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# CAPACITANCE CALCULATIONS FOR SYSTEM OF FINITE CONDUCTING PATHS

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Наведено результати теоретичного визначення ємності для системи з прямокутними провідними каналами в плівкових мікросхемах. Отримані результати є продовженням попередніх досліджень. У першому випадку розглядаються два паралельні провідні канали з визначеною шириною і довжиною, які розміщені на одному боці діелектричної підкладки невизначених розмірів. В іншому випадку ці канали відхиляються один від одного на кут 20. Визначення ємності для обох випадків базується на розв'язанні тривимірної крайової задачі. Електричний потенціал подається у формі інтегралів Фур'є, які задовольняють рівняння Лапласа. Виходячи з цього, числовими методами розв'язується рівняння розподілу електричного заряду. На цій основі розраховується величина ємності.

Results of theoretical capacitance determination for the rectangular conductive path systems in film microcircuit are presented in this paper. It is a continuation of earlier investigations. In the first order two parallel conductive paths with determined width and length located on the same side of dielectric microcircuit substrate of infinite dimensions is here considered. Next these paths are deflected in relation to each other at an  $2\alpha$ -angle The capacitance determination of both systems is based on the solution of three-dimensional boundary problem. Electrical potential is presented in form of Fourier integrals, satisfying the Laplace's equation. Resulting from here the equation system of electric charge distribution has next been solved by application of numerical method. On this basis the capacitance value has been calculated.

### **1. INTRODUCTION**

The design of film microcircuits differs radically in scope and procedure from conventional circuit design techniques. Coupling effects become here sufficiently significant. Therefore, taking into consideration the criteria of electromagnetic compatibility, reliability or tolerance aspects, an identification and analysis of interelement influences between particular elements of the microcircuits are very important. From this point of view knowledge about parasitic or usefully coupling capacitance among various conductors is also necessary.

Capacitive couplings appear in the all thin-film thick-film structures and integrated circuits and generally may have deleterious effect on the over-all circuit performance by introducing stray capacitance between various paths. It is necessary to have the means for estimation these parasitic capacities in order to take into account their effect on the complete circuit function [1]. From the other side, for example in capacitive film sensor systems where capacitive couplings perform a useful function, the knowledge about capