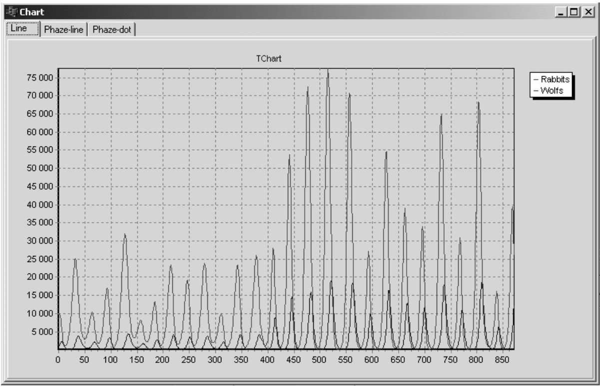

Рис. 2.

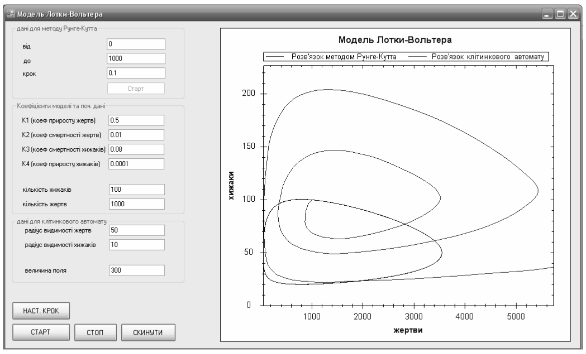

Рис. 3.

Результати, які отримані при розв’язуванні математичним методом і при моделюванні, зображаються кривими у фазовому просторі, що дає змогу легко проаналізувати їх і визначити які корективи необхідно внести до вхідних даних, щоб покращити отриманий результат.

Така структура програми дає змогу досить ретельно аналізувати процес моделювання.

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Evaluation of lower natural frequencies of axisymmetric vibrations of laminate circular plates clamped on the boundary

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In the paper the possibility of using Cauchy’s influence function method in the case of material nonhomogeneity as layered structures was showed. For evaluation the lower natural frequencies of axisymmetric vibrations of laminate circular plates the double-sided estimators of Bernstein-Keropian tables and simplified analytical model of plates vibration were used, obtained results were compared with results obtained from finite element method. The difference growing between obtained results during frequency growing was noticed, but differences were in tolerance limit. Proposed algorithm of evaluating lower natural frequencies of axisymmetric vibrations can be used as approximative and can find an application in degradation degree evaluation of composite laminates.

Key words – clamped circular plates, laminate, Cauchy’s influence function method.

I. Introduction

Modern engineering constructions used more frequently unconventional construction materials such as composites. Significant part of it is layered structures with polymer matrix like GFRP and CFRP laminates. Main advantage of polymer laminates is in its lightweight with simultaneously high durability properties, which creates a possibility to use it in most responsible applications applied in aircraft and aerospace industries, automotive industry, military industry and many others. The behavior of it must be predictable in all phases of exploitation and in research it must also take into consideration some additional phenomena (e.g. crack initiation and propagation [1,2], self-activating temperature [3], etc.).

One of the main criteria of degradation degree evaluation is dynamic behavior of the structures. Based on frequency responses it is a possibility to detect faults in layered structures using algorithm based on wavelet transform [4]. In this paper were investigated vibrations of clamped circular plate. This model can represent many responsible constructional elements like bottoms of cylindrical containers, rotors etc. For evaluation of natural frequencies of vibrations the Cauchy’s influence function method and Bernstein-Keropian tables were used. Previous works shows, that proposed method gives good results for circular clamped plates with homogeneous and inhomogeneous materials [5,6] and additional properties like thickness variability [7,8] and additional masses [9]. In this paper the lower frequencies of axisymmetrical vibrations were investigated, which can find an application in degradation degree evaluation of layered structures.

