Methods of comparing interval objects in intelligent computer systems

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Abstract. Problems of expert knowledge representation by means of generalized interval estimates approach and using methods of comparing interval alternatives in the framework of intelligent computer systems are considered. The problems are common in economy, engineering and in other domains. Necessity of multi criteria approach to comparing problem that is taking into account both preference criteria and risk ones is shown. It is proposed to use a multi-steps approach to decision-making concerning choice of preferable interval alternatives. It is based on consistent using of different comparing methods: new collective risk estimating techniques, ―mean-risk" approach (for interval-probability situations) and Savage method (for full uncertainty situations).

Keywords: interval alternatives, risk estimating techniques, collective risk assessment, ―mean-risk approach", generalized interval estimates

1 Introduction

Intelligent systems are a staple of artificial intelligence research. Characteristic features of intelligent computer systems are, among others, availability of the subsystem of knowledge representation and subsystem of problem solving. Problem solving should also include decision making as the process of the best suitable alternative choice out of multiple alternatives set that due to complexity and uncertainty decision-making requires human involvement in such processes. An important role in practice plays problems of comparing and choice of alternatives with numerical quality indicators, which due to uncertainty have interval representations (so-called interval alternatives – IA). To include such objects in the knowledge and model bases of intelligent systems they should be describe by special methods of representation and analysis.

As to representation problem it should be borne in mind that in many cases quite difficult to expressexpert knowledge of an uncertain parameter byusing the only interval estimate. Indeed, a too-wideinterval reduces the value of expert knowledge while atoo-narrow interval often causes errors of forecasting. To overcome this difficulty an approach of generalized interval estimates (GIE) was proposed [1, 2]. The approach provides expert tools for expressing expert knowledge about problem parameters by specifying a set of intervals. The set characterizes the inaccuracy of lengthand location of the interval estimate for a parameter of problem. To represent knowledge about parameters of a problem an expert firstly forms apolyinterval estimate (PIE). For this purpose theexpert specifies several characteristic intervals of the set of interval estimates and then constructs a PIE from these intervals. The simplestexample of PIE construction is the specification ofsome interval as the initial estimate of a parameter and extending it (not necessarily symmetrically) on bothsides of the initial boundaries. It is natural to assume that allintervals between the initial and the extended intervals are included in the set of interval estimates and characterize the expert knowledge of the parameter. Theseintervals are possible realizations (scenarios) of theanalyzed parameter.In this case, the PIE of a parameter D is visuallyrepresented by the curvilinear trapezoid $X = D$, $Y = h$in the plane, which is constructed from the expertestimates for the trapezoid bases. On the axis of ordinates $h \in [0, 1]$ the ordered left boundaries of theintervals from the set are marked. The greatest base ofthe trapezoid (the base interval) corresponds to $h = 0$, and the least base (the miniinterval) corresponds to$h = 1$. Thus, the PIE is determined by the positionsand lengths of the least (upper) interval $[D_{lu}, D_{ru}]$) andthe base (lower) interval $[D_{ld}, D_{rd}]$) in the set of intervals, as well as by the shape of the lateral sides of thetrapezoid. In applications these sides can be assumedto be rectilinear. The h axis can be interpreted not onlyas the axis of marks of the intervals forming the PIE.Its interpretation depends on the problem under consideration. So this axis has an obvious physicalmeaning in problems with dependent variables. Forexample, in the problem of estimating the dependenceof the amount of extracted oil on their price each point estimate for the price on the h axis of thePIE corresponds to an interval estimate of reserves onthe D axis.The expert judgement about the chance that a certain value of the analyzed parameter is realized may be expressed by specifying a density of the joint probability distribution function $f(D, h) = f_1(h)f_2(D|h)$ on the PIE. We refer tothus obtained construction as a generalized interval estimate for the quantity D. In the general case thedensities $f_2(D|h)$ defined on different (with respect to$h$) strips of the PIE may belong to different families ofdistributions.There are some ways of applying the GIEapproach to decision making problems. We will focus here only on one possibility. Specifically, we can obtain anaveraged density $f(D)$ (density of marginal distribution) on the baseandthus go to the model of well-known monointerval case. One can see that $f(D)$on the baseinterval is a probability mixture of distributions onGIE intervalscenarios with mixing function determined by the PIE distribution on the h axis. The most common in practice boundariesof PIE arerectilinear. Note that averaged probability distributions received by this way generalize known distributions. Forexample, if to set uniform distributions on both PIEaxes, averaging yields a generalized uniform distribution whose properties are much richer that those of thestandard uniform distribution.

The GIE approach is at the junction of several scientific domains, such as the theory of decision making, knowledge engineering, probability theory, andsupport systems for expert decisions. This approachexpands the possibilities of more completely revealingexpert knowledge of initial data, provides a more adequate allowance for uncertainty, and improves thequality of decisions.

Thus speaking about the problem of decision-making in relation to the choice of a preferable object from a set of IA we can confine ourselves to the task of comparing mono interval estimates. Quite often the problems of comparing IA belong to a class of problems of unique (non-repeating) choice. Therefore preference chances of tested alternatives in comparing with others are as a rule unknown.In these circumstances expert/DM may either abandon the use of hypotheses about mentioned chances or involve different approaches for the formalization of the knowledge.Such well-known methods of decision-making under the game with nature, or methods under pure uncertainty, as methods by Wald, Hurwicz, Savage may be used in the first case [3].Such methods of comparingascomparison on value of mathematical expectations of quality indicators of IA, method of stochastic dominance, ―mean– risk" method [3] and the method of collective risk estimation [4]become available for the usein the second case.The purpose of the further part of this paperis to compare the various approaches and methods of comparing IA and to identify their place in the decision-making process for the choice of the preferred interval objects.

2 Comparing interval alternatives under interval-probabilistic uncertainty

It is natural to assume that each interval estimate includes all possible, up to the available knowledge, point implementations of studied parameter. But in the future, when the uncertainty is removed, this quantity will receive certain the only numeric value. Let assume also for definiteness that such situations take place when the greater values of the quality indicators are preferable than small values. Note that problem of IA comparing due to its nature cannot be exhaustively solved by purely mathematical methods. Indeed in the general case when compared alternatives have non-zero intersection in principle one cannot with certainty conclude, which alternative in their set will be preferable. Any alternative may be ―better" in the future, at the time of ―removal" of uncertainty, when the interval estimates are replaced by point (exact) values of quality indicator. So at the time of the comparison can be judged only on the chances that one alternative will be preferable to others. Therefore always there is an irremovable risk that in fact namely another alternative but not tested one would be better.Thus formal methods of comparison cannot guarantee choice of truly the best object in the process of comparing. It means that such problems are problems of the decision-making theory as the decision-making processes have to include preferences of decision makers (DM) and take account of their risk tolerance. Therefore comparison, which is adequate to essence of the problem, should be based on at least two criteria, measure of preference of alternative in relation to others in their group and risk measure. Human involvement in the decision-making process can determine not only the choice of the tools of describing the uncertainty but also, due to previous experience and knowledge of human being, the choice of the comparison methods that lead to the result indicators, which are familiar to the expert or DMs.

Each of the available methods of comparison allows calculate its measures of alternative preference in relation to the other alternatives in their group and (but not always) their risk measures. For some configurations, i.e. relative locations, of compared alternatives and types of uncertainty describing predictions of different methods are equivalent but for others not. In this regard it should be stated that at present there is no approach transcending all others in quality of recommendations obtained on its basis. Each method has its advantages and disadvantages.Joint using of different methods on various successive stages of the decision-making process is probably the best way to combine the power of formal methods and knowledge of experts and DMs. Therefore decision-making for comparing of IA is both science and art.

Let assume that all alternatives are comparable on preference (system of alternatives is full). If disjunction containing comparable interval alternatives is not rigorous then choice of preferred objects depends on the chances of preferences of the disjunction members. Similar relations of disjunction members are based on the degree of assurance in the truth of the hypotheses about preference an alternative from their set, which is tested by DM. Such relations may be called by relations including the risk. Assume that from all possibilities of uncertainty description the tools of distribution functions, similar to tools that used in the probability theory, is selected here for the quantification of preference chances for compared interval alternatives or subsets of values contained in them. This apparatus is the most familiar, in our experience, expert practitioners. It is important because expert analysis of practical problems is most productive when it is conducted in the usual for domain experts' language with using terminology understandable to them.

