Analysis of Robust Stability of Electromechanical Systems, Described by Fractional Order Transfer Function

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***Abstract* – The majority of electromechanical systems are characterized by parametric uncertainty. This paper develops the approach to determine the stability of control object, which is an important step in the selection process of fractional controller parameters during its robust synthesis of desired quality of the transition process. The research of transfer functions of electromechanical systems on the stability of fractional order differs from traditional systems. The method of approach to the definition of stability suitable for stability analysis of electromechanical systems, described by transfer functions of both fractional and integer order, has been examined. Informative parameter for quality control and stability of electromechanical systems has been established, which allows to obtain information about the stability, oscillation and the quality of the transition process of an output coordinate of electromechanical system in the first approximation.**

Key words – stability, robustness, electromechanical system, fractional order transfer function.

I. Introduction

Robust stability of integer order systems has been examined in the number of works [1]. The question of stability of fractional order systems has been considered in [2,3] regardless of the uncertainty of its parameters, that is why the question of robust stability of fractional order electromechanical systems (EMS), and developing approaches to its operational control with the prospect of developing self-tuning systems remains topical. This paper develops the approach to the creation of "restrictive zone" of control object stability in the selection process of fractional controller parameters during its robust synthesis of desired quality for the transition process.

Most EMS are characterized by parameter uncertainty and therefore the problem of stability of the system can be reduced to characteristic polynomial robust stability [1]. Research of fractional order EMS on the stability differs from traditional systems, which are described by transfer functions (TF) of integer order, in particular, stable system of fractional order may have roots in the right half of the complex plane.

II. Stability Analysis

In order to develop rational variant of alternative operational analysis of EMS robust stability condition in the construction of self-tuning systems, below there are considered the following options of approaches with the use of:

- complex ω plane of Riemann surface,
- Kharitonov theorem.

These studies should result in the particular informative parameter concerning EMS in the area of stable, unstable functioning or on the boundaries of these zones, and approaching of this parameter to one of these zones to predict the nature of the transition process.

The first approach is based on the research presented in [2]. According to this approach, EMS characteristic equation is written in the form of fractional order general linear system:

$$a_n s^{\alpha_n} + \ldots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0} \equiv \sum_{i=0}^{n} a_i s^{\alpha_i} = 0, \qquad (1)$$

rewritten in the form

$$\sum_{i=0}^{n} a_i s^{\frac{\mu_i}{\upsilon_i}} = 0,$$

and transformed in the ω plane

$$\sum_{i=0}^{n} a_i \omega^i = 0, \qquad (2)$$

where $\omega = s^{\frac{k}{m}}$, m is the lowest multiple υ_i common.

Next, stability analysis is conducted in the following sequence.

1) To set a_i it is necessary to calculate the roots of equation (2) and find their absolute phase $|\phi_\omega|$.

2) The roots of the original sheet of Riemann surface ω plane must comply with roots on the s-plane and can be obtained by finding all the roots which lie in $|\phi_\omega| < \pi/m$ with the further application of the reverse transformation $s = \omega^m$. The sector with $|\phi_\omega| > \pi/m$ no has no physical meaning.

3) Stability condition $\pi/2m < |\phi_\omega| < \pi/m$. Condition of oscillations $|\phi_\omega| = \pi/2m$, otherwise the system is unstable. If there are no roots on the physical s-plane, the system will always be stable [2].

The search of the informative parameter for implementing robust control in the framework of this approach to stability analysis is exemplified by a linear system of fractional order, described by transfer function

$$W(s) = \frac{1}{0.8s^{2.2} + 0.5s^z + 1}, \qquad (3)$$

by changing the value of the fractional exponent z = 0.9; 1.9; 1.7.

The corresponding characteristic system equation:

$$P(s): 0.8s^{2,2} + 0.5s^z + 1 = 0 \Rightarrow$$

$$\Rightarrow 0.8s^{\frac{22}{10}} + 0.5s^{\frac{z}{10}} + 1 = 0$$

at m=10, $\omega = s^{1/10}$ polynomial has the form

$$P(\omega): 0.8\omega^{22} + 0.5\omega^{10z} + 1 = 0.$$

Let us consider just the root ω_i with lowest module angles and their respective angles ϕ_i. The results are summarized in Table 1 where TF (3) with the change of z is checked, and Fig. 1 shows the placement of the roots in the complex ω plane of Riemann surface of function on condition of stability $\pi / 2m < |\phi_{\omega}| < \pi / m$, i.e. $0.157 < |\phi_{\omega}| < 0.314$ rad.