II.Problem description

Assuming Kirchhoff-Love's theory, thin clamped circular plate, with the radius *R*, flexural rigidity D_0 and thickness described by power functions of the radial coordinate *r* for general case of thickness variability. In case of constant thickness of plate (*R = r*, so D_0, $h_0 = const$), parameter of thickness variability *m = 0* [10].

$$D = D_0\left(\frac{r}{R}\right)^m, \quad h = h_0\left(\frac{r}{R}\right)^{\frac{m}{3}}, \quad 0 < r \le R, \qquad D_0 = \frac{Eh_0^3}{12(1-\nu^2)}. \tag{1}$$

Investigation of free axi-symmetrical vibrations, such as plate, consists of the boundary problem [11]

$$L_0[u] - pr^{-\frac{2}{3}m}u = 0, \quad p = \frac{\rho h_0}{D_0}R^{\frac{2}{3}m}\omega^2, \tag{2}$$

$$u(R) = 0, \qquad u'(R) = 0, \tag{3}$$

$$L_0[u] \equiv u^{IV} + 2r^{-1}(m+1)u^{III} + r^{-2}\left(m^2 + m + \nu m - 1\right)u^{II} + r^{-3}(m-1)(\nu m - 1)u^{I}, \tag{4}$$

where: *u* – flexural amplitude, *u=u(r)*, *ρ* – density, *ω* – frequency parameter, *ν* – Poisson's ratio, *E* – Young's flexural modulus.

Taking into consideration constant thickness of the plate the differential operator (4) assumes form (5):

$$L_0[u] \equiv u^{IV} + \frac{2}{r}u^{III} - \frac{1}{r^2}u^{II} + \frac{1}{r^3}u^{I}. \tag{5}$$

After equaling (5) to zero and solving obtained equation four independent roots can be obtained and based on it the influence function can be constructed. For constant thickness it has the next form [11]:

$$K_0(r,\alpha) = \frac{1}{4}\alpha\left[\alpha^2 - r^2 + \left(\alpha^2 + r^2\right)\ln\frac{r}{\alpha}\right]. \tag{6}$$

Equation (2) simplifies to linear form. Then, elements of power series were obtained from (7) and (8). Closed form of power series can be written as (9) [12].

$$U = 1 + pu_1 + p^2u_2 + \ldots, \qquad V = r + pv_1 + p^2v_2 + \ldots, \tag{7}$$

$$u_i = \int_0^r K_0(r,s)s^{-2}u_{i-1}(s)ds; \; u_0 = 1, \qquad v_i = \int_0^r K_0(r,s)s^{-2}v_{i-1}(s)ds; \; v_0 = r, \tag{8}$$

$$U = \sum_{k=0}^{\infty}(-1)^k p^k r^{2k}\prod_{i=1}^{k}\alpha_i, \qquad V = \sum_{k=0}^{\infty}(-1)^k p^k r^{2k-1}\prod_{i=1}^{k}\alpha_i. \tag{9}$$

According to equation (2) limited by zero, functions (8) and series (9) we can acquire equation (10). Based on (10) the characteristic determinant can be built, which is reconstructed to characteristic series (11).

$$u = C_1U + C_2V. \tag{10}$$

$$\begin{vmatrix} U & V \\ U' & V' \end{vmatrix} = 1 - a_1\left(pR^2\right) + a_2\left(pR^2\right)^2 - \ldots = 0. \tag{11}$$

From (11) the *a* coefficients were calculated, from which double-sided estimators of coefficient of natural frequency of vibrations were obtained based on Bernstein-Keropian tables [13]

$$(\gamma_1)_- = \sqrt{\frac{a_0}{\sqrt{a_1^2 - 2a_0a_2}}}, \qquad (\gamma_1)_+ = \sqrt{\frac{2a_0}{a_1 + \sqrt{a_1^2 - 4a_0a_2}}}, \tag{12}$$

where: $a_0 = 1$, $a_1 = 1/96$, $a_2 = 1/122880$ [11].

Based on (12) the basic frequency of vibrations was calculated:

$$\omega_1 = \gamma\frac{1}{R^2}\sqrt{\frac{D_0}{\rho h_0}}, \tag{13}$$

where $\gamma = 10{,}214$ [8].

