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MODELLING OF THE ARTICULATED VEHICLE DYNAMICS TAKING INTO ACCOUNT THE ELASTICALLY FASTENED FREIGHT

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Abstract. In this article, some methodological statements of the formation of mathematical models in the problems of machine dynamics are elucidated. The presence of a cushioned freight in the form of the fastened vehicle complicates the system, so that the equations describing its oscillations are too complicated and unsuitable for practical usage. It is shown that with all the complexity of the research object, it is expedient to form the mathematical model at the level of the design diagram (model) with a relatively small number of degrees of freedom. The peculiarities of mathematical modelling of the motion processes of specialized articulated vehicles for transportation of cars, namely, formation of the design diagram (model), making the necessary assumptions, choosing the independent coordinates and writing down the second-order Lagrange equations describing the system oscillations, are presented. While modelling, the most typical case, namely, the motion over the unit prominence (road shock), the effect of which on the wheels of the articulated vehicle is of the impact type, is considered. The choosing of the initial perturbation in the form of a unit impact is caused by the possibility of obtaining an impact characteristic, which is defined as the reaction of the system on the δ-function. In addition, under the impact action, it is possible to carry out the direct visual analysis of non-interfered single-frequency oscillatory processes, which allows to quickly obtain the approximate express-evaluation of the amplitude values of the process, frequencies of oscillations, as well as the oscillations damping decrement of the two-section articulated vehicle, if it is necessary.

The mathematical model of the articulated vehicle consisting of a tractor and a semi-trailer in the form of a linear system having 9 degrees of freedom is developed. It consists of three cushioned masses, namely, the frame of the truck tractor, the frame of the semi-trailer and three non-cushioned masses, which are axles with wheels as an assembly. Both sections of this system are connected by the saddle-coupling element that is not absolutely rigid. While moving, the frames of the truck tractor and of the semi-trailer performs oscillatory motion in the vertical plane and angular oscillations about an axis passing through the centre of masses. The axles with wheels in the assembly oscillate only in the vertical plane. The peculiarity of the model is that the elastic and damping properties of the suspension of vehicle fixed on the semi-trailer are taken into account. The solution of the mathematical model allows to obtain the impact characteristic, to calculate the amplitude-frequency characteristic and to obtain the required (unknown) values of the root-mean-square accelerations of the non-cushioned masses.

The technology of systemic consideration of the problems of formation of mathematical models in the form of block diagrams (structure charts) which are equivalent in dynamic relation to the systems of automatic control is offered.

Keywords: articulated vehicle, dynamics, freight, transportation, cushioned mass, truck tractor, semi-trailer, suspension, impact characteristic, mathematical model.

Introduction

The works of many scientists are dedicated to analysis of suspension vibration of wheeled vehicles, to evaluation of motion evenness and their vibration-proof features [1]. The following problems were the most interesting to be solved by the researchers: studying the dynamic characteristics of the system; calculation of statistical characteristics of oscillations during the system motion, when it is influenced by random disturbances caused by the road shocks; research of the influence of structural parameters on oscillation processes and their optimization; studying the dynamics of certain elements of the system in order to obtain simpler equivalent schemes for considering their operation; unification into a single complex of some separate objects of the system road-tire-vehicle-driver with models of other parts of this system. At present, the development or improvement of the mathematical model of motion and its implementation as a software product are the most actual problems, where the special attention is to be payed to describing the solution of practical issues, especially of the issues of complementation and changes of the calculation scheme according to the predefined criteria, in order to increase the rate of the work, to reduce the number of experimental models and eventually to improve the quality and efficiency of experimental and design work at the design stage.

Problem statement

The investigation of vibratory processes in the specialized wheeled vehicles transporting specific freights, including cushioned ones, are of the important problems because the requirements to the road transport are becoming increasingly rigorous.

At present, very few scientific investigations has been carried out on the problems elucidated. In the main, they were conducted for trucks (lorries) and general-purpose articulated vehicles [2]. In most cases, the study of vertical dynamics of the specialized vehicles with the cushioned freights was carried out without taking into account the effect of the vehicle body oscillations [3]. In the work [4] the attempt to investigate this issue by simulating the body of the vehicle as a particle mass was made.

However, the fastening of the freight defines the behaviour of such a wheeled vehicle in terms of stability and evenness of motion on the road. Typical fastening of the vehicle consists in rigid fixing of wheels to a frame using special spanning belts around each wheel. This causes extra loadings acting upon the frame due to the vibrations of the system vehicle-freight while its transporting. This loadings influence the stability and evenness of motion of the truck tractor. Therefore, it is obviously expedient to study the dynamics of vertical vibrations of such a wheeled vehicle, taking into account the supplementary cushioned freight.

Analysis of modern information sources

A specialized articulated vehicle for cars transportation is a complex multi-mass mechanical system with many degrees of freedom. At the same time, the problem of determination of some parameters of the system's vibrations is very time-consuming one. Nevertheless, in order to improve the quality of designing and debugging of structure of the articulated vehicles, it is necessary to ensure high accuracy and self-descriptiveness of calculations while forming a mathematical model, which should define the basic groups of dynamic parameters. However, as of today, the capabilities of computer technologies give a modern scientist and engineer large possibilities for selection of various tools for simulation of dynamic systems of arbitrary complexity. There are block simulation languages (Simulink, EASY5, System Build, etc.) and physical modelling languages (Modelica, Dymola, Omola, etc.).

