Mathematical Model for Investigation of Wave Processes in High-Voltage Transformers

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***Abstract* – A mathematical model intended for research of wave processes in high-voltage transformers was developed. The variable separation method was used to solve the integral-differential equations describing the process under research. The voltage distribution in the transformer windings under the influence of the impulse overvoltages was analyzed.**

Key words – transformer winding, mathematical modelling, impulse overvoltage, partial derivatives, initial conditions, boundary conditions.

I. Introduction

In general the mathematical modelling of wave processes transformers can be classified respectively to mathematical methods used for analysis of the problem. These approaches are known as "white box", "black box" and "grey box" modelling methods.

The "white box" modelling method requires formation of separate mathematical models of different elements of the electric power system with consideration of all parameters of its equivalent circuit. Such an approach allows us to analyse the process inside the element under research.

The "black box" approach does not require construction of the mathematical model for every element of electric power system. In this approach the coordinates of element's model are calculated or measured on the input and output terminals of the oblect. For this purpose the method of state variables is used because it allows to analyse and synthesize electric power system from the point of view of electromagnetic compatibility of all its elements. It makes it possible to define currents and voltages at any time moment.

The "Grey box" method is a combination of "White box" and "Black box" approaches as it was introduced in [3].

The main reason to develop the mathematical model of the transformer based on "white box" modelling method is to calculate the voltage along the transformer winding under the impulse overvoltages of different shapes and free oscillations inside the winding that allows to coordinate the insulation of the windings. Coordination of insulation is a set of measures which prevent the damaging of the insulation of all electric power system elements with minimal costs.

The mathematical analysis of internal voltage oscillations in the transformer windings can be divided into the next stages:

1) formation of differential equations describing the voltage fluctuations based on equivalent circuit of the transformer winding;

2) formation of the boundary conditions at the windings ends;

3) determination of the initial voltage distribution along the winding under the pulse overvoltage;

4) determination of the final voltage distribution;

5) initial voltage distribution along the winding voltage to be equal to the final distribution functions plus free oscillations for the initial point ($t = 0$);

6) solving of the equations under influence of the wave with an arbitrary shape.

Voltage oscillations in the transformer windings cause appearing of overvoltages relatively to the ground as well as between the winding coils and transformer windings. These amplitudes and gradient overvoltages can be reduced by:

1) decreasing of internal oscillating overvoltages in the transformer winding;

2) influencing the overvoltage applied to the winding;

3) influencing the transformer neutral voltages.

These three factors are very important during the transformer circuits protection from overvoltages.

Therefore the goal of the research is to develop the mathematical models intended for the research of wave processes in the windings of high voltage two- windings transformers influenced by impulse overvoltasges based on solving of differential equations in partial derivatives by separation of variables. During the study of wave processes in the transformer windings it is necessary to consider the relationship between the propagation time of electromagnetic wave along the entire length of the winding and time interval when current and voltage change more evidently in comparing with their full changes during process to be considered. Voltage oscillation in the coil arises with the frequency corresponding to the whole winding, and with its own frequency of the coil. These oscillations depend on the inductive and capacitive coupling in the winding. Free oscillations are characterized by high frequencies and they almost completely damp.

In the paper a new approach to form a mathematical model which will consider the distributed parameters and allow to conduct calculation of pulse overvoltages with considering of all parameters of the equivalent circuit. In order to study the wave processes in transformers while external overvoltages take place it is necessary to simulate the shape of the wave. Empirical equation, which describes the shape of the pulse wave is [1, 2]:

$$e = E(e^{-at} - e^{-bt}), \quad (1)$$

where E is maximmum value of the wave potential; a and b are parameters of the wave and t is a time parameter.

Impulse wave is characterized by the following parameters: the maximum value of the wave potential (amplitude); the length of the front; part of the wave from 0 up to E_{max}; the length of the wave; part of the wave from E_{max} to the half of the amplitude; and by the sign of the amplitude.

The wave tail can be reached approximately for 20 ms. Therefore it is necessary to select parameters of the equation (1), and if a and b have real values then the equation (1) can represent the wave with rounded front and exponential tail.

