# **Numerical solution of parametric optimization of the resonant vibro-impact system with technological limitations**

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*Abstract – The purpose of research: development of method of optimization of the resonance vibro-impact system stiffness parameters with technological limitations of kinematic and dynamic characteristics. The synthesis is based on a numerical dynamic analysis which uses methods of tackling nonlinear differential system of equations. The discrete algorithm of choice-based optimization of synthesized coefficients of asymmetric lump linear stiffness characteristic with limitations imposed on technological parameters and system frequency characteristics is proposed. Optimal parameters of a vibroimpact machine are determined by the maximum of power criteria, which is the ratio of maximum acceleration of the working mass to the current value of the power consumption of a mechanical system.* 

Кеу words – vibro-impact system, resonance, vibratory machine, optimization with limitations, stiffness characteristic.

# I. Introduction

Vibrotechnologies are based on detailed analysis of complex nanoprocesses, is directly related to the study and introduction of the nonlinear effects in oscillatory systems, which have significant influence on complex physical and mechanical transformation in the technological structures of mediums. The result is the increase in use of the vibration systems in non-traditional production areas, and also the increase in technological effectiveness of the machines in traditional areas of their usage [1]. The considerable amount of researches are based on application of multi-frequency, especially highfrequency vibrations during surface and volumetric processing, concrete production, screening etc., which are caused by the usage of vibro-impact systems [2].

# II. Problem statement

Inertial, stiffness-damping and power parameters of the system are in charge of vibro-impact mode formation, particularly there is a specific coordination between them. The definition of rational (optimal), stiffness and force parameters of vibro-impact systems should be based on an appropriate dynamic analysis. Regardless of considerable subject popularity of nonlinear equations, there are not integral techniques to synthesize appropriate modes and thus be a practical instrument during the development of vibro-impact system of a certain type which has additional limitations.

# III. Analysis of the latest research

In particular, multi-frequency of vibro-impact systems is one of the most essential factors, which proves their efficiency. The dynamic features of lump and linear systems are discussed in various scientific works which initiate the dependency to form their frequency and kinematics characteristics [2, 3]. Recently, proper nonlinear systems have been introduced on the level of resonance schemes [4], including the machines with electromagnetic and electromechanical drive [5, 6]. Traditional resonance machines with electromagnetic drive have mild nonlinear properties, which are mainly caused by parametric connections between force and kinematics characteristics [7]. Therefore, there is a tendency to use nonlinear qualities in resonance modes.

## IV. Purpose of the article

The article aims at a generalized approach to the synthesis of stiffness parameters of two-mass resonance vibration machine on the condition that a vibro-impact mode is being used. The accent is put on the frequency characteristics of nonlinear systems. In particular, a type of the system performance on synthesized resonance mode has been approved. Besides, meeting the requirements of the basic technological conditions (at which the machine runs), accordance with kinematic characteristics of vibro-impact mode and necessary technological resistance of the system (which works on a resonance mode providing a proper value of resonance zone width) is anticipated.

# V. The main statement

The construction of system is structured into two-mass and determined by inertial characteristics of oscillatory masses – mass  $m_1$  is working, mass  $m_2$  is reactive. The masses are connected by an elastic-dissipative element. The system disturbance acquires an impulsive nature. Construction is based by vibro-insulator.



Fig. 1. Vibratory system's structural scheme

The proposed system is designed to mould products made of concrete at high-frequency (50 Hz and over), where one of the essential characteristic of vibro-impact machine mode is an asymmetry coefficient [2] of working mass acceleration:

$$
k_a = \frac{a_{1max}}{|a_{1min}|},
$$

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 $a_{1max}$ ,  $a_{1min}$  – maximal positive value and minimal negative value of working mass acceleration, the recommended value for this process is  $k_1 = 3...6$ . Since the inertial parameters of both oscillatory masses  $m_1$  and  $m<sub>2</sub>$  are known, a stiffness characteristic is determined by variable stiffness coefficient c . The bilinear stiffness characteristic will have the following form:

$$
C = \begin{cases} C_{1}, & x \ge 0 \\ C_{2}, & x < 0 \end{cases}
$$
 (1)