Simulation is the most effective and universal method of research and evaluation of dynamic systems, the behaviour of which depends on random factors. For studying the problems of dynamics, there have been developed one- [5], two- [6] and multi-mass mathematical models of vibrations of wheeled vehicles [7]. They are presented in the form of the systems of differential equations, which represent the peculiarities of the machine design and the interrelation of its separate parts taking into account some assumptions. Regarding the usage of simulation modelling, the implementation of mathematical models of wheeled vehicles in MATLAB SIMULINK software and their structural models are presented in the paper [8], where the modelling process is also outlined.

The investigation of the vertical dynamics directly affects the speed-, weight-, design- and strength-parameters of the articulated vehicle, and therefore, it is a very urgent problem.

The purpose and problems of research

The aim of the work is to improve the mathematical model of vibrations of the articulated vehicle with cushioned freight, which, unlike the existing one [12], would allow taking into account the effect of elastic fastening of the car, namely the vibrations of its front and rear suspensions in order to carry out the further study of its motion evenness.

Main material presentation

It is more useful to represent the simplified calculation diagram of the articulated vehicle as a flat one in the form of linear system having 9 degrees of freedom (Fig. 1). It consists of three cushioned masses, i.e. of the frames of the truck tractor and semi-trailer, and of three non-cushioned masses, which are axles with wheels as an assembly. Both sections of this system are connected by the saddle-coupling element that is not absolutely rigid. While moving, the frames of the truck tractor and of the semi-trailer performs oscillatory motion in the vertical plane and angular oscillations about an axis passing through the centre of masses. The axles with wheels in the assembly oscillate only in the vertical plane.