Let us first briefly describe methods for interval-probability comparing. In the method of collective risk estimating [4], when direct calculations of preference chances of alternatives in comparison with others in their group is produced, compared objects are viewed as interconnected community. Because tool of distribution functions was selected for quantification of preference chances and associated risks, the problem of comparing can be analyzed in the framework of probability logic approach [5]. In accordance with this approach in addition to the truth or falsity of logical statements intermediate logical values are possible. They are interpreted as chances of truth. The use of this approach to IA comparing allows calculating both the chances of alternative preferences and associated risks. Risk of choice of an alternative in their group as the preferred depends on the relative position of alternatives (configuration of alternatives) and on the number of compared objects. An interaction of compared objects leads to a ―collective effect", which consists in the fact that the properties of objects of interacting components of the system are significantly different from those of relatively independent objects. Therefore risk of making the wrong decision during choosing a preferred object increases with the growing number of compared alternatives. The matter looks at a rigorous language as follows.Suppose that there are KIAI_i, $i = 1, 2,\dots, K$ with the same interval quality indicators and dimensionless quantity $C(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\dots, I_K))$ is the chances in the truth of a testable hypothesis of preference that the alternative I_i is more preferable than all at once alternatives $(I_1, I_2,.., I_{i-1}, I_{i+1},\dots, I_K)$ from initially given their set (I_i is ―better" of others ―as a whole"), $\succ$ is symbol of preference. The term ―all at once" means here that

$I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K) \equiv (I_i \succ I_1) \wedge (I_i \succ I_2) \wedge (I_i \succ I_3) \wedge \ldots \wedge (I_i \succ I_{i+1}) \wedge \ldots \wedge (I_i \succ I_K)$,

where $\equiv$ and $\wedge$ are symbols of equivalence and conjunction respectively.Risk that I_i would not preferred in reality will be measured by means of dimensionless quantity $R_s(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K))$ complementing previous chances to unity so that

$$Rs(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K)) = 1 - C(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K)).$$

As can be seen $R_s(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K))$ is degree of assurance in the truth of a hypothesis, which is opposite to the testable hypothesis of preference. Equivalently $R_s(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K)) = C(\neg(I_i \succ (I_1, I_2,.., I_{i-1}, I_{i+1},\ldots, I_K)))$, where $\neg$ is symbol of negation. One may show that the following relations hold for chances

$$C(I_1 \succ (I_2, I_3,\ldots, I_K)) + C(I_2 \succ (I_1, I_3,\ldots, I_K)) + C(I_3 \succ (I_1, I_2, I_4,\ldots, I_K)) +...+C(I_K \succ (I_1, I_2,\ldots, I_{K-1})) = 1$$

and for risks

$$R_s(I_1 \succ (I_2, I_3,\ldots, I_K)) + R_s(I_2 \succ (I_1, I_3,\ldots, I_K)) + R_s(I_3 \succ (I_1, I_2, I_4,\ldots, I_K)) +...+ R_s(I_K \succ (I_1, I_2,\ldots, I_{K-1}))= K - 1.$$

The natural desire of DM is reduce the risk when deciding. A possibility to do so is to reduce the number of compared alternatives as the method of collective risk estimating suggests. Therefore before deciding on preferred alternative choice it‘s useful to conduct a preliminary analysis of their initial set to reduce the calculated risk. Some recipes for this are given some later. By reducing the number of intervals in their initial set one may increase the calculated preference chances of analyzed alternative and decrease risks.Can other methods of comparing help to reduce the dimension of the problem?

As to mathematical expectation of quality indicator as random variable it should note that this criterion is adequate for problems of repetitive choice. At the same time the problems under uncertainty deal mainly with situations of unique choice. This requires, in general, rejection of average estimates, or, if they are used, the mandatory accounting as no less important criterion estimations of risk calculated on the basis of certain indicators. Method ―mean – risk" does so. Compared alternatives are considered here as isolated, not ―interactive" objects. Value of preference criterion is calculatedfor each alternative separately and then a risk indicator is computed again separately for each object. The problem of comparing is solved then as a two-criterion task. Mathematical expectation value is here the criterion of preference. Such indicators as variance, left and right semivariances, left and right mean semideviations and the others act as risk criteria [6]. Let note that the calculated values of the comparison criteria for these methods do not depend on the number of analyzed alternatives. Method ―mean – risk" can be used to reduce number of compared IA and to diminish risk of false decision about preferences [4].