TABLE 1

RESULTS OF STABILITY CONDITION OF LINEAR SYSTEMS STUDY

N	TF $W(s)$	$\omega_{21,22}$,	$\lvert\phi_{21,22}\rvert$, rad.	State of system
1	$\frac{1}{0.8s^{2.2}+0.5s^{0.9}+1}$	1.0045± 0.1684i	0.1661	stable
2	$\frac{1}{0.8s^{2.2}+0.5s^{1.9}+1}$	0.9774± 0.1486i	0.151	unstable
3	$\frac{1}{0.8s^{2.2}+0.5s^{1.7}+1}$	0.9786± 0.1546i	0.157	on the stability boundary, oscillating

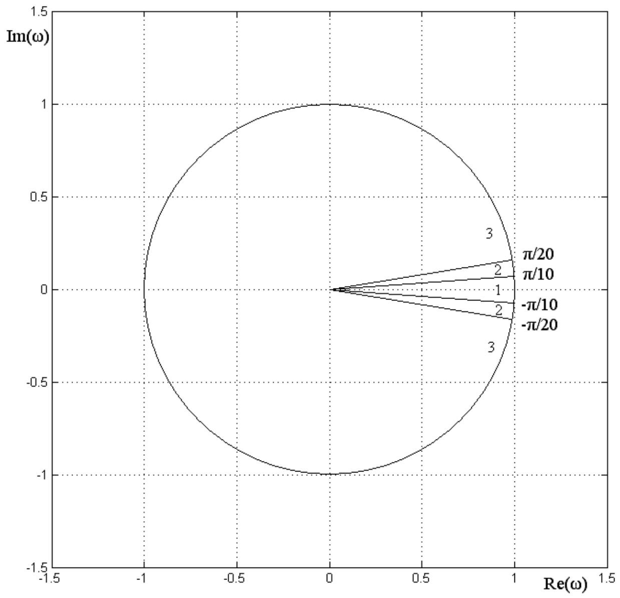

Fig. 1. Complex ω plane of Riemann surface of function $\omega = s^{1/10}$ with indicated zones of stability / instability: 1 – unstable zone; 2 – stable zone; 3 – the zone has no physical meaning.

Fig. 1 shows complex ω plane of Riemann surface of function $\omega = s^{1/10}$ with indicated zones of stability / instability (1 – unstable zone; 2 – stable zone; 3 – the zone, which has no physical meaning). Thus, the change in z degree leads to changes in the roots of a polynomial and, consequently, state of the system.

Reliability of the results is checked by studying polynomials (3) in MATLAB Simulink environment by using Ninteger packet. Figure 2 shows the results of these studies, including the "a" - a reaction to a single jump for TF (1), "b" - TF (2) and "c" - TF (3).

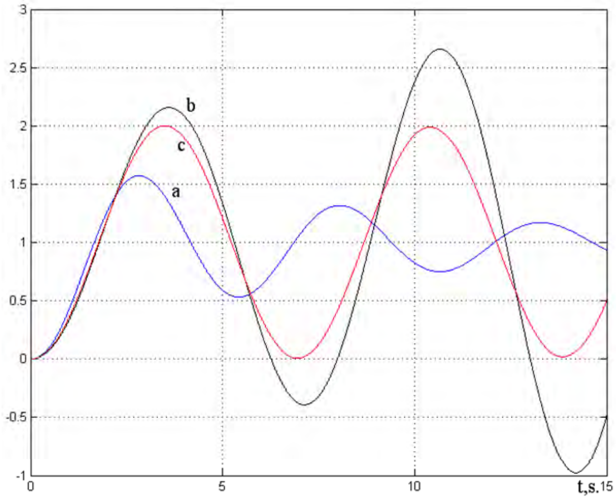

Fig. 2. Transition function units EMS with TF (1) – curve "a", (2) – curve "b" and (3) – curve "c".

So, robust stability analysis of fractional order linear systems with regards to this approach makes it possible to monitor the placement of the obtained roots of polynomials in the complex ω plane of Riemann surface of function $\omega = s^{1/10}$ (Fig. 1) and thus to control the condition of stability.

Conclusion

1. The method of approach to the definition of stability as considered in the article is suitable for stability analysis of EMS, described by both fractional and integer order transfer functions.

2. Parameter $|\phi_{n-1,n}|$ is informative to control quality and stability of EMS. If the parameter $|\phi_{n-1,n}|$ = 0.314 rad., then in the transition function of output EMS coordinate the overshoot is missing. With the reduction $|\phi_{n-1,n}|$ from 0.314 rad. overshoot is growing and at $|\phi_{n-1,n}|$ = 0.157 rad. the oscillating mode arises. That is, the value of the parameters $|\phi_{n-1,n}|$ can provide information about the stability, oscillation and the quality of transition of output EMS coordinate in the first approximation.

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