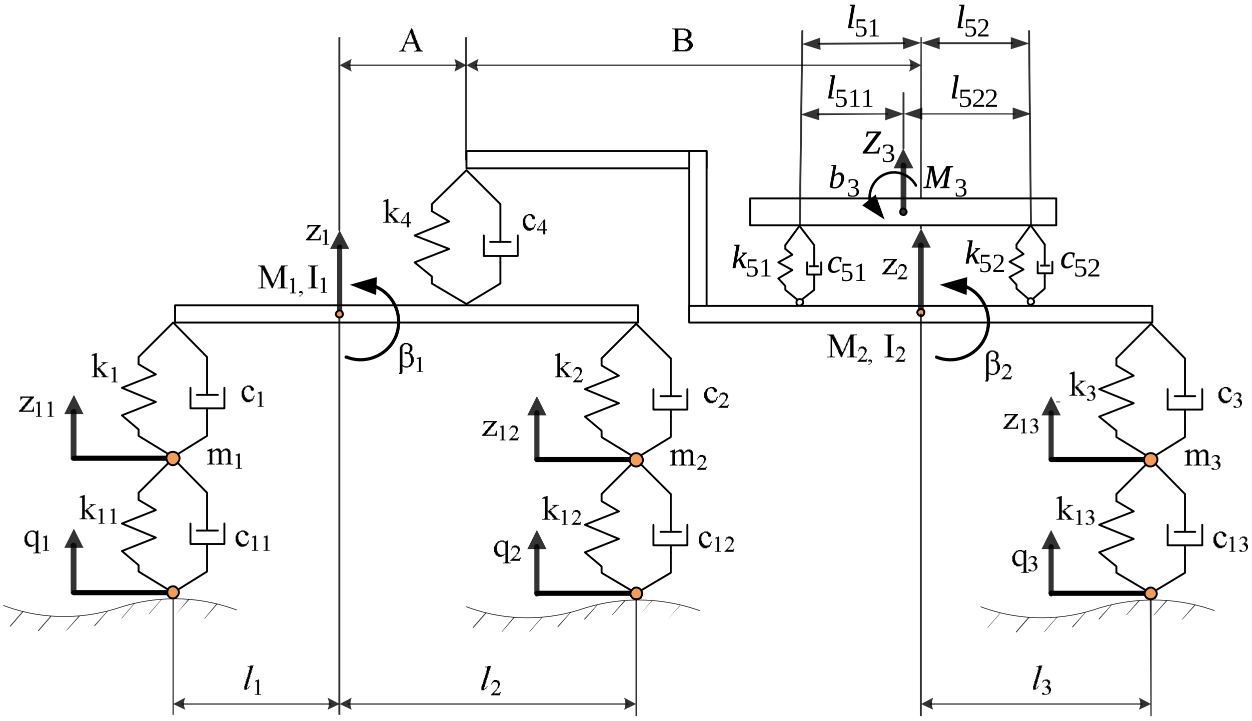

Fig. 1. *Simplified calculation diagram of the articulated vehicle*

In Fig. 1, we used the following notations: z_1, z_2, z_3 are vertical displacements of the cushioned masses of the truck tractor, the semi-trailer and the car being transported (in meters); z_{11}, z_{12}, z_{13} are vertical displacements of the wheels of the truck tractor (in meters); q_1, q_2, q_3 are the disturbances caused by the road shocks in the form of the barrier of rectangular cross-section, which act upon the wheels of the articulated vehicle (in meters); b_1, b_2, b_3 are the angular displacements of the truck tractor and of the semi-trailer about the axis, which is perpendicular to the plane of the figure; I_1, I_2, I_3 are the moments of inertia of the truck tractor, the semi-trailer and the car, which is being transported, about the axis, which is perpendicular to the plane of the figure (in kg/m^2) (2265,5 kg/m^2, 10235,0 kg/m^2 and 24735 kg/m^2, correspondingly); M_1, M_2, M_3 are the masses of the truck tractor, the semi-trailer and the car being transported (in kg) (1818,2 kg, 14283,2 kg and 4825,0 kg, correspondingly); m_1, m_2, m_3 are the masses of the axles of the truck trailer and of the semi-trailer (in kg) (279,7 kg, 524,5 kg and 524,5 kg, correspondingly); l_1, l_2 are the distances between the centers of masses of the truck trailer and its front and rear wheels, correspondingly (in meters) (1,5 m and 4,5 m, correspondingly); l_3 is the distance between the center of masses of the semi-trailer and its rear wheels (in meters) (6,0 m); l_{511}, l_{522} are the distances between the center of mass of the car being transported and its front and rear wheels, correspondingly (in meters) (2,0 m and 2,6 m, correspondingly); l_{51}, l_{52} are the distances between the center of masses of the semi-trailer and the front and rear wheel of the car being transported (in meters) (2,6 m and 2,0 m, correspondingly); A, B are the distances from the centers of masses of the truck tractor and the semi-trailer to the axis of the pin (pivot) (in meters) (3,0 m and 5,6 m, correspondingly); k_1, k_2, k_3, k_{51}, k_{52} are the coefficients of stiffness of the suspensions of the truck tractor, semi-trailer, and the car being transported (in Newtons per meter) (19325,1 N/m, 126096,0 N/m, 126096,0 N/m, 465300 N/m and 1970100 N/m, correspondingly); c_1, c_2, c_3, c_{51}, c_{52} are the coefficients of damping of the suspensions of the truck tractor, semi-trailer, and the car being transported (in Newtons × seconds per meter) (262,7 N·s/m, 262,7 N·s/m, 1260,9 N·s/m, 14980 N·s/m and 27600 N·s/m, correspondingly); k_{11}, k_{12}, k_{13} are the coefficients of tires stiffness (in Newtons per meter) (78810,0 N/m, 157620,0 N/m and 157620,0 N/m, correspondingly); c_{11}, c_{12}, c_{13} are the coefficients of tires damping (in Newtons × seconds per meter) (78,8 N·s/m, 157,6 N·s/m and 157,6 N·s/m, correspondingly); k_4 is the coefficient of stiffness of the saddle-coupling device of the articulated vehicle (in Newtons per meter) (1751334,5 N/m); c_4 is the coefficient of damping of the saddle-coupling device of the articulated vehicle (in Newtons × seconds per meter) (17513,3 N·s/m).

The mathematical model (1) of the system (Fig. 1) is intended to study the vertical vibrations of cushioned and non-cushioned masses of the articulated vehicle with the cushioned freight under various types of external disturbances. This model is a system of differential equations of elastic and damping forces and of equations of moments acting on the cushioned masses. Using the second-order Lagrange equations for the simplified calculation diagram (Fig. 1), the system of differential equations describing the vibrations of the vehicle may be presented as follows:

$$
\begin{aligned}
\ddot{z}_1 &= \left(-\frac{1}{M_1}\right)\cdot\Big[k_1\cdot(z_1+l_1\cdot b_1-z_{11})+c_1\cdot\left(\dot{z}_1+l_1\cdot\dot{b}_1-\dot{z}_{11}\right)+k_2\cdot(z_1-l_2\cdot b_1-z_{12})+\\
&+c_2\cdot\left(\dot{z}_1-l_2\cdot\dot{b}_1-\dot{z}_{12}\right)+k_4\cdot(z_1-A\cdot b_1-z_2-B\cdot b_2)+c_4\cdot\left(\dot{z}_1-A\cdot\dot{b}_1-\dot{z}_2-B\cdot\dot{b}_2\right)\Big];\\
\ddot{b}_1 &= \left(-\frac{1}{I_1}\right)\cdot\Big[l_1\cdot\left(k_1\cdot(z_1+l_1\cdot b_1-z_{11})+c_1\cdot\left(\dot{z}_1+l_1\cdot\dot{b}_1-\dot{z}_{11}\right)\right)+l_2\cdot\Big(k_2\cdot(z_1-l_2\cdot b_1-z_{12})+\\
&+c_2\cdot\left(\dot{z}_1-l_2\cdot\dot{b}_1-\dot{z}_{12}\right)\Big)-A\cdot\left(k_4\cdot(z_1-A\cdot b_1-z_2-B\cdot b_2)+c_4\cdot\left(\dot{z}_1-A\cdot\dot{b}_1-\dot{z}_2-B\cdot\dot{b}_2\right)\right)\Big];\\
\ddot{z}_2 &= \left(-\frac{1}{M_2}\right)\cdot\Big[k_3\cdot(z_2-l_3\cdot b_2-z_{13})+c_3\cdot\left(\dot{z}_2-l_3\cdot\dot{b}_2-\dot{z}_{13}\right)+\\
&+k_4\cdot(z_2+B\cdot b_2-z_1+A\cdot b_1)+c_4\cdot\left(\dot{z}_2+B\cdot\dot{b}_2-\dot{z}_1+A\cdot\dot{b}_1\right)+\\
&+k_{51}\cdot\left[(z_3-b_3\cdot l_{511})+(z_2-b_2\cdot l_{51})\right]+c_{51}\cdot\left[\left(\dot{z}_3-\dot{b}_3\cdot l_{511}\right)+\left(\dot{z}_2-\dot{b}_2\cdot l_{51}\right)\right]+\\
&+k_{52}\cdot\left[(z_3+b_3\cdot l_{522})-(z_2-b_2\cdot l_{52})\right]+c_{52}\cdot\left[\left(\dot{z}_3+\dot{b}_3\cdot l_{522}\right)-\left(\dot{z}_2-\dot{b}_2\cdot l_{52}\right)\right]\Big];\\
\ddot{b}_2 &= \left(-\frac{1}{I_2}\right)\cdot\Big[-l_3\cdot\left(k_3\cdot(z_2-l_3\cdot b_2-z_{13})+c_3\cdot\left(\dot{z}_2-l_3\cdot\dot{b}_2-\dot{z}_{13}\right)\right)+\\
&+B\cdot\left(k_4\cdot(z_2+B\cdot b_2-z_1+A\cdot b_1)+c_4\cdot\left(\dot{z}_2+B\cdot\dot{b}_2-\dot{z}_1+A\cdot\dot{b}_1\right)\right)-\\
&-l_{51}\Big[k_{51}\cdot\left[(z_3-b_3\cdot l_{511})+(z_2-b_2\cdot l_{51})\right]+c_{51}\cdot\left[\left(\dot{z}_3-\dot{b}_3\cdot l_{511}\right)+\left(\dot{z}_2-\dot{b}_2\cdot l_{51}\right)\right]\Big]+\\
&+l_{52}\Big[k_{52}\cdot\left[(z_3+b_3\cdot l_{522})-(z_2-b_2\cdot l_{52})\right]+c_{52}\cdot\left[\left(\dot{z}_3+\dot{b}_3\cdot l_{522}\right)-\left(\dot{z}_2-\dot{b}_2\cdot l_{52}\right)\right]\Big]\Big];
\end{aligned}
\tag{1}
$$