II. Mathematical Model

Equations in partial derivatives which describe the wave processes in the transformer windings can be written down as [3-6]

$$-\partial i_n / \partial x = g_0 u_n + C_0 \partial u_n / \partial t - \\ -C_{M0} \partial^3 u_n / (\partial x^2 \partial t), \qquad (2)$$

$$-\partial u_n / \partial x = r_0 i_n + L_0 \partial i_n / \partial t. \qquad (3)$$

Let us use in equations (2) and (3) one variable $u(x,t)$ For this purpose the equation (2) should be differetiated by t and multiplied by L_0:

$$L_0 \partial^2 i_n / (\partial t \partial x) + L_0 g_0 \partial u_n / \partial t + \\ + L_0 C_0 \partial^2 u_n / \partial t^2 - \\ - L_0 C_{M0} \partial^4 u_n / (\partial x^2 \partial t^2) = 0, \qquad (4)$$

and the equation (3) was differentiated by x

$$\partial^2 u_n / \partial x^2 + r_0 \partial i_n / \partial t + L_0 \partial^2 i_n / (\partial t \partial x) = 0. \qquad (5)$$

Let us subtract the equation (5) from equation (4). As a result of this operation we have

$$-L_0 C_{M0} \partial^4 u_n / (\partial x^2 \partial t^2) + L_0 C_0 \partial^2 u_n / \partial t^2 - \\ - \partial^2 u_n / \partial x^2 + L_0 g_0 \partial u_n / \partial t - r_0 \partial i_n / \partial x = 0. \qquad (6)$$

Let us substitute into the equation (6) instead of the derivative di_n/dx its value from the equation (2). After this operation we can obtain an equation which contain just one unknown function $u(x,t)$ or in other words we have

$$-L_0 C_{M0} \partial^4 u_n / (\partial x^2 \partial t^2) - \\ - r_0 C_{M0} \partial^3 u_n / (\partial x^2 \partial t) + \\ + L_0 C_0 \partial^2 u_n / \partial t^2 - \partial^2 u_n / \partial x^2 + \\ (r_0 C_0 + L_0 g_0) \partial u_n / \partial t + r_0 g_0 u_n = 0. \qquad (7)$$

To solve the equation (7) let us use the method of variable separation.

We will consider that $a = L_0 C_{M0}$, $b = r_0 C_M$, $c = L_0 C_0$, $h = r_0 C_0 + L_0 g_0$, $s = r_0 g_0$.

In such a case

$$-a \partial^4 u_n / (\partial x^2 \partial t^2) - b \partial^3 u_n / (\partial x^2 \partial t) + \\ + c \partial^2 u_n / \partial t^2 - \partial^2 u_n / \partial x^2 + h \partial u_n / \partial t + s u_n = 0. \qquad (8)$$

Initial and boundary conditions can be written down as follows:

$$u|_{t=0} = u_0(x), \ x \in (0, l); \qquad (9)$$

$$\partial u / \partial t|_{t=0} = u_1(x); \qquad (10)$$

$$u|_{x=0} = f_0(t), \ t > 0; \qquad (11)$$

$$u|_{x=l} = f_1(t). \qquad (12)$$

We have a concorance of conditions

$$f_0(0) = u_0(0); \quad f_1(0) = u_1(l); \\ f_0'(0) = u_1(0); \quad f_1'(0) = u_1(l). \qquad (13)$$

We should find functions $A(t)$ and $B(t)$ to ensure the homogeneus conditions (11) (12) when substitition of

$$u(x,t) = V(x,t) + A(t) + xB(t), \qquad (14)$$

will give for $V(x,t)$ homogeneus conditions (11)-(12),

$$V|_{x=0} = 0; \ V|_{x=l} = 0. \qquad (15)$$

So, according to (14) we have that

$$u|_{x=0} = V|_{x=0} + A(t) = f_0(t);$$

$$u|_{x=l} = V|_{x=l} + A(t) + lB(t) = f_1(t),$$

from where we can see that

$$A(t) = f_0(t); \\ B(t) = 1/l(f_1(t) - f_0(t). \qquad (16)$$

Instead of (9)-(10) the following conditions apper

$$u|_{t=0} = V|_{t=0} + A(0) - xB(0) = u_0(x); \\ \partial u / \partial t|_{t=0} = \partial V / \partial t|_{t=0} + A'(0) + \\ + xB'(0) = u_1(x), \qquad (17)$$

or

$$V|_{t=0} = u_0(x) - A(0) - xB(0) \equiv V_0(x); \\ \partial V / \partial t|_{t=0} = u_1(x) - A'(0) - xB'(0) \equiv V_1(x). \qquad (18)$$

An equation for the variable V we can obtain after substitution of equation (14) into the equation (8)

$$-a \partial^4 V / (\partial x^2 \partial t^2) - b \partial^3 V / (\partial x^2 \partial t) + \\ + c(\partial^2 V / \partial t^2 + A''(t) + xB''(t)) - \\ - \partial^2 V / \partial x^2 + h(\partial V / \partial t + A'(t) + \\ + xB'(t)) + s(V + A(t) + xB(t)) = 0, \qquad (19)$$

or

$$-a \partial^4 V / (\partial x^2 \partial t^2) - b \partial^3 V / (\partial x^2 \partial t) + \\ + c \partial^2 V / \partial t^2 - \partial^2 V / \partial x^2 + \\ + h \partial V / \partial t + sV = F(x,t), \qquad (20)$$

where $F(x,t)$ is known right part, namely

$$F(x,t) = -cA''(t) - cxB''(t) - hA'(t) - \\ - hxB'(t) - sA(t) - sxB(t). \qquad (21)$$

Equations (8)-(12) where replaced by equations (20), (18), (15).

A solution should be found out in the form of Fourier series

$$V(x,t) = \sum_{k=1}^{\infty} c_k(t) \sin \pi k x / l, \ 0 < x < l. \qquad (22)$$

The condition (15) is fulfilled if $C_k(t)$ fast enough approaches zero if $k \to \infty$.

Let us expand functions $V_0(x)$ and $V_1(x)$ in a Fourier series, hence

$$V_0(x)=\sum_{k=1}^{\infty}\alpha_k\sin(\pi kx/l);\quad V_1(x)=\sum_{k=1}^{\infty}\beta_k\sin(\pi kx/l). \tag{23}$$

Conditions (18) for the function (22) will be fulfilled if

$$C_k(0)=\alpha_k;\quad C'_k(0)=\beta_k, \tag{24}$$

where $k = 0,1,\ldots$

Using 22 transform the left side 19 so

$$-a\partial^4V(x,t)/(\partial x^2\partial t^2)-c\partial^3V(x,t)/dt^2= =\sum_{k=1}^{100}(a(\pi k/l)^2+c)\times \times\partial^2C_k(dt^2)/(\partial t^2)\sin(\pi kx/l)); \tag{25}$$

$$-b\partial^3V(x,t)/(\partial^2udt)-h\partial V(x,t)/dt= =\sum_{k=1}^{100}(b(\pi k/l)^2+h)\times \times\partial C_k(t)/(\partial t^2)\sin(\pi kx/l); \tag{26}$$

$$-\partial^2V(x,t)/(\partial x^2)-s\partial V(x,t)/dt= =\sum_{k=1}^{100}(b(\pi k/l)^2+s)C_k(t)\sin(\pi kx/l). \tag{27}$$

We introduce the notation:

$$a_1=(\pi k/l)^2+c;\quad b_1=(\pi k/l)^2+h;\quad c_1=(\pi k/l)^2+s. \tag{28}$$

The result is equation

$$a_1d^2C_k(t)/(dt)+ +b_1dC_k(t)/dt+c_1C_{k(t)}=\gamma_k(t), \tag{29}$$

or

$$a_1e_1(t)+b_1e_2(t)+c_1=0. \tag{30}$$

Write characteristic equation

$$a_1\lambda^2+b_1\lambda+c_1=0, \tag{31}$$

at $k\to\infty$ get

$$\lambda^2+\lambda+1=0. \tag{32}$$

Solve the solution of equation as

$$\lambda_{1,2}=\delta+j\theta;\quad e_1(t)=e^{\delta t}\cos(\theta\cdot t);\quad e_2(t)=e^{\delta t}\sin(\theta\cdot t).$$

According to the method of variation of arbitrary constants we obtain C_k as

$$C_k(t)=B_1(t)e_1(t)+B_2(t)e_2(t). \tag{33}$$

Function $B_1(t)$ and $B_2(t)$ looking from the system of equations

$$\frac{dB_1(t)}{dt}e_1(t)+\frac{dB_2(t)}{dt}e_2(t)=0;\quad \frac{dB_1(t)}{dt}\frac{de_1(t)}{dt}+\frac{dB_2(t)}{dt}\frac{de_2(t)}{dt}=\frac{1}{a_1}\gamma_k(t), \tag{34}$$

entering a designation

$$\Delta=\begin{vmatrix}e_1(t) & e_2(t)\\ de_1(t)/dt & de_2(t)/dt\end{vmatrix},$$

