axisymmetrical natural frequencies of vibrations of plates. Also, the method gives acceptable results for all types of laminates (symmetrical, non-symmetrical, antisymmetrical and with mid-ply symmetry.)

References

[1] A. Katunin, W. Moczulski "The conception of a methodology of degradation degree evaluation of laminates", *Maintenance and Reliability*, vol. 1 (41), pp. 33-38, Warsaw 2009.

[2] W. Moczulski, A. Katunin "Research on model-based and experimental evaluation of degradation of composite laminates", *4th International Congress on Technical Diagnostics "Diagnostics '2008"*, p. 130, Olsztyn 2008.

[3] A. Katunin "About self-activating temperature modeling In laminate", *2nd Scientific Conference „Computer Methods – 2008"*, p. 21-24, Gliwice 2008 (in Polish).

[4] A. Katunin, W. Moczulski "Faults detection in layered composite structures using wavelet transform", *8th International Technical Systems Degradation Seminar*, Liptovský Mikuláš 2009.

[5] J. Jaroszewicz, L. Zoryj, A. Katunin "Analysis of axisymmetrical vibrations of multilayered circular plates with constant and linear variable thickness", *4th Conference "Energy in Science and Technology"*, pp. 81-92, Bialystok-Suwalki 2005 (in Polish).

[6] A. Katunin, "Analysis of basic frequency of axisymmetrical vibrations of circular composite plate", *1st Scientific Conference „Computer Methods – 2007"*, pp. 33-36, Gliwice 2007 (in Polish).

[7] L. Zoryj, J. Jaroszewicz "Main frequencies of axial symmetric vibrations of the thin plates with variable parameters distribution", *Mechanical Engineering*, Lviv, vol. 5, pp. 37-38, 2002 (in Ukrainian).

[8] J. Jaroszewicz, L. Zoryj, A. Katunin "Double-sided estimators of natural frequencies of axisymmetric vibrations of circular plates with variable thickness", *3rd Conference "Energy in Science and Technology"*, Suwalki, pp. 45-56, 2004 (in Polish).

[9] J. Jaroszewicz, L. Zoryj, A. Katunin "Influence of additional mass rings on frequencies of axi-symmetrical vibrations of clamped circular plates of linearly variable thickness", *Journal of theoretical and applied mechanics*, 44, 4, pp. 867-880, Warsaw 2006.

[10] J. Jaroszewicz, L. Zoryj "Methods of analysis axisymmetrical vibrations of circular plates using Cauchy's influence function method", 120 p., Bialystok 2005.

[11] L. Zoryj, J. Jaroszewicz "Investigation of the effect of axial loads on the traverse vibrations a vertical cantilever with variable parameters", *International Applied Mechanics*, vol. 36, 9, p. 1242-1251, 2000.

[12] A. Katunin "Analysis of the influence of thickness parameter on frequencies of axi-symmetrical vibrations of clamped circular plates with linearly and nonlinearly variable thickness", *2nd International Conference of Young Scientists "Computer Science and Engineering 2007"*, pp. 159-162, Lviv 2007.

[13] S.A. Bernstein, K.K. Keropian "Evaluation of frequencies of vibrations of rod systems using spectral function method", *Gosstrojizdat*, 281 p., 1960.

New mathematical approach to the treatment of positron annihilation lifetime data for humidity-sensitive ceramics

Halyna Klym[1,2], Adam Ingram[3], Oleh Shpotyuk[2]

[1]Specialized Computer Systems Dpt., Lviv Polytechnic National University, UKRAINE, Lviv, S. Bandery str., 12, E-mail: klymha@yahoo.com

[2]Scientific Research Company "Carat", UKRAINE, Lviv, Stryjska str, 202

[3]Opole University of Technology, POLAND, Opole, Ozimska str., 75

***Abstract* – The new mathematical approach to the treatment of positron annihilation lifetime data in humidity-sensitive $MgAl_2O_4$ ceramics are proposed. It is shown that water-sorption processes in these ceramics leads to increase in positron trapping rates of extended defects located near intergranual boundaries. The fixation of direct positron lifetime components allows refining the most significant changes in positron trapping rate.**

Keywords – mathematical treatment, water-sorption process, intergranual boundaries, positron trapping.