$$\ddot{z}_3 = \left(\frac{1}{M_3}\right)\cdot\Big[k_{51}\cdot\big[(z_3 - b_3\cdot l_{511}) + (z_2 - b_2\cdot l_{51})\big] + c_{51}\cdot\big[(\dot{z}_3 - \dot{b}_3\cdot l_{511}) + (\dot{z}_2 - \dot{b}_2\cdot l_{51})\big] +$$

$$+ k_{52}\cdot\big[(z_3 + b_3\cdot l_{522}) - (z_2 - b_2\cdot l_{52})\big] + c_{52}\cdot\big[(\dot{z}_3 + \dot{b}_3\cdot l_{522}) - (\dot{z}_2 - \dot{b}_2\cdot l_{52})\big]\Big];$$

$$\ddot{b}_3 = \left(\frac{1}{I_3}\right)\cdot\Big[-l_{11}\cdot\Big[k_{51}\cdot\big[(z_3 - b_3\cdot l_{511}) + (z_2 - b_2\cdot l_{51})\big] + c_{51}\cdot\big[(\dot{z}_3 - \dot{b}_3\cdot l_{511}) +$$

$$+ (\dot{z}_2 - \dot{b}_2\cdot l_{51})\big]\Big] + l_{22}\Big[k_{52}\cdot\big[(z_3 + b_3\cdot l_{522}) - (z_2 - b_2\cdot l_{52})\big] +$$

$$+ c_{52}\cdot\big[(\dot{z}_3 + \dot{b}_3\cdot l_{522}) - (\dot{z}_2 - \dot{b}_2\cdot l_{52})\big]\Big]\Big].$$

Thus, the mathematical description of the dynamic system “road-vehicle-cushioned freight” is obtained. The obtained mathematical model for the articulated vehicle with the cushioned freight placed on the semi-trailer consists of nine differential equations, which are the functions of the generalized coordinates.

While carrying out the calculations of free vibrations of the two-section articulated vehicle, the most typical case, namely, the motion over the unit prominence (road shock), the effect of which on the wheels of the articulated vehicle is of the impact type, is considered. The choosing of the initial perturbation in the form of a unit impact is caused by the possibility of obtaining an impact characteristic, which is defined as the reaction of the system on the δ-function.

While investigating the vibratory processes that occur when moving over a unit prominence (road shock), we assume that while moving to the road shock, the state of the system is a completely predefined by the values of the coordinates and their first derivatives (velocities). In the other words, the initial conditions at the time moment when the vehicle starts moving over the prominence are known, since only the results that are obtained from this moment are of greatest interest.

As the most convenient programming software, a simulation package for dynamic systems Simulink [9] of the useful computing complex MathWorks Matlab is offered. The Simulink package is specifically designed to study the dynamical systems used in various engineering calculations and has been well-proven over the course of 30 years of its use. For the solution of systems of differential equations of the vehicle motion, the Runge-Kutta method of 4–5 orders was used as the most universal one.

The initial material for the structural modelling of the system of differential equations while investigating the oscillations of the dynamic system is the simulation scheme. It shows the corresponding blocks of the Simulink libraries and the links between them. By connecting the blocks correctly with each other, we obtain a scheme for the implementation of the mathematical model. In Fig. 2, the block diagram of Simulink model of the uniform motion of the articulated vehicle with the cushioned freight is shown.

The block diagram consists of eleven sub-systems of the lower level of the hierarchy, each of which is a separate equation of the mathematical model. To the subsystems of the block diagram, the input signals in the form of generalized coordinates in accordance with the variables of each equation are applied. The output signal of each of the subsystems is the left side of the corresponding equation.

While forming the block diagram in the MATLAB SIMULINK software, the following block types were used: Constant (sets the constant level of the signal), Product (multiplies the current signals values), Integrator (integrates the input signal), Transport Delay (ensures the delay of the input signal for the defined period of time), Derivative (performs a numerical differentiation of the input signal), Gain (performs multiplication of the current values of signals on constant coefficient), Sum (performs addition of current values of signals), Signal Builder (forms a linear signal of arbitrary form using a user graphical interface).

The peculiarity of such an approach is that the computer program, which results in the solution of a system of differential equations in the form of an array of numerical values, is not formed. In this case, according to the system of differential equations, the block diagram is formed using a set of typical blocks of the SIMULINK library. For example, the block diagram of Subsystem 8 is shown in Figs. 3 and 4. It is based on the first two equations of the mathematical model (1).

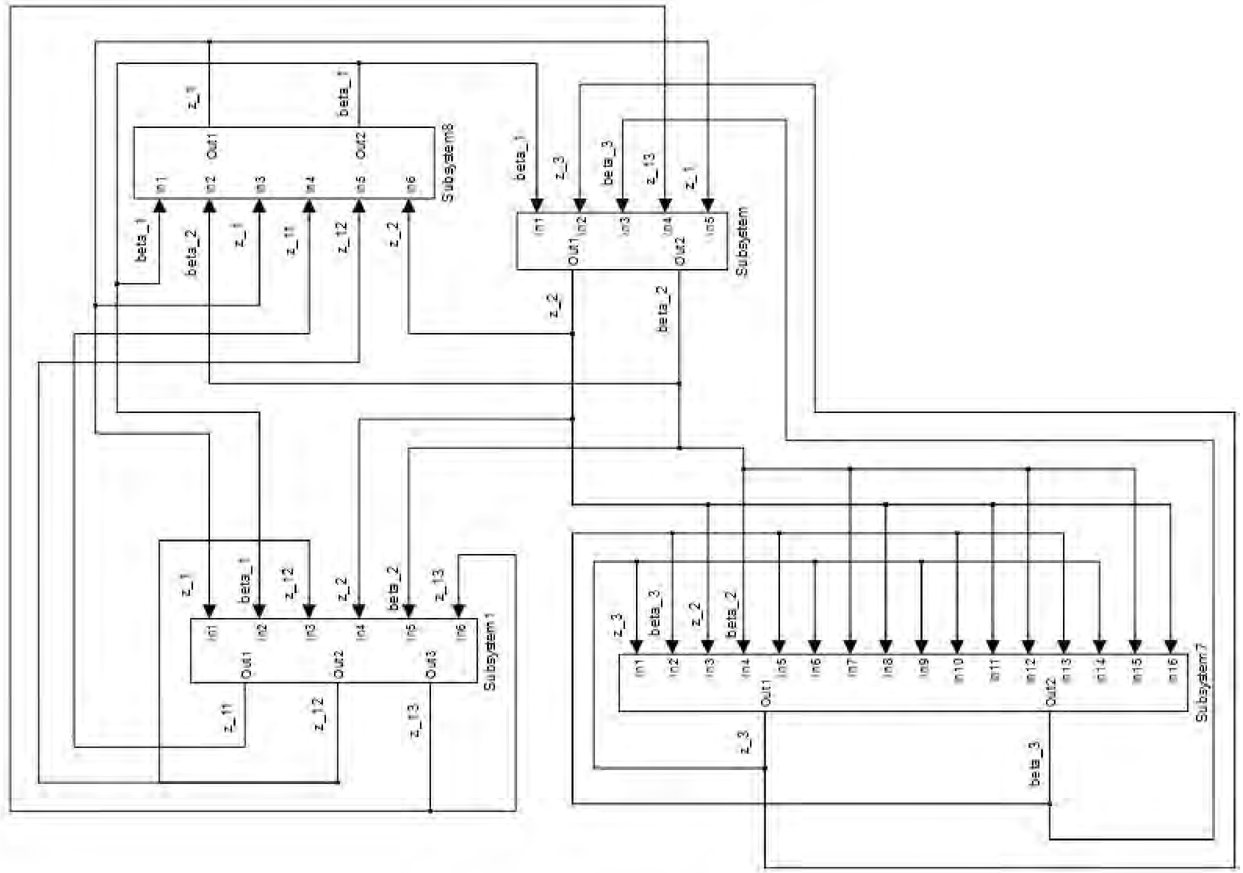

Fig. 2. *Simulink block diagram for uniform motion of the articulated vehicle with the cushioned freight*