In laminates case with transversal isotropic model form of (1) must be changed, because rigidity D_0 reconstruct to matrix (14):

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}, \tag{14}$$

where

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\left\{Q_{ij}\right\}_k\left(z_k^3 - z_{k-1}^3\right), \; (i,j = 1,2,3), \tag{15}$$

z – distance of investigated layer from laminate mid-ply. According to (14), (15) and taking into consideration layers orientation in laminate the scalar approximative value of rigidity can be constructed:

$$D = \sum_{k=1}^{n}\frac{z_k^3 - z_{k-1}^3}{6(1-\nu_{12}\nu_{21})}\times \left[\begin{matrix} 4\sin^2\theta\cos^2\theta\left(E_2\nu_{21} + 2G_{12}(1-\nu_{12}\nu_{21})\right) + \\ + \left(\sin^4\theta + \cos^4\theta\right)\left(E_1 + E_2\right) \end{matrix}\right], \tag{16}$$

In (16) only the values of elements of rigidity matrix in main directions are took into consideration.

III.Using Bernstein-Keropian estimators for lower frequencies

Bernstein-Keropian tables [13] give an opportunity to obtain the estimators of three lower natural frequencies and lower estimator for fourth frequency of plate vibrations. Firstly, B_i coefficients were calculated using (17). Then, based on ratio B_2 / B_1^2 values from tables can be noticed. Using (18) and (19) estimators of coefficients of lower natural frequencies can be obtained.

$$B_1 = \frac{a_1}{a_0}, \quad B_2 = \left(\frac{a_1}{a_0}\right)^2 - 2\frac{a_1 a_2}{a_0^2} \qquad (17)$$

$$(\gamma_i)_- = \sqrt[4]{\frac{\varphi_i}{B_1}}, \ (\gamma_i)_+ = \sqrt[4]{\frac{\beta_i}{B_1}}, \ \gamma_4 \approx \sqrt[4]{\frac{\psi}{B_1}} \qquad (18)$$

$$\omega_i = (\gamma_i)_\pm^2 \frac{1}{R^2}\sqrt{\frac{D_0}{\rho h_0}}; \quad (i = 2,3,4) \qquad (19)$$

IV. Results analysis and recapitulation

24-layered GFRP laminate circular clamped plate with structural formula (20) was investigated. Geometric parameters and material properties of the plate were presented in Table I.

$$[0/60/-60/-60/60/0]_{4S}, \qquad (20)$$

TABLE I

GEOMETRIC PARAMETERS AND MATERIAL PROPERTIES

E_1, GPa	E_2, GPa	G_{12}, GPa	v_{12}
38,283	10,141	3,533	0,366
ρ, kg/m^3	**R**, m	**h**, m	h_0, m
1794	0,19	0,00528	0,00022

Based on Table I and (16)-(19) estimators of coefficients of natural frequencies and natural frequencies of vibration were calculated. Obtained results presented by (21) and (22).

$$\begin{gathered} 1412{,}8377 < \omega_1 < 1414{,}9852 \\ 4745{,}6169 < \omega_2 < 5903{,}4312 \\ 8545{,}2206 < \omega_3 < 15166{,}7474, \\ \omega_4 < 19011{,}1413 \end{gathered} \qquad (21)$$

$$\begin{gathered} 224{,}86 < f_1 < 225{,}202 \\ 755{,}288 < f_2 < 939{,}56 \\ 1360{,}014 < f_3 < 2413{,}863. \\ f_4 < 3025{,}717 \end{gathered} \qquad (22)$$

For comparison of obtained results the numerical model based on FEM was built by using MSC.Patran/Nastran software. Defining the laminate the geometrical model was built, which was divided to 1380 finite elements and in according to Table I material properties were defined. Then, Laminate Builder Tool was used for defining and orientation of layers according to (20). Analysis type was defined as normal modes analysis. Analytically obtained values of natural frequencies (22) were averaged and compared with FEM results. The comparison was presented in Table II. Modes of axisymmerical vibrations of above-mentioned cases presented on Figs.1-4.

