# Mathematical Modeling and Experimental Study of Impulse Lines of Flowmeters


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***Abstract* – *Mathematical model of an impulse line with a pressure transducer (PT) was built. This model provides the possibility to simulate and analyze the transient processes in an impulse line. Experimental studies of the step response curves of an impulse line with PT were carried out with application of a high-frequency analog-to-digital converter of the pressure signal in the PT chamber. The simulated step response curves were compared to the experimental ones. Maximum relative deviation of the simulated step response curves from the experimental ones is 6.3 %.***



Key words – impulse line, mathematical model, step response, experimental study, simulation.


## I. Introduction

The accuracy of flow rate measurement is a very important issue today especially for fluid energy carriers flow rate and volume measurement. The differential pressure method is one of the most widespread methods for flow rate measurement. The flow meters based on this method are simple in construction and they do not need individual calibration. However, flow pulsations may take place in the measuring sections of the pipes. These pulsations may lead to a negative impact on the accuracy of flow rate and volume measurement since they may cause the additional errors of measurement.

Impulse lines may be a source of the error of measurement. Based on the experimental and theoretical study [1, 2] it was defined that an impulse line (IL) together with the chamber of a pressure transducer is an oscillating system where resonance may take place. The resonance in the impulse line leads to the additional error of flow rate and volume measurement.

## II. Mathematical Model

In order to develop the mathematical model of the IL it was considered that the pressure signal from the pipe (or from the carrier ring of an orifice plate) was transferred by means of the IL to the chamber of the PT (or the differential pressure transducer (DPT)) and there was a connecting nipple (throttle) between them. The internal diameter of the connecting nipple (throttle) is smaller than the internal diameter of the IL. That is why the nipple was considered as a cylindrical throttle. The following assumptions were made during the development of the mathematical model:

- the temperature of the gas in the IL is the same along the IL and does not vary in time;
- the volume of the chamber of PT is constant;
- the molar mass and the compressibility factor of the gas are constant within the time of investigation of the pressure pulsations.

The mathematical model was developed based on the following equations and formulae:

- the modified Mendeleev-Clapeyron equation of state for defining the mass of gas in the chamber of the PT;
- the Newton's second law for the description of the gas movement in the IL;
- the Poiseuille equation for the laminar regime of flow;
- the Darcy-Weisbach equation for the turbulent regime of flow;
- other equations of the classical fluid dynamics theory.

The mathematical model of IL was built and studied for the following three cases:

- laminar flow in the IL and laminar flow through the nipple (throttle);
- turbulent flow in the IL and turbulent flow through the nipple (throttle);
- laminar flow in the IL and turbulent flow through the nipple (throttle).

Based on the investigation results it was found that the mathematical model of IL for the laminar flow in the IL and laminar flow through the nipple (throttle) provides transient processes similar to those derived experimentally.

By analyzing the results of experimental study (see below) it was found that the transient processes in the IL should be described by means of a system of three differential equations. That is why one more equation was introduced into the initial system of two differential equations. The third equation was the equation for the pressure measuring transducer.

The mathematical model of IL can be presented by the following system of three linear differential equations

$$\begin{cases} T_{MT}\dfrac{dP_{Meas}}{dt}+P_{Meas}=P_{Ch}, \\ A\dfrac{dP_{Ch}}{dt}=Q\cdot\rho, \\ T_1\dfrac{dQ}{dt}+Q\cdot(1+\dfrac{K_L}{K_{L_N}})=K_L\cdot\dfrac{(P_{in}-P_{Ch})}{\rho}, \end{cases} \quad (1)$$

where $T_{MT}$ is time constant of PT, $P_{Meas}$ is measured pressure by PT (output signal), $P_{Ch}$ is pressure in the chamber of PT, $Q$ is gas flow rate in IL, ρ is gas density, $P_{in}$ is pressure at the input of the IL.

The system of differential equations (1) was reduced to the following transfer function:

$$W(p)=\frac{1}{(k_3T_{33})^3p^3+(k_2T_{32})^2p^2+k_1T_{31}p+1}. \quad (2)$$

This transfer function was applied for simulation of transient processes in IL.