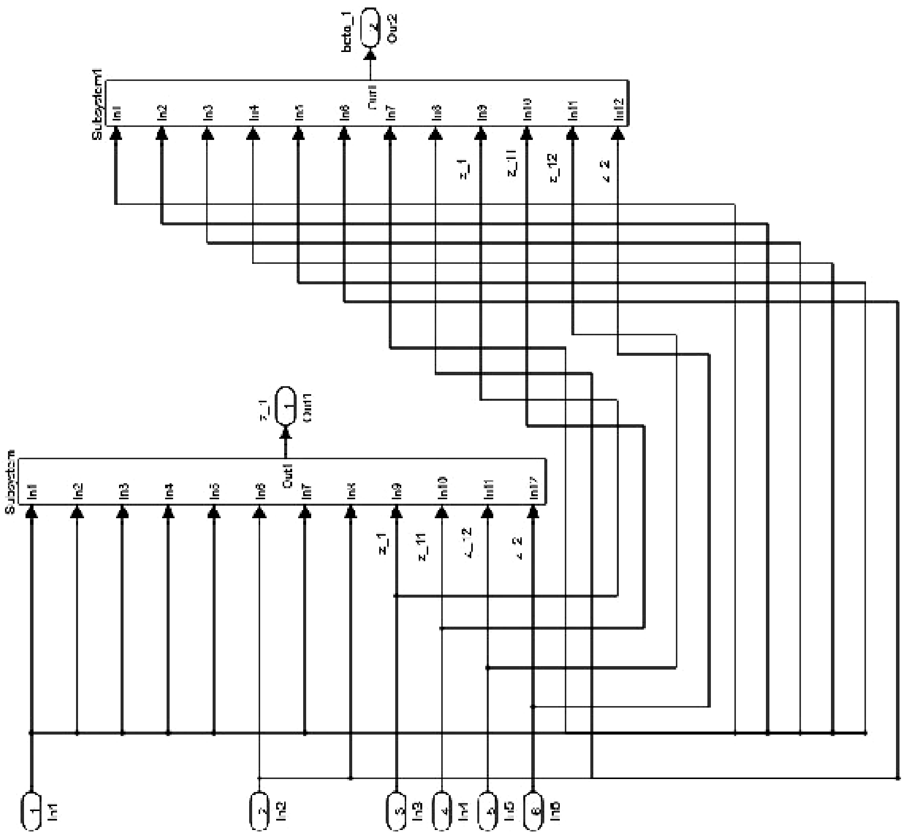

Fig. 3. *Simulink block diagram of the Subsystem 8 based on the first two equations of the mathematical model (1)*

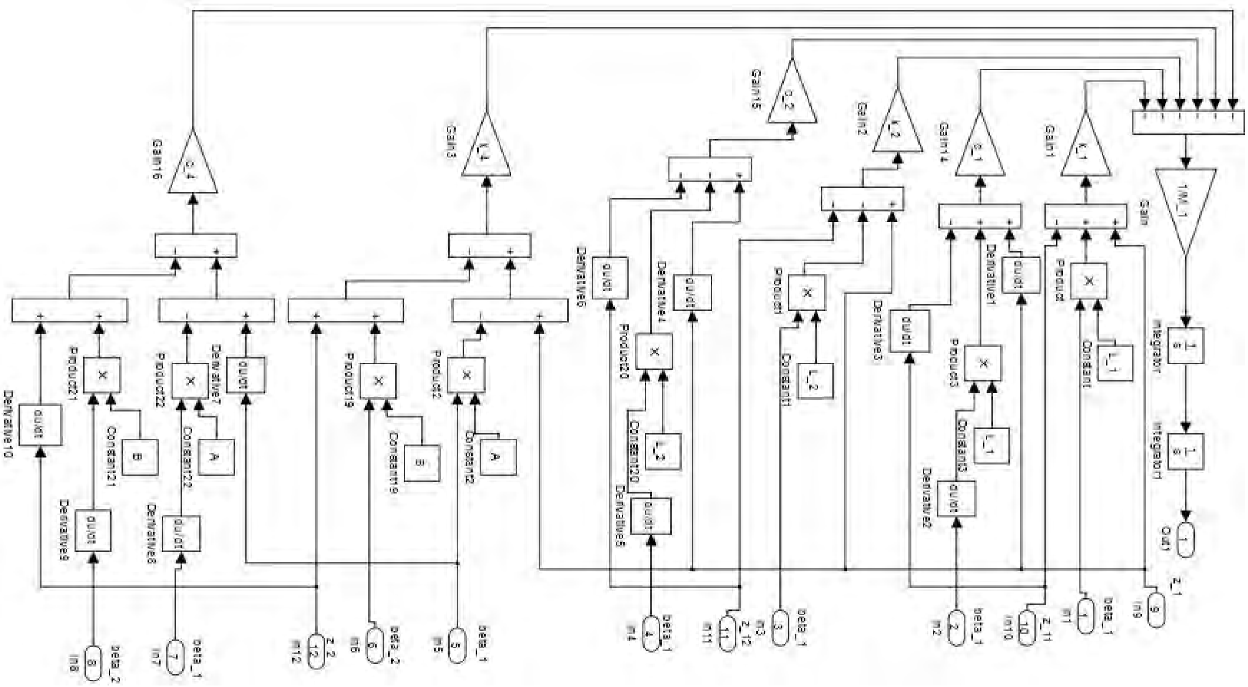

Fig. 4. *Simulink block diagram of the Subsystem of the Subsystem 8 for solving the first equation of the mathematical model (1)*

The block diagram thus created allows: to carry out dynamic investigations of vibrations of the cushioned and non-cushioned masses of the system (Fig. 1); to visually monitor the vibratory processes in real time scale; to quickly and usefully change the parameters of the oscillatory module and vibrations disturbance; to perform the simultaneous stop of simulation model and dynamical plots of oscillations parameters at any time moment, with the subsequent restoration of their simulation; to easily change the shape of the perturbation by replacing the Simulink blocks.

The obtained program allows to define amplitude-frequency and phase-frequency characteristics of any concentrated mass, as well as root-mean-square values of accelerations in the dialogue mode. This allows quick choosing of characteristics in order to obtain the required amplitude-frequency and phase-frequency characteristics, as well as to predict the vibration loading parameters in order to ensure the compliance with GOST 12.1.012-90. The specific graphic dependences of the vertical vibrations of the centers of masses of the considered system (Fig. 1) are shown in Fig. 5.