In the methods of stochastic dominance pairwise comparison of alternatives carried out only on the basis of the behavior of the distribution functions defined on intervals-carriers, without taking into account the numerical characteristics of the firsts. If to define interval objects $I_i = [L_i, R_i]$ by the left L_i and right R_i boundaries, $L_i<R_i$, then there are, up to permutations, the four different configurations of compared alternatives pairs: coinciding intervals; intervals without intersection; configurations of right shift and embedded intervals. As to comparing problem the second configuration has no interest. One can show thatalready for a pair of the compared intervals this comparison method does not allow to reduce the dimensionality of the problem.They say that the IAI_1, where integral distribution F_1 of the random variable X_1 is given, dominates (by probability) IA I_2, where distribution F_2 of the random variable X_2 is given, if for a set of possible point implementations $I_1 \mathrm{U} I_2$ for any point implementation x chances $F_1(X_1<x)$ are not more than chances $F_2(X_2<x)$, and at least for one point implementation they are smaller. In other words the graph of the distribution function F_1 for alternative I_1 lies always below the graph of the distribution function F_2, possibly coinciding with the first in some parts. In the case of right shift configurations, when $L_2<L_1<R_2<R_1$, for uniform distributions alternative I_1 dominates alternative I_2 by probability. Indeed if by definition $\Delta I_i = R_i - L_i$, $i = 1, 2$ then distribution functions F_i are intersected at a point $I_{int} = (L_2\Delta I_1 - L_1\Delta I_2)/(L_2\Delta I_1 - L_1\Delta I_2)$ besides the case $\Delta I_1 = \Delta I_2$ when they are parallel. One can verify that the inequality $L_1<I_{int}<R_2$ is not met for the right shift configurations, and therefore in area $[L_2, R_1]$ $F_1 \le F_2$. Therefore the first alternative dominates the second by probability. This conclusion is valid for any scope of the uncertainty zone $[L_1, R_2]$ for point implementations of IA that is a significant disadvantage of the method. May DM therefore always select as the preferred first alternative due to the dominance of the second by probability? It seems that not because the adoption of this requirement means the neglect of the risk of making a wrong decision on the preference. DM can but should not make such a choice. Thus using of the dominance by probability principle to eliminate certain alternatives from their set for decreasing their number to reduce collective risk is problematic.

Thus in the framework of methods for interval-probabilistic uncertainty may recommend using of the method of collective risk estimating to evaluate integral risks for each alternative in the group and find a subset of the ―best" alternatives as the alternatives with the highest chances at pairwise comparisons in the set of compared alternatives. Then this narrowed set of alternatives may be evaluated according to the criteria of preference and risk, which are based on methodof ―mean – risk" approach.

3 Comparing interval alternatives under pure uncertainty

Expert does not attempt to specify the distributions of chanceson interval-carriers under pure uncertainty. Instead values of the quality indicators are forecasted usually for small number of possible different states of nature.These values are in a certain interval of values. By this, however, is limited the similarity of comparing methods under pure uncertainty and other methods based on interval representation parameters of the problem when, relatively speaking, the number of the states of nature, which are taken into account, is infinitely large. For the first case the uncertainty is given by means of indication of the nature states, which are essential for the expert, but values of the quality indicators for the statesare calculated as deterministic.This leads to a certain coarsening of the real problem and to refusal of the possibility for quantitative estimating risk that does not allow for an adequate analysis of the problem. At the same time these methods may be useful in some cases as a means of express-analysis of the problem. The results of such analysis have usually simple interpretation that is essential for DM- practitioners.