I. Introduction

The spinel-structured $MgAl_2O_4$ ceramics are perspective materials for humidity sensors mainly due to a uniform porous structure, which promotes effective adsorption of great number of water molecules [1]. Recently, it was shown that the amount of adsorbed water in these ceramics affects not only their electrical conductivity, but also positron trapping modes of extended defects tested with positron annihilation lifetime (PAL) spectroscopy [1,2]. The positrons injected in the studied $MgAl_2O_4$ ceramics underwent two positron trapping with two components in positron lifetimes and ortho-positronium o-Ps decaying, these parameters being obtained with a so-called three-component mathematical fitting procedure. Within this approach, the shortest component of the deconvoluted PAL spectra with positron lifetime τ_1 reflects mainly microstructure specificity of the spinel ceramics and the middle component with positron lifetime τ_2 corresponds to

extended defects located near intergranual boundaries. The third component with lifetime τ_3 is due to "pick-off" annihilation of o-Ps in the nanopores. It is established that the adsorbed water molecules act catalytically on positron trapping in $MgAl_2O_4$ ceramics, do not changing significantly o-Ps decaying modes [2].

To refine the most significant changes in positron trapping in $MgAl_2O_4$ ceramics caused by water sorption, a new mathematical approach to the treatment of experimental PAL data should be developed in such a way to accumulate the catalytic effect in some non-direct trapping parameters, while other direct components (the reduced bulk and defect-related lifetimes, in the first hand) being left nearly constant.

II. Experimantal

The PAL measurements were performed with an ORTEC spectrometer with ^{22}Na source placed between two ceramic samples (see Fig. 1) at 20 ^{o}C within row of relative humidity (RH) of 25-60-98-60-25 % using humidistat PID+ (see Fig.2).

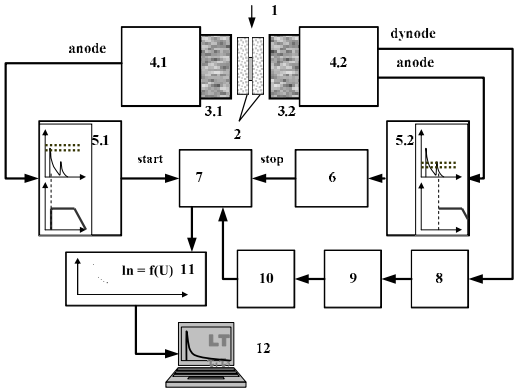

Fig.1. Block-scheme of conventional sample-source "sandwich" arrangement for PAL measurements using the ORTEC apparatus [1]:
1 – foil-covered ^{22}Na source, 2 – two identical samples, 3.1 and 3.2 – scintillators of γ-quanta (plastic KL detectors), 4.1 and 4.2 – photomultipliers (model RCA 8575), 5.1 and 5.2 – constant fraction discriminators (model 473A), 6 – delay line (model 425A), 7 – time-pulse height converter (model 467), 8 – preamplifier (model 113), 9 – amplifier (model 471), 10 – single channel analyzer (model 455), 11 – multichannel analyzer (model 6420B), 12 – personal computer.

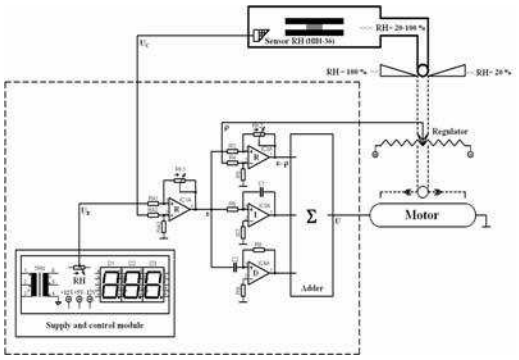

Fig.2. Schematic of humidity control instrument PID+:
U_B – turn-on voltage, which proportional to desired value of RH; U_c – output voltage of humidity sensor, which proportional to desired value of RH in the chamber; ε(t) – difference between the set and necessary values: $\varepsilon \sim U_B - U_c$; ρ(t) – regulator voltage, which proportional to the given portion of air in humidity; R – difference input in amplification regulation: k_1 (RK1), k_2 (RK2); I – integration input in the general time regulation T_i (R6, C1), D –difference input in detection loss regulation T_d (R8, C2), Σ –adder input from input voltage controlling by voltage amplifier:

$$u(t) = k_1\left[\left(\varepsilon(t) - \rho(t)\frac{k_2}{k_1}\right) + \frac{1}{T_i}\int_0^{T_i}\varepsilon(t)dt + T_d\frac{de(t)}{dt}\right], \quad (1)$$

where k_1=RK1/*RA*1, k_2=*RK*2/*R*3, T_i=*C*1·*R*6 and T_d=*C*2·*R*8. These values for measuring chamber were experimental selected based on previous calculations in accordance with Zieglera-Nicholsa criteria.

The selection of corresponding values for measuring chamber permit to investigation of samples at constant values of RH in the range of 25-60 % with an accuracy of ± 0,5 % and 60-98 % з with an accuracy of ± 1 %. The obtained PAL data were mathematically treated within three-component fitting procedure with fixed first and second positron lifetimes using LT computer program. Using formalism for two-state positron trapping model [1], the following parameters describing positron lifetime spectra can be calculated according to equations (2):

$$\kappa_d = \frac{I_2}{I_1}\left(\frac{1}{\tau_b} - \frac{1}{\tau_2}\right), \quad \tau_b = \frac{I_1 + I_2}{\frac{I_1}{\tau_1} + \frac{I_2}{\tau_2}}, \quad \tau_{av} = \frac{\tau_1 I_1 + \tau_2 I_2}{I_1 + I_2}, \quad (2)$$

where κ_d is positron trapping rate in defect, τ_b – positron lifetime in defect-free bulk and $\tau_{av.}$ – average positron lifetime. The difference (τ_2 - τ_b) can be accepted as a size measure of extended defects, as well as the τ_2/τ_b ratio represents the nature of these defects [2].

III. Results

It is established that the adsorbed water molecules act catalytically on positron trapping modes in $MgAl_2O_4$ ceramics, do not changing significantly o-Ps decaying modes [2]. Nevertheless, refining the most considerable changes in positron trapping in the studied ceramics caused by water sorption is a difficult problem through a large quantity of arbitrary fitting parameters at treatment of PAL spectra. This task can be permitted due to the treatment of experimental PAL data at the fixed values of lifetimes τ_1 and τ_2. It is established that lifetime τ_1 reflects mainly microstructure specificity of $MgAl_2O_4$ ceramics [2]. The adsorption processes are not change the structure of these ceramics. The lifetime τ_2 corresponds to extended defects near intergranual boundaries where ceramics are more defective. It is shown that the positrons are trapped in the same extended defects in $MgAl_2O_4$ independently of the content of absorbed water in their nanoporous [2].

Thus, the lifetimes of the first and the second PAL components (τ_1 and τ_2) at the treatment of experimental PAL data can be considered nearly constant. Within this mathematical approach, all changes in the fitting

parameters of these components will be reflected in their intensities (I_1 and I_2). The third longest component with lifetime τ_3 is non-fixed. The treatment of experimental data was carried out at fixed lifetime values of τ_1=0.17-0.20 ns and τ_2=0.36-38 ns. The best FIT was obtained at constant lifetimes τ_1 = 0.17 ns and τ_2 = 0.37 ns. Within this approach I_1 and I_2 intensities of the direct PAL components are changes dependently from amount of adsorbed water in the studied ceramics. So increasing of RH from 25 to 98 % result in decreasing of I_1 intensity and increasing of I_2 intensity. The changing of RH from 98 to 25 % reflects inverse to the previous direction in I_1 and I_2 intensities (see table 1). The positron trapping in water-filled defects reflecting the second component with I_2 intensity occurs more intensive.