The synthesis of that stiffness parameters is carried out based on the grounds of the influence of the synthesized coefficients on the kinematic characteristics. Thus, motion equation of a mechanical oscillatory system has a view:

$$
\begin{cases}\nm_1(1 + k_{\text{load}}) \ddot{x}_1(t) + c_{\text{is}} \cdot x_1(t) + b \cdot (\dot{x}_1(t) - \dot{x}_2(t)) + \\
\begin{cases}\nc_1 \cdot (x_1(t) - x_2(t)), \\
\text{if } x_1(t) - x_2(t) \ge 0 \\
\vdots \\
c_2 \cdot (x_1(t) - x_2(t)), \\
\text{if } x_1(t) - x_2(t) < 0\n\end{cases} = f(t); \\
m_2 \ddot{x}_2(t) - b \cdot (\dot{x}_1(t) - \dot{x}_2(t)) - \\
\begin{cases}\nc_1 \cdot (x_1(t) - x_2(t)), \\
\text{if } x_1(t) - x_2(t) \ge 0 \\
c_2 \cdot (x_1(t) - x_2(t)), \\
\text{if } x_1(t) - x_2(t) < 0\n\end{cases} = -f(t).\n\end{cases} \tag{2}
$$

where,  $k_{load}$  – coefficient of technological capacity [2]; (t) 4  $f(t) = F \sin\left(\frac{\omega t}{2}\right)$ J  $\left(\frac{\omega t}{2}\right)$ J  $= F \sin \left( \frac{\omega t}{2} \right)^4$  – idealized law of impulsive electromagnetic system disturbance with an amplitude force value is carried out F (this law is based on imitating modeling);  $b = 2 \frac{m_1 m_2}{g}$  $1 + \mathrm{III}$  $1 \cdot m_2$  $m_1 + m$  $b=2\frac{m_1 \cdot m_2}{m_1 \cdot m_2}$  $^{+}$  $= 2 \frac{m_1 \cdot m_2}{\sqrt{\omega}}$   $\zeta \omega$  – a coefficient of viscous friction;  $\zeta$  – damping parameters (nonmetric damping

coefficient) of driven resonance system as a part of crippling damping for a given system;  $c_1$ ,  $c_2$  and  $c_{is}$  – stiffness parameters of the basic springs and isolators;  $\omega$  – disturbance frequency.

Model's parameters numerical value are:

$$
m_1 = 60 \text{kg}; m_2 = 10,5 \text{kg}; k_{\text{load}} = 0.25 ;
$$
  

$$
\varsigma = 0.2; F = 1.2 \text{ kN}; c_{\text{is}} = 2 \times 10^4 \text{ N/m};
$$
  

$$
\omega = 314 \text{ rad/s}; b = 1.12 \times 10^3 \text{ N} \cdot \text{s/m}.
$$

Synthesized stiffness parameters are unknown and defined based on relations:

$$
c_1 = \frac{m_1 m_2}{m_1 + m_2} \cdot \omega_{01}^{2},
$$
  
\n
$$
c_2 = \frac{m_1 m_2}{m_1 + m_2} \cdot \omega_{02}^{2},
$$
\n(3)

where  $\omega_{01} = \Theta \cdot \frac{\omega}{z}$ ;  $\omega_{02} = \Lambda \cdot \omega_{01} = \Lambda \cdot \Theta \cdot \frac{\omega}{z}$  – proper

oscillation frequency values by stiffness coefficients;  $\Theta$  – frequency coefficient;

 $\Lambda$  – proper frequency relations;  $\sigma$  – resonance settings (takes  $z = 1$ ). Based on this, unknown stiffness characteristics will be defined by the value of synthesized coefficients  $\Theta$  and  $\Lambda$ :

$$
c_1(\Theta) = \frac{m_1 m_2}{m_1 + m_2} \cdot (\Theta \cdot \omega)^2,
$$
  
\n
$$
c_2(A, \Theta) = \frac{m_1 m_2}{m_1 + m_2} \cdot (A \cdot \Theta \cdot \omega)^2.
$$
 (4)

It certainly simplifies the task in contrast to the synthesis by stiffness parameters, which are located in considerably much wider ranges.

The fixed value of proper oscillation frequency for gapless stiffness characteristic  $(Eq. (1))$  is defined by appropriate synthesized parameters:

$$
\Omega_0 = \frac{2\omega_{01}\omega_{02}}{\omega_{01} + \omega_{02}} = \frac{2\Theta A}{A + 1}\omega.
$$
 (5)

Researched kinematic characteristics essentially depend on synthesis coefficients (fig. 2), accomplished in accordance with kinematic characteristics of a vibro-impact mode and fixed value of proper oscillation frequency (of a resonance zone width) in the appropriate range:

$$
10\text{m/s}^2 < |a_{1\text{min}}(\Theta, \Lambda)| < 15\text{m/s}^2, \\
3 < k_a(\Theta, \Lambda) < 6, \\
\Omega_0 \min < \Omega_0(\Theta, \Lambda) < \Omega_0 \max,\n\tag{6}
$$

An algorithm for the selection of synthesized coefficient and the calculation of determinant parameters was carried out to tackle the problem (Eq. 6).

The feature of the given algorithms is to meet technological conditions and to search for the maximum of maximal value of working mass acceleration, low power and on the set frequency band. An objective function of the optimization has a view of specific energy accelerationpower criteria, which also depends on factors synthesis

$$
\xi_{a}(\Theta, \Lambda) = \frac{a_{1\max}(\Theta, \Lambda)}{p_{d}(\Theta, \Lambda)} \to \max. \tag{7}
$$

 $p_d = \text{stdev}[f(t) \cdot (v_1(t, \Theta, A)) - v_2(t, \Theta, A)]$  – power of mechanical oscillation system.

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Fig. 2. An objective function (a) and technological limitation (b)-(d) and their dependence on factors synthesis

For solutions of optimization problem was developed the program in MathCAD (fig. 3). Synthesized factors  $\Theta$  and  $\Lambda$  that provide relevant kinematic constraints are the main results of the program.

fstmin:=  
\nr ← rows(almost) - 1  
\nc ← cols(almost) - 1  
\n  
\n
$$
v ← \zeta_{a_{0,0}}
$$
  
\nfor m ∈ 0..r  
\nfor n ∈ 0..c  
\nif  $v < \zeta_{m,n}$   
\n $3 < ka_{m,n} < 6 \land 10 < almin_{m,n} < 15 \land 55 < \frac{\Omega_{m,n}}{2\pi} < 70$   
\n  
\n $v ← \zeta_{a_{m,n}}$   
\n $d ← ka_{m,n}$   
\ne ← almin<sub>m,n</sub>  
\nf ←  $\Theta_m$   
\ng ←  $\Lambda_n$   
\n $\Omega_{m,n}$   
\nh ←  $\frac{\Omega_{0_{m,n}}}{2\pi}$   
\n(v d e f g h)  
\nfstmin = (0.253 3.715 13.982 0.85 4.5 69.545)

Fig. 3. Program optimization problem of brute parameter's values

The vector of the parameters which are involved in the synthesis by program at the fig.3 in accordance with the main resonance near 50Hz within the possible

residence of proper oscillation frequency  
50 Hz 
$$
<\frac{\Omega_0(\Theta, \Lambda)}{2\pi}
$$
 < 70 Hz has the following form:

$$
\mathbf{X} = \begin{pmatrix} \xi_a \\ k_a \\ a_{1\text{min}} \\ \Theta \\ A \\ \frac{\Omega_0}{2\pi} \end{pmatrix} = \begin{pmatrix} 0.253 \left[ (\text{m/s}^2)/\text{W} \right] \\ 3.715 \left[ - \right] \\ -13.982 \left[ \text{m/s}^2 \right] \\ 0.85 \left[ - \right] \\ 4.5 \left[ - \right] \\ 69.55 \left[ \text{Hz} \right] \end{pmatrix}.
$$

The main kinematics' characteristic in time – acceleration of working mass is at the fig. 4. It has a pronounced asymmetric view with calculated coefficient  $k_a$ .



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Frequency dependence of main kinematics and energy parameters are one of the most characteristic's of the resonance systems. The calculated kinematic constraints and power that depend on the frequency of disturbance are on the fig. 5. The chart for maximal value of acceleration (Fig.5, a) indicate near resonance mode, defined by high energy efficiency.



Fig. 5. Frequency dependence of technological limitations (a) and acceleration-power criteria of a system (b)

# **Conclusion**

The parametric synthesis model of asymmetric lumplinear stiffness characteristic is proposed depending on introduced synthesis coefficients, which determine kinematic and dynamic (in particular frequency) features of vibro-impact system. Optimization problem based on energy criteria, allowing to operate main characteristics of the machine. The results obtained by numerical calculations of differential equations. Also used a program with the evaluation of limitations and the objective function for selection of parameters. The technique is useful for designing of process-type resonant vibration machines.

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