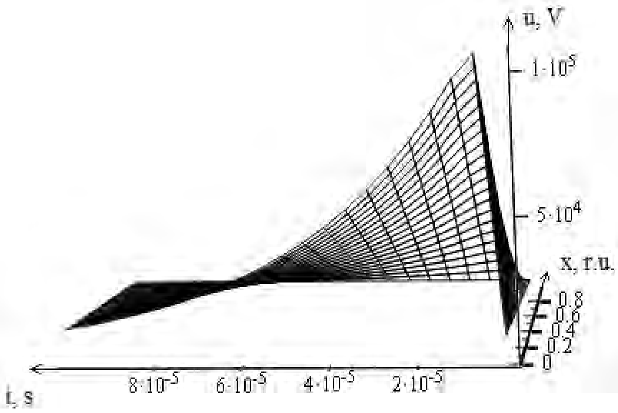
and find

$$dB_1(t)/dt=-\frac{1}{a_1\Delta}\gamma_k(t)e_2(t);\quad dB_2(t)/dt=-\frac{1}{a_1\Delta}\gamma_k(t)e_1(t). \tag{35}$$

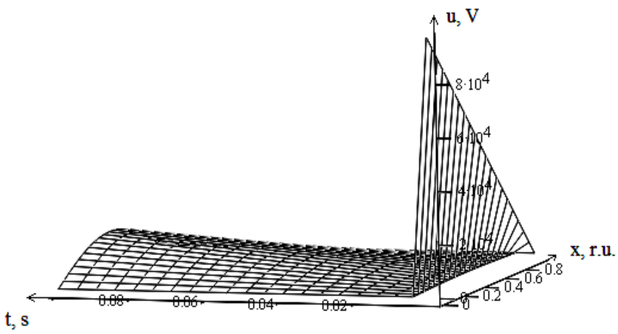
$B_1(t)$ and $B_2(t)$ get as

$$B_1(t)=\int_0^t dB_1(t)/dt+B_1(t)_{t=0};$$

$$B_2(t)=\int_0^t dB_2(t)/dt+B_2(t)_{t=0}.$$



a) $t = 1\cdot10^{-7}\ldots\ 1\cdot10^{-4}$ s



b) $t = 1\cdot10^{-4}\ldots$ 0.1 s

Fig. 1. The voltage distribution along winding of transformer for variable values of time

Conclusion

Derived mathematical approach makes it possible to investigate the wave processes in the windings of the high-voltage transformers taking into account the dependence of voltage and current on the winding length and time. Under research of wave processes in the windings of transformers it is necessary to take into account the ratio between the time interval of propagation of electromagnetic waves along the entire length of the winding and an interval of time when current and voltage are changed more significantly in comparing with their full change in the process that was under research.

References

[1] O. Kyrylenko, M. Seheda, O. Butkevych and T. Mazur, "Matematychne modeliuvannia v elektroenergetytsi" ["Mathematical modeling in electric power engineering"], Lviv, Ukraine: Lviv Polytechnic National University Publ., 2013, 608 pages.

[2] M. Seheda, "Matematychne modeliuvannia khvyliovych procesiv y transformatorah" ["Mathematical Modeling of Wave Processes in Transformers"], Tekhnichna elektrodynamika - Technical electrodynamics, No.3, pp.47-49, 2002.

[3] Electrical Transient Interaction between Transformers and the Power System. CIGRE. April 2014.

[4] M. Seheda, Y. Cheremnykh and T. Mazur, "Matematychne modeliuvannia vilnykh kolyvan naprugy y transformatornykh obvytkah z vrahuvanniam mizhvyt-kovyh vzajemoinduktyvnostej za umovy vplyvy perenapgug" ["Mathematical modeling of free voltage oscillations on transformer windings taking into account interwinding mutual induction under surge overvoltages"], Naukovyi visnyk natsionalnoho hirnychoho universytetu - Scientific Bulletin of National Mining University, No.1(133), pp.68-76, 2013.