TABLE 1

PAL CHARACTERISTICS OF CERAMICS

RH, %	FITTING PARAMETERS					
	τ_1, ns	I_1, a.u.	τ_2, ns	I_2, a.u.	τ_3, ns	I_3, a.u.
25	0.17	0.85	0.37	0.14	2.37	0.01
60	0.17	0.83	0.37	0.16	2.81	0.01
98	0.17	0.81	0.37	0.17	2.42	0.01
60	0.17	0.83	0.37	0.16	2.34	0.01
25	0.17	0.84	0.37	0.15	2.38	0.01
RH, %	Positron trapping modes					
	τ_{av}, ns	τ_b, ns	κ_d, ns^{-1}	τ_2 - τ_b, ns	τ_2/τ_b	
25	0.20	0.18	0.44	0.18	2.01	
60	0.20	0.19	0.52	0.18	1.98	
98	0.21	0.19	0.56	0.18	1.97	
60	0.20	0.19	0.50	0.18	1.99	
25	0.20	0.19	0.49	0.18	2.00	

The lifetimes τ_3 are closed to ~2.3-2.8 ns (see table 1). The input of this third component is not change and intensity closed to 0.01. Thus, this channel is non-significant to water sorption-desorption processes. The positron trapping modes such as the average τ_{av}, defect-free bulk τ_b and difference τ_2 - τ_b are non-changed with RH. In addition, the positron trapping centre (τ_2/τ_b) is formed on a typical for $MgAl_2O_4$ ceramics level of ~1.9 [2], which testify to the same nature of trapping sites whichever the content of absorbed water. In contrast, most significant changes in positron trapping in $MgAl_2O_4$ ceramics caused by water sorption reflect in positron trapping rate in defect κ_d. Thus, the catalytic water-sorption effect in the studied spinel-structured ceramics is accumulated in non-direct trapping κ_d parameter.

Conclusion

The mathematical treatment of experimental PAL data at constant values of reduced bulk and defect-related lifetimes allow to refine the most significant changes caused by absorbed water in the spinel-structured $MgAl_2O_4$ ceramics

References

[1] R. Krause-Rehberg and H.S. Leipner, "Positron Annihilation in Semiconductors. Defect Studies, Springer-Verlag, Berlin-Heidelberg-New York, 1999, p 378.

[2] H. Klym, A. Ingram, O. Shpotyuk, J. Filipecki and I. Hadzaman, "Extended positron-trapping defects in insulating $MgAl_2O_4$ spinel-type ceramics", Phys. Stat. Sol. C, vol. 4, No 3, pp. 715-718, 2007.

Моделювання однієї оберненої задачі до задачі Стефана

Олена Кобильська

Кафедра інформатики та вищої математики, Кременчуцький державний політехнічний університет ім. Михайла Остроградського, УКРАЇНА, м.Кременчук, вул.Першотравнева, 20, E-mail: leca91@ya.ru

Abstract – A mathematical model is considered as a problem of Steafan for equation of heat conductivity. The decision of problem is found in space of gridding functions of the Rote method and by a report to nonlinear integral equation of Gammershteyn type with a basis of Grina's functions. Basic parameter of control the temperature field defined. The numeral calculations of the temperature distributing are conducted.

Ключові слова – problem of Steafan, mathematical model, heat equation.

I. Вступ

У роботі розглянута математична модель у вигляді задачі Стефана для рівняння теплопровідності. Розв'язок задачі знайдено у просторі сіткових функцій методом Роте та шляхом зведення до системи нелінійних інтегральних рівнянь типу Гаммерштейна з ядром у вигляді функцій Гріна. Постановка задачі викликана необхідністю створення системи керування температурним полем рухомого середовища.

II. Постановка проблеми

У порошковій металургії виробництво стрижнів та дроту із тугоплавких металів, наприклад вольфраму, відбувається одночасно з процесом підігрівання металу [1]. Це викликано тим, що більшість тугоплавких металів не деформується при кімнатній температурі. Перед пластичною деформацією дріт, що рухається через зону нагрівання довжиною $\xi(t) = L - v(t)t$ зі швидкістю $v(t) \neq 0$, розігрівається електричним струмом I_0 до технологічної температури T_1, а потім потрапляє у пристрій для деформування. При цьому до одного кінця зони нагрівання підведений нерухомий струмопідвід, а до іншого рухомий.

Розглянемо дріт у вигляді рухомого ізотропного середовища зі сталими теплофізичними характеристиками, що рухається через зону нагрівання зі швидкістю $v(t) \neq 0$. Проблема керування температур-