As a disturbance from the road, the prominence (road shock) of the rectangular cross-section with a height of 0.1 m and a width of 0.25 m was used. This form of road prominence allows us to determine whether such typical disturbance leads to intense continuous oscillations, that is, to instability of motion. This approach allows us to analyze the system motion stability at each specific speed when there are vertical prominences (shocks) on the road [10].

In order to determine the amplitude-frequency characteristic and phase-frequency characteristic of the system, the system was diagnosed for forced oscillations. The amplitude-frequency and phase-frequency characteristics are graphical representations of the functions of motion or accelerations of the cushioned and non-cushioned masses of the articulated vehicle, which gives a visual representation of the oscillatory process. The graphical dependencies of the amplitude-frequency characteristic and phase-frequency characteristic ensure an opportunity to present the effect of various design parameters of the suspension and of the cushioned freight on the oscillations, and also to allow to study the zones of high-frequency and low-frequency resonances. Matlab allows you to automatically obtain the amplitude-frequency characteristic and phase-frequency characteristic. In order to form the amplitude-frequency characteristic, into the Simulink block diagram, we load the “Spectrum Analyzer” block, which performs the comparison of the output and the input signals by amplitude and phase (Fig. 6).

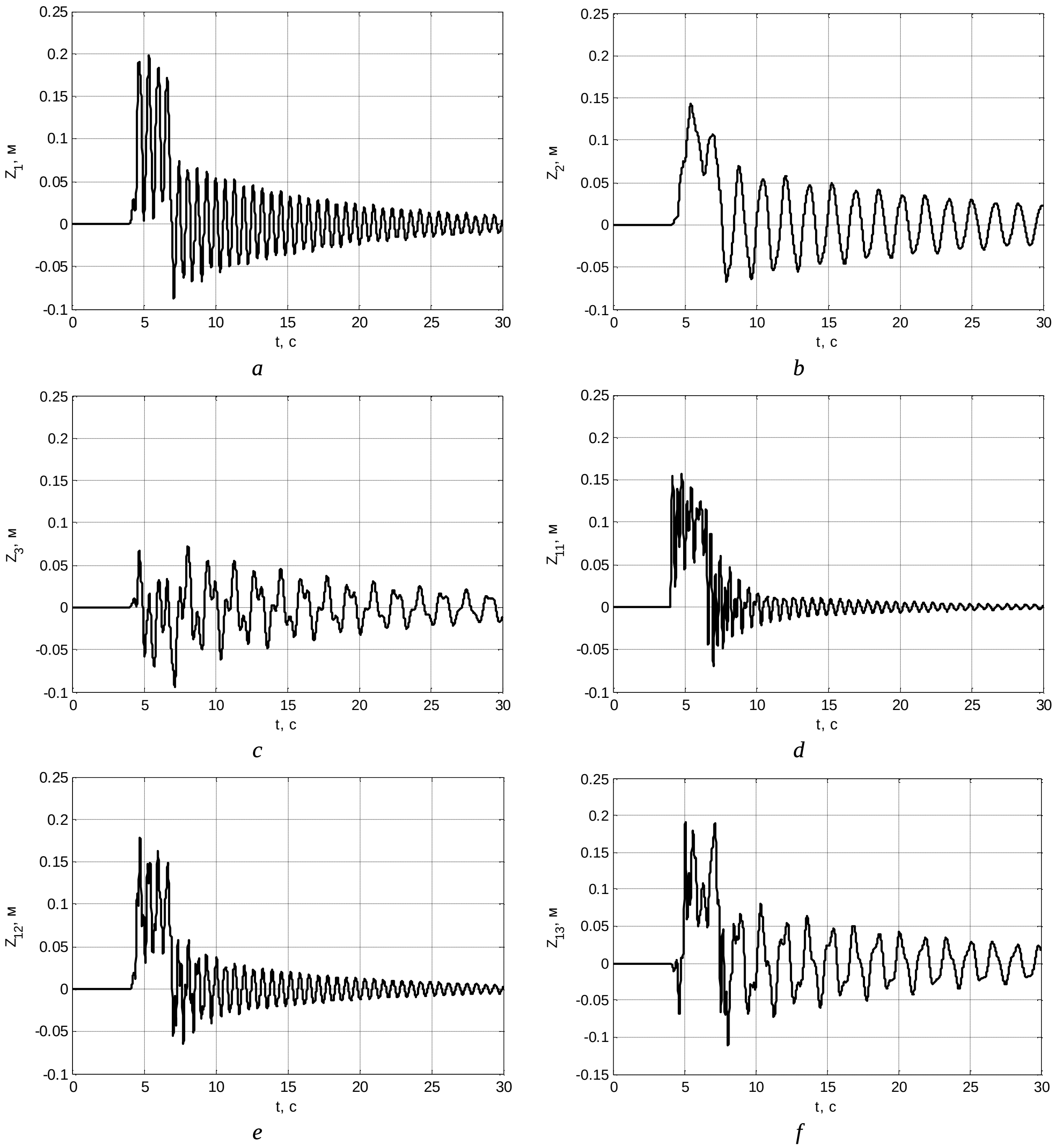

***Fig. 5.** Vibrations of the centre of masses of the system in a vertical plane: a – vibrations of the centre of masses of the truck tractor; b – vibrations of the centre of masses of the semi-trailer; c – vibrations of the centre of masses of the cushioned freight; d, e, f – vibrations of the non-cushioned masses*

Conclusions

The results of computer simulation show that for the “truck tractor – semi-trailer” system, the consideration of the elastic properties of the freight may lead to a significant increase / decrease of the frequency and amplitude of vertical vibrations of all units and sections of the system. It should be noted that this effect becomes apparent much more significantly than for the “truck tractor – trailer” system [11]. This is due to the fact that this system consists of sections that cannot be considered separately (in the matrix of generalized masses, the non-diagonal components are considerable and this leads to the dynamic redistribution of the energy of vibrations).

Thus, in this paper, the mathematical model of the vibrations of the articulated vehicle with the cushioned freight has been improved. Unlike the existing ones [12], this model made it possible to take into account the influence of the elastic fastening of the vehicle being transported, namely, the vibrations of its front and rear suspensions for further study of its motion evenness.

The technique and the method of implementation of the mathematical model of the articulated vehicle with a help of the modern software product for solving urgent problems of dynamics are shown.

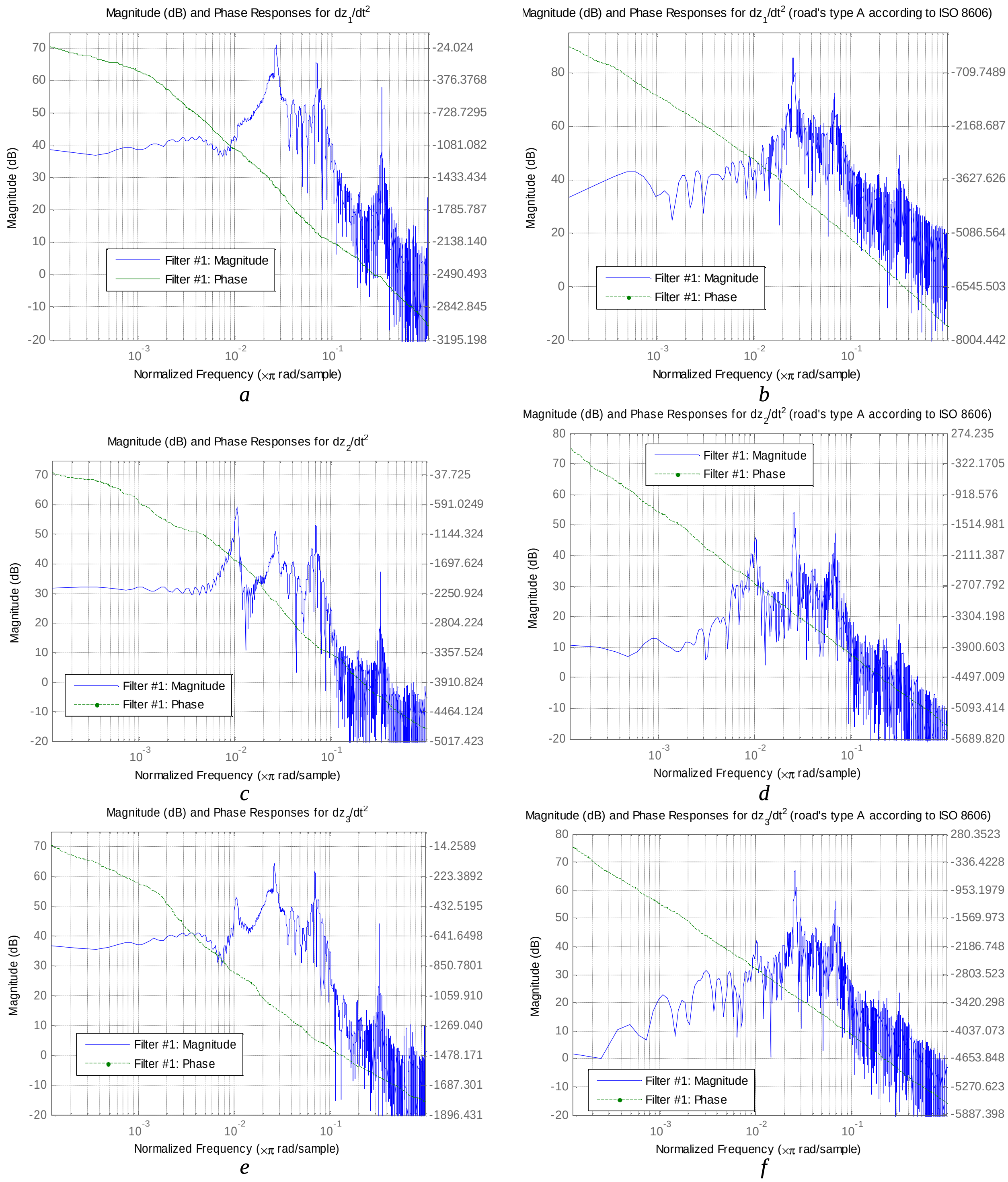

Fig. 6. *Amplitude-frequency characteristics and phase-frequency characteristics of the articulated vehicle with the cushioned freight: a, b – for the truck tractor when moving over the unit road shock of the rectangular cross-section and while moving along the road of A category according to the Standard ISO 8606; c, d – for the semi-trailer under the same motion conditions; e, f – for the cushioned freight under the same motion conditions*

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