## III. Experimental Study

The experimental study of the step response curves of the impulse lines with the pressure transducer was carried out by means of a rig where air was used as a working gas. There were the following main devices and components in the rig:

- impulse line with the internal diameter of 4 mm and with the following three different lengths ($L_{IL}$): 2.1 m; 4.2 m; 6.3 m;
- electro-magnetic valve ZGRCH6UP, Bürkert;

- pressure transducer PPS.3-PN (overpressure), Techprylad, the upper limit of measurement is 10 kPa, the main error of measurement is 0.15 %;
- differential pressure transducer Sitrans P DSIII (applied for pressure measurement), Siemens, the upper limit of measurement is 10 kPa, the main error of measurement is 0.15 %;
- analog-to-digital converter NI USB-6009, National Instruments, the resolution is 12 bit, the sampling frequency is 48 kHz;
- compressor UK25-16M with the maximum output pressure of 343.2 kPa;
- pressure stabilizer SAD-305;
- vessel ($D$ = 180 mm, $H$ = 310 mm, $V$ = 0.0079 $m^3$).

Three series of tests were carried out for the ILs of different lengths (2.1 m; 4.2 m; 6.3 m). There were 10 tests in each series for the IL of a particular length, i.e. 10 transient processes were obtained for the same step change of the pressure for each IL. The tests were carried out in the following way:

1) The electro-magnetic valve was opened and the pressure in the whole system (vessel and the IL) was set to 4 kPa.

2) The electro-magnetic valve was closed.

3) New value of the pressure (8 kPa) was set in the vessel by means of the pressure stabilizer.

4) The electro-magnetic valve was opened abruptly. The transient process in the chamber of the PT took place and the electric output signal of PT was recorded.

5) To carry out the next test the pressure in the whole system was reduced down to 4 kPa and steps 2-4 were accomplished.

The value of the step change of pressure at the input of the IL in each test was 4 kPa.

At the result of processing and averaging the 10 transient processes for each IL the averaged step response curves were obtained. The values of the averaged curves for each specific moment of time were obtained as a mean arithmetic value of the 10 transient processes in the specific moment of time.

## IV. Comparison of Experimental and Simulation Results

The values of time constants $T_{31}$, $T_{32}$, $T_{33}$ and correction factors $k_1$, $k_2$, $k_3$ in (2) were calculated for the parameters of the experimental test rig. The results of calculation are presented in Table 1 and Table 2.

The simulated transient processes in IL on the basis of the mathematical model (2) were compared to the experimental step response curves. The comparison for $L_{IL}$ = 2.1 m is presented in Fig.1. The same comparison was made for other values of impulse line length ($L_{IL}$).

TABLE 1

TIME CONSTANTS $T_{31}$, $T_{32}$, $T_{33}$

| $L_{IL}$, m | $T_{31}$, s | $T_{32}$, s | $T_{33}$, s |
|---|---|---|---|
| 2.1 | 0.0074 | 0.0106 | 0.0075 |
| 4.2 | 0.0105 | 0.0150 | 0.0095 |
| 6.3 | 0.0135 | 0.0184 | 0.0108 |

TABLE 2

CORRECTION FACTORS $k_1$, $k_2$, $k_3$

| $L_{IL}$, m | $k_1$ | $k_2$ | $k_3$ |
|---|---|---|---|
| 2.1 | 0.5541 | 0.2736 | 0.3600 |
| 4.2 | 0.5143 | 0.2800 | 0.3579 |
| 6.3 | 0.5185 | 0.2772 | 0.3889 |

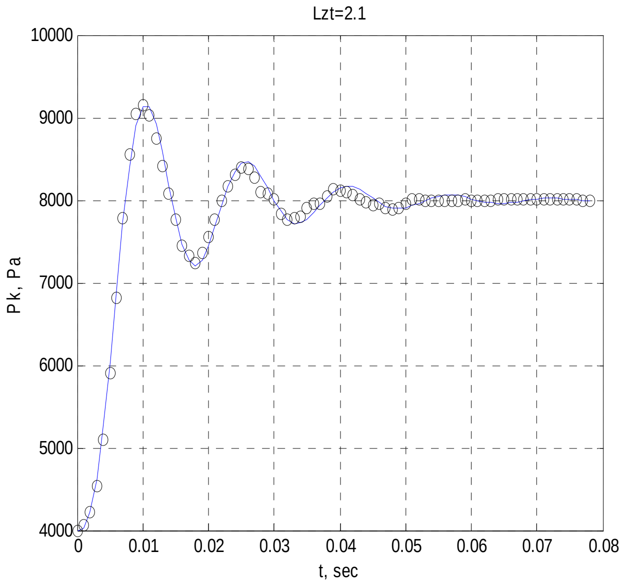


Fig. 1. Comparison of simulated transient process (-) with the experimental step response curve (o)

As we can see from Fig.1 the simulated transient process is close to the experimental step response curve. The maximum relative deviation of the simulated curve from the experimental one is 6.3 %.

## Conclusion

Mathematical model of an impulse line was built. Simulated transient processes were compared to the experimental step response curves.

## References

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