Let us restrict ourselves to the three states of nature: unfavorable states, which correspond to the left borders L_i of IA quality indicators; favorable, which correspond to the right boundary $R_i$of quality indicators; and neutral ones, which correspond to some internal pointsIn_i<$R_i$of IA quality indicators.If an expert has decided to use Wald method for IA comparing, then$I_i \succ I_j$ifL_i>L_j.The advantage of the Wald method consists in rapidity and visibility of the results. The disadvantages are rooted in the incomplete using of information about IA, information about the problem concerning only unfavorable state of nature, as well as in the actual refusal of the risk estimating during decision-making.But the risk, which isdepend on the configuration of compared IA, may be quite large. So IA $I_1$is chosen as the preferred on the Wald method in the configuration of the right shift.However entering the point implementations in the interval [L_1, R_2] (area of I_1 and I_2intersection) does not guarantee that I_1 will be the best. Here the greater the length of the mentioned interval the higher the risk of error in the choice of the preferred object.In the case of a pair of embedded intervals Wald method selects I_2: $I_2 \succ I_1$. If the left boundary of I_2 is positive and DM avoids the risk one can agreewith this conclusion. But if the length ofinterval [R_2, R_1] is rather big and additionally to take into account that part of the point implementations in [L_2, R_2] favored to choice of I_1 then, even with a small propensity of DM to risk, he may prefer I_1.Thus the use of the Wald method for comparing IA requires additional analysis and taking into account the specifics of mutual location of interval estimates.The absence in the framework of the method of a risk indicator, which is required by content of the problem, is a disadvantagethat reduces the possibility of applying the method.

By resorting to Hurwicz method, expert uses two estimates that delimit values of IA quality indicators and correspond to unfavorable and favorable states of nature.However Hurwicz approach is not limited by considerationof quality indicators only for these boundary states of nature.In fact the method takes into account all possible states corresponding to the values of quality indicators within the interval estimation.By this Hurwicz approach differs from all other methods of pure uncertainty. According to Hurwicz interval indicator $I = [L, R]$ is replaced by a point indicator $T(\lambda)$, which is equivalent to the initial interval estimate on expert opinionwhen IAare compared.The value of T is determined by expert choice of the parameter λ, which reflects the expert knowledge and referred to as Hurwicz ―pessimism – optimism"factor. Then for $0 < \lambda < 1$

$$T(\lambda) = (1 - \lambda)L + \lambda R.$$

In a situation where the larger value of the quality indicator corresponds to a more preferred state $\lambda = 1$ corresponds to unrestrained optimism of DM and $\lambda = 0$ to pessimism. These limit values of λ should be reversed to the opposite situation. It is believed that under comparing of IA preference should give to the alternative with the best (highest or lowest) value of $T(\lambda)$. In the Hurwicz method also no place for estimating risk of making a wrong decision about preference of IA. Here there is also a disadvantage associated with the complexity of justifying the value of λ in concrete problems of IAcomparing.Let us take attention in this connection on the fact that using recommended sometimes values of the ―pessimism – optimism" factor, for example, $\lambda = 1/3$, in some cases insufficiently productive.So $I_1 \succ I_2$for all permissible identical for the compared IA values of $\lambda \in [0, 1]$in the configurations of the right shift ($L_2 < L_1 < R_2 < R_1$).Indeed since

$$T_1 - T_2 = \lambda(R_1 - R_2) + (1 - \lambda)(L_2 - L_1),$$

then $T_1 > T_2$ for these configurations. Therefore some experts have to show their knowledge use various different values of λfor differentcompared IA. This opens the way for arbitrariness in the choice of the preferred IA. These remarks apply also to other configurations in the case of application of Hurwicz method for comparing IA.

Let expert decided to bring all available information about IA that meets all three states of nature and to use Savage method.In accordance with this method we have for pair of IA in configuration of right shift: for the unfavorable state of nature $MAX_{UF}(I_1, I_2) = L_1$; forthe neutral state of nature $MAX_N(I_1, I_2) = MAX(In_1, In_2)$;for the favorable state of nature $MAX_F(I_1, I_2) = R_1$. We havethen the following regret matrix (Table 1) for configuration of right shift (RS):

Table 1. Regret matrix(for RS)

	UF	N	F
I_1	0	$\mathrm{MAX}(In_1, In_2) - In_1$	0
I_2	$L_1 - L_2$	$\mathrm{MAX}(In_1, In_2) - In_2$	$R_1 - R_2$

One can see now that $I_1 \succ I_2$ in accordance with the Savage criterion for $In_1 \geq In_2$(natural case ofuniform changing the value of the quality indicator with changing states of nature). We receive for$In_1 < In_2$ (such condition can be set by an expert):

$$I_1 \succ I_2, \text{ if } In_2 - In_1 < MAX(L_1 - L_2, R_1 - R_2)$$

$$I_2 \succ I_1, \text{ if } In_2 - In_1 > MAX(L_1 - L_2, R_1 - R_2).$$

Similarly, in the configuration of embedded intervals we have: forthe unfavorable state of nature $MAX_{UF}(I_1, I_2) = L_2$; for the neutral state of nature $MAX_N(I_1, I_2) = MAX(In_1, In_2)$;for the favorable state of nature $MAX_F(I_1, I_2) = R_1$. We havethen the following regret matrix(Table 2) for configuration of embedded intervals (EI):

Table 2. Regret matrix(for EI)

	UF	N	F
I_1	$L_2 - L_1$	$\mathrm{MAX}(In_1, In_2) - In_1$	0
I_2	0	$\mathrm{MAX}(In_1, In_2) - In_2$	$R_1 - R_2$

Hence, if$In_1 \geq In_2$then

$$I_1 \succ I_2, \text{ if } L_2 - L_1 < MAX(In_1 - In_2, R_1 - R_2),$$

$$I_2 \succ I_1, \text{ if } L_2 - L_1 > MAX(In_1 - In_2, R_1 - R_2).$$

Similarly, for $In_1 < In_2$ one can receive:

$$I_1 \succ I_2, \text{ if} R_1 - R_2 < MAX(L_2 - L_1, In_2 - In_1),$$

$$I_2 \succ I_1, \text{ if} R_1 - R_2 > MAX(L_2 - L_1, In_2 - In_1).$$

For coincide intervals Wald method leads to the conclusion that compared objects are equivalenton preference. The results for Savage method depend on the position of the point estimates corresponding to the neutral state of nature. Namely,

$$I_1 \succ I_2, \text{ if } In_1 \geq In_2; I_2 \succ I_1, \text{ if } In_1 < In_2.$$

Thus one can see that Savage method is the most suitable for comparing IA under pure uncertainty.The method permits to use the knowledge of expertsbetter than by other methods of this class. It takes into account the values of quality indicators for the many states of nature, as well as DM preferences in predicting values of quality indicators at interior points of the interval estimates.

4 Conclusion

Formal methods of comparison of interval alternatives as components of intellectual computer systems for information-analytical support of the decision-making process cannot guarantee choice of truly the best object as the result of comparing procedure. The results using of such methods can serve for DM only as a guideline, kind of a hint in the decision-making. At present there is no approach transcending all others in quality of recommendations obtained on its basis. Each of the available methods has its advantages and disadvantages. Each of method allows calculating its measures of alternative preference in relation to the other alternatives in their set and some as well as their risk measures.Presence of the collective effect in groups of compared alternatives is manifested primarily in reducing value of preference chances for each alternative with respect to its chances under pair-wise comparison. This leads to a quantitative increasing risk value of selection as preferred alternative such one, which may not actually be per se later, at the time of removal of uncertainty.However seeing that the perception of risk is individual and can vary from one DM to another the risk value resulting from the use of formal methods is nothing more than a calculated risk, which can serve only as an estimate for the DMs. To reduce the calculated risk shouldtry to reduce the number of comparable alternatives, to take into account a possibility of joint using of different comparing methods as well as to combine the power of formal methods and knowledge of experts and DMs.In this regard the following procedure can be recommended. Firstly, to use the method of collective risk estimating to evaluate integral risks for each alternative in the group and find the ―best" alternatives as the alternatives with the highest chances at pairwise comparisons in the set of compared alternatives. Then this narrowing set of alternatives should be evaluated according to the criteria of preference and risk, which are based on methods of ―mean – risk" approach. For situation of pure uncertainty Savage method may be recommended as tool of comparing because the method permits more fully to use the knowledge of expertsthan in the framework of other methods of this class.

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