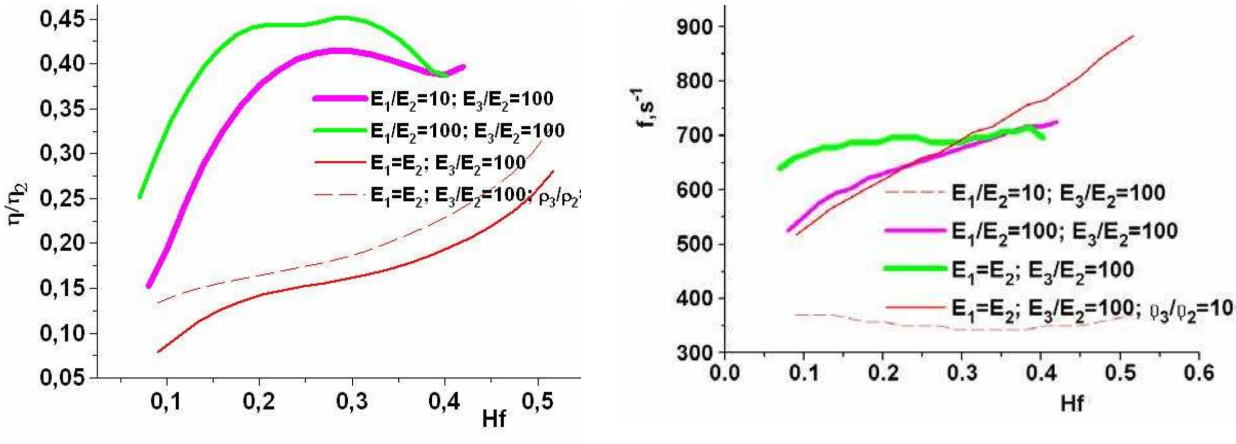


Fig. 5. Equivalent bending rigidity for sandwich

Finally let us consider the influence of damping layer position on the damping and rigid properties of the lamina. In Fig. 6 the damping properties are presented for various damping layer position. Here Hf is a thickness of cover layers (3). The results presented in a nondimensional form, were $E_2 = 1, \eta_2 = 1$.



(a) (b)

Fig. 6 Lamina properties for various damping layer position: (a) – the loss factors for the fife-layered beam with damping interlayer; (b) the first eigen-frequency for the five-layered beam with damping interlayer

In Fig. 6a. the relation of loss factors for the five-layered beam with damping interlayer coefficient η_2 is shown. The depth of the cover rigid layer Hf and layer material rigid module relations Ei/Ej were changed. In Fig. 6b. the first eigenfrequency for this parameter is presented. In Fig.7 difference of damping properties behavior in different frequency ranges may be seen.

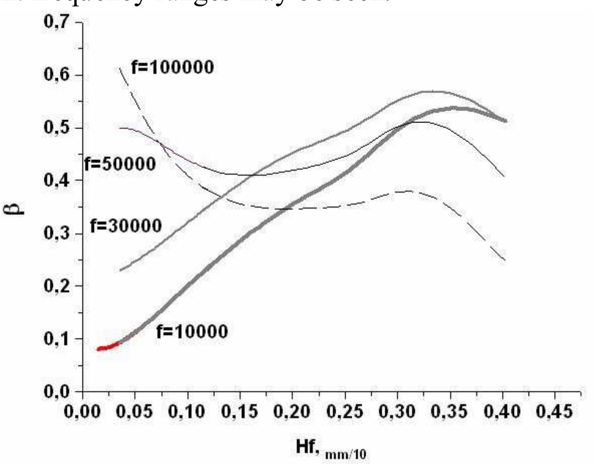


Fig. 7. The influence of geometry on the damping in various frequency ranges

Acoustical properties

When a panel is excited acoustically, the frequency at which the speed of the forced bending wave in the panel is equal to the speed of the free bending wave in the panel is called the coincidence frequency. It is expected that sound power transmission coefficient is very high at the coincidence frequency of the panel. An expression for estimating the coincidence frequency of a panel considering the transverse shear deformation is derived in an earlier study [5]. But the expression for the coincidence frequency is derived without considering the sound power transmission coefficient of the panel. Instead it is derived by equating the wavelength of the trace wave in the panel to the wavelength of the free bending wave in the panel. From the present results it can be seen that the frequency at which the sound power transmission coefficient is the maximum is the coincidence frequency of the panel. Same expression for coincidence frequency is obtained from both the approaches. This confirms the correctness of the expression for the sound power transmission coefficient derived here. To get a better insight into the transmission loss characteristics, transmission loss of panels for different structural parameters are presented.

Consider a panel with an incident acoustic field (Fig.8).

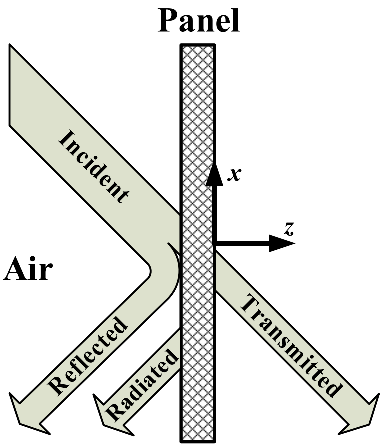


Fig.8. Panel in plain acoustic field (plain wave)

An external excitation in the form of a plane sound wave of frequency is assumed to be incident on the top skin. Sound power transmission coefficient is defined as the ratio of the intensity of the transmitted sound to the intensity of the incident sound. If P_i is the intensity of the incident sound wave and P_t is the intensity of the transmitted sound wave, the sound power transmission coefficient t is defined by $\tau = P_t / P_i$.

The beam acts as a partition in the air of specific acoustic impedances, where ρ and c are the density and speed of sound in the air. Also, a sound transmission loss, STL, is defied which is the common index $Stl = 10\log\left(\tau^{-1}\right)$ [21] .

The net acoustic pressure is

$$q = p_i + p_r + p_{rad} - p_t \tag{12}$$

We take the same form for all this acoustic pressures as for displacements (8) (on the plate surfaces)

$$p_i = p_{i0}e^{i\omega t}e^{ikx\sin\varphi},\quad p_r = p_{r0}e^{i\omega t}e^{ikx\sin\varphi},\quad p_{rad} = p_{rad0}e^{i\omega t}e^{ikx\sin\varphi},\quad p_t = p_{t0}e^{i\omega t}e^{ikx\sin\varphi} \tag{13}$$

Since the medium present on both sides of the plate has the same properties, the sound power radiated to both sides of the panel are equal and hence $p_t = p_{rad}$. The requirement of the continuity of the particle velocity necessitates that $p_i = p_r$. Using the above results and substituting Eq. (7) in Eq. (11), the external force on the plate due to acoustic excitation becomes

$$q = 2\left(p_i - p_t\right) \tag{14}$$

The amplitude of the displacement of the plate is related to the amplitude of the transmitted sound wave by the expression

$$w = \frac{p_t \cos\varphi}{i\omega\rho_a c_a} \tag{15}$$

Substituting Eqs. (14), (15) by Eq. (9) yields

$$\tau = \left|1 - i\frac{\Phi\cos\varphi}{2\rho_a c_a \omega}\right|^2,\quad \Phi = \frac{\left(SGk_s\right)^2}{EJk_s^2 + SG - \rho I\omega^2} - SGk_s^2 + S\rho\omega^2 \tag{16}$$

Let us now consider some numerical examples, ($E = 200MPa$, $G = 50MPa$, $\rho = 200\,kg/m^3$, $h = 0.0254m$) the transmission loss (***TL***) or (Stl) [21] function:

$$T = 10\log\left|\tau^{-1}\right| \tag{17}$$

are presented here for the light foam material beam. In Fig.9. ***TL*** is presented for various angles of incident sound waves (given in rad.).

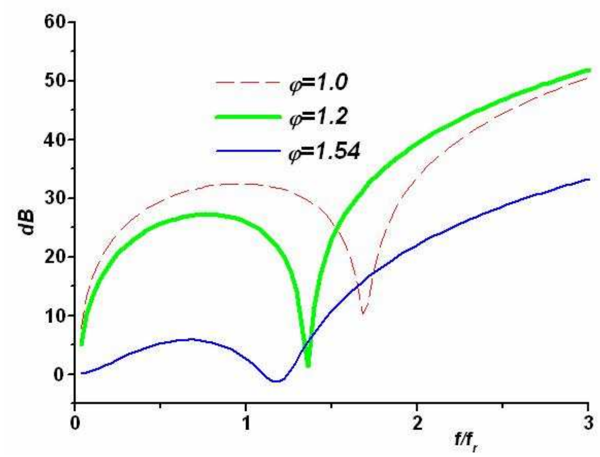


Fig.9. ***TL*** *for various values angles* φ *of incident sound wave*

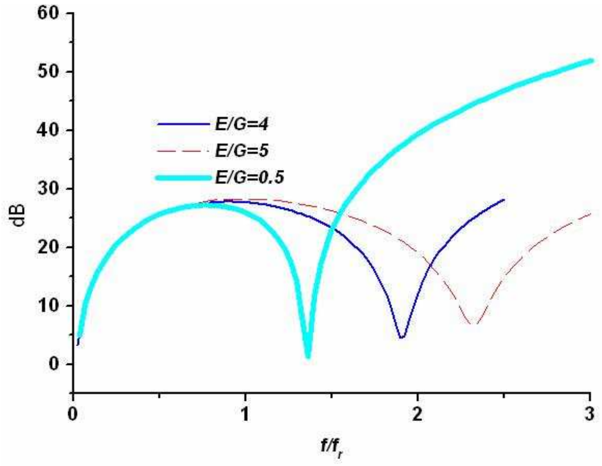


Fig.10. ***TL*** *for various* ***G*** *(*$\varphi = 1.2 rad$

In Fig. 10. ***TL*** is presented for various shear coefficients ***G*** . In Fig. 11. the influence of damping is presented. The viscous frequency of independent bending mode and shear mode of damping are considered

$$E = E_0(1. + iDempE),\ G = G_0(1. + iDempG) \tag{18}$$

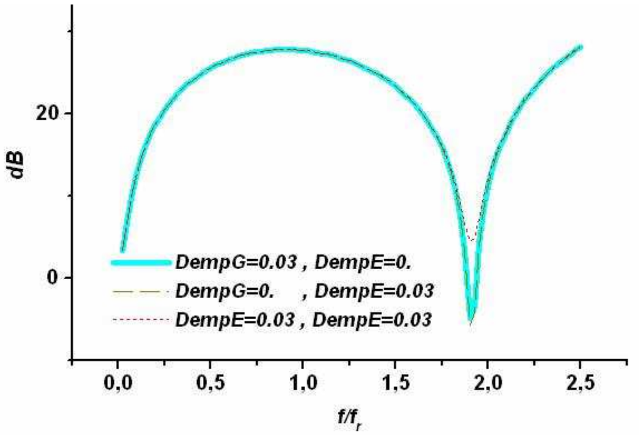


Fig.11. Damping dependent TL

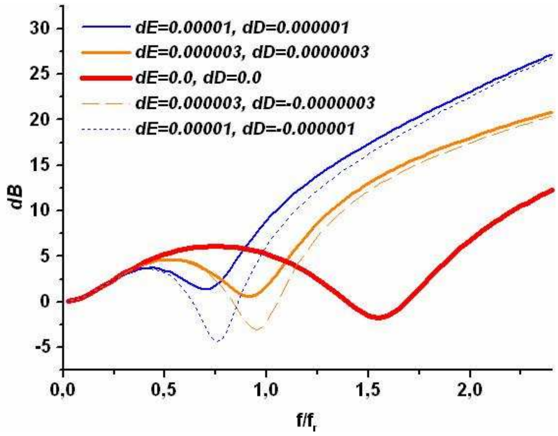


Fig.12. TL for frequency dependent coefficients

At last the influence of frequency on rigidity and damping is under discussion. In Fig.12. ***TL*** function is presented for parameter and for varying with frequency parameters.

$$E_f = E\left(1. + dE * f^2\right),\ G_f = G\left(1. + dE * f^2\right),$$
$$DempE_f = DempE\left(1. + dD * f^2\right),\ DempG_f = DempG\left(1. + dD * f^2\right) \tag{19}$$

Simple quadratic approximations are applied here. In Fig.17. ***TL*** are presented for the sandwich type beam with the soft core (positive values of ***dD)*** and the soft cover sheets (negative dD) (see Fig.4-12).

Beam with the DVA

One-mass DVA. Damped DVAs are applied to allow for energy dissipation, thereby motivating the term ‘absorber’. These realistic absorbers furthermore reduce their sensitivities to parameter variations from optimal values and reduce the primary system motions at the resonance frequencies, while increasing their effective bandwidth, as compared to their undamped examples. Consider the viscously damped DVA (Fig.13) with an elastic and a viscous damping elements, between the masses. Detailed descriptions of the fundamentals of the such DVA's are given in [22-24].

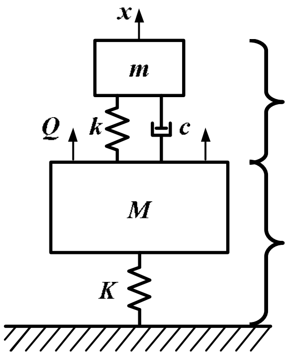


Fig.13 . One-mass DVA

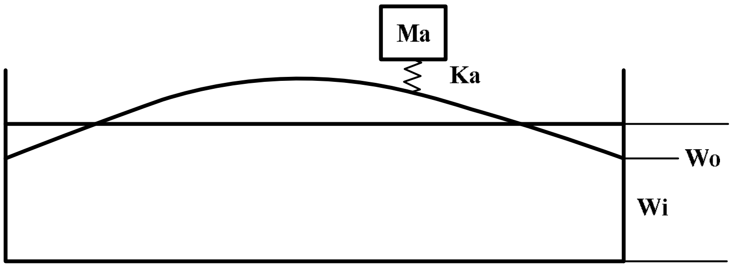


Fig.14. Beam with the DVA

Let us now consider a layered beam with locally attached DVA (Fig.14). Taking into account only the first form of vibration we obtain the similar as in (7) set of equations. Only one additional equation on DVA is applied:

$$-M_A\omega^2 w_A + (K_A + i\omega C_A)(w_A - w) = 0. \qquad (20)$$

In Fig. 15 TL is presented for DVA parameters.

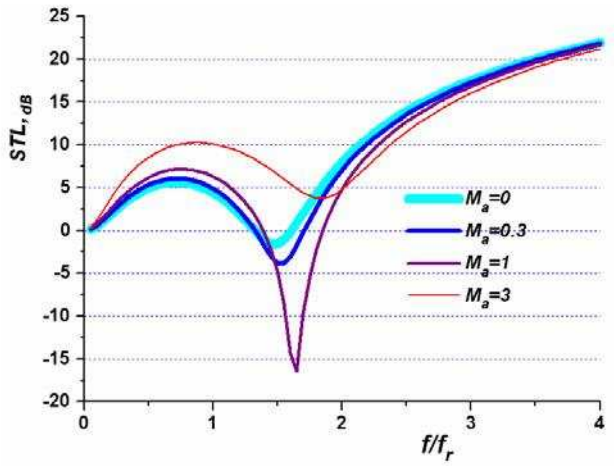


Fig.15 The DVA's mass influence

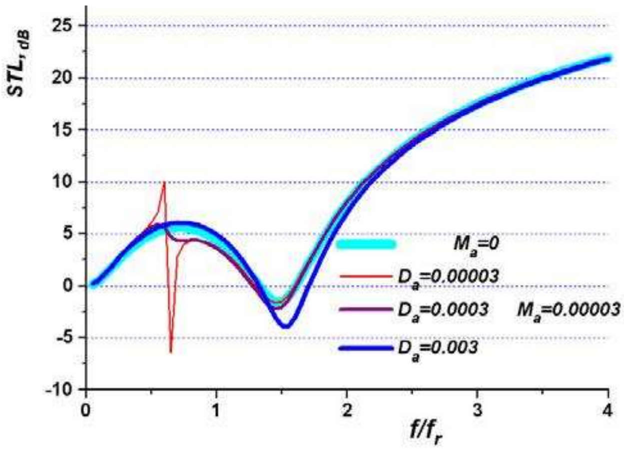


Fig.16 Damping influence

In Fig.16 the DVA's mass influence on ***TL*** is presented. In Fig.16 the influence of damping in DVA is presented.

Concluding remarks

The present paper is the first attempt at proposing a novel procedure to derive damping and sound isolation parameters for forced vibrating sandwich plates. The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical ore approximation method. The same multistage technique may be applied for multi-scale optimization problems. In the future the extension of the present approach to various sandwich plates will be performed in order to test various experimental conditions.

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