TABLE II

COMPARISON OF ANALYTICAL AND FEM RESULTS

Freq. #	f_1, Hz	f_2, Hz	f_3, Hz	f_4, Hz
Analyt.	225,031	847,424	1886,94	3025,72
FEM	237,484	920,028	2045,69	3596,35
Diff, %	5,24	7,89	7,76	15,87

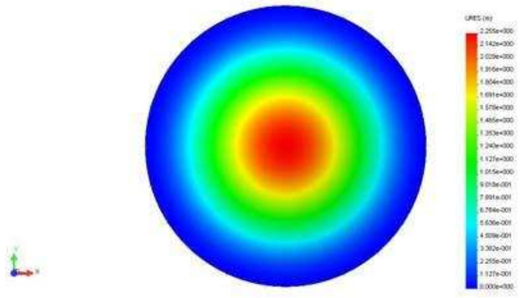

Fig.1. First natural mode of plate vibrations (first axisymmetrical mode)

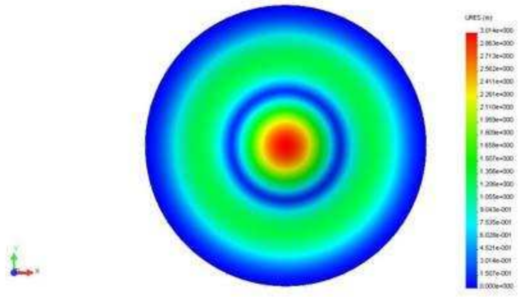

Fig.2. Sixth natural mode of plate vibrations (second axisymmetrical mode)

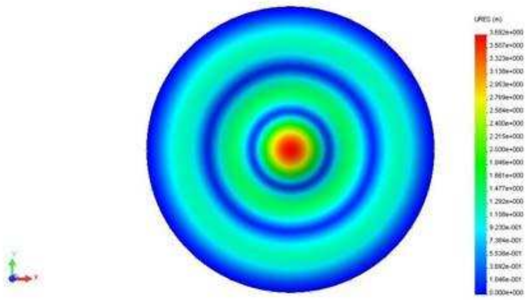

Fig.3. Fifteenth natural mode of plate vibrations (third axisymmetrical mode)

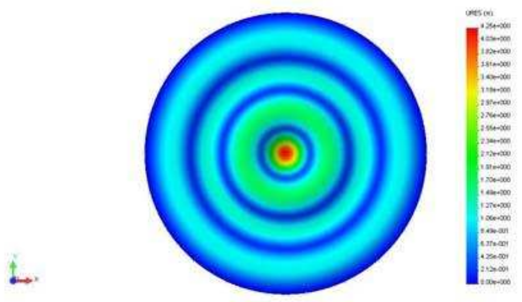

Fig.4. Thirtieth natural mode of plate vibrations (fourth axisymmetrical mode)

Analyzing obtained results (Table II) it is possibility to affirm, that proposed algorithm of evaluating lower frequencies of vibrations of layered circular plates allows to approximative obtaining values of frequencies, because differences between analytical and FEM results gives differences in acceptability bounds. The disadvantage of the method is impossibility of obtaining nonaxisymmetrical natural frequencies of vibrations of plates. Also, the method gives acceptable results for all types of laminates (symmetrical, non-symmetrical, antisymmetrical and with mid-ply symmetry.)

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New mathematical approach to the treatment of positron annihilation lifetime data for humidity-sensitive ceramics

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Abstract – The new mathematical approach to the treatment of positron annihilation lifetime data in humidity-sensitive $MgAl_2O_4$ ceramics are proposed. It is shown that water-sorption processes in these ceramics leads to increase in positron trapping rates of extended defects located near intergranual boundaries. The fixation of direct positron lifetime components allows refining the most significant changes in positron trapping rate.

Keywords – mathematical treatment, water-sorption process, intergranual boundaries, positron trapping.

I. Introduction

The spinel-structured $MgAl_2O_4$ ceramics are perspective materials for humidity sensors mainly due to a uniform porous structure, which promotes effective adsorption of great number of water molecules [1]. Recently, it was shown that the amount of adsorbed water in these ceramics affects not only their electrical conductivity, but also positron trapping modes of extended defects tested with positron annihilation lifetime (PAL) spectroscopy [1,2]. The positrons injected in the studied $MgAl_2O_4$ ceramics underwent two positron trapping with two components in positron lifetimes and ortho-positronium o-Ps decaying, these parameters being obtained with a so-called three-component mathematical fitting procedure. Within this approach, the shortest component of the deconvoluted PAL spectra with positron lifetime τ_1 reflects mainly microstructure specificity of the spinel ceramics and the middle component with positron lifetime τ_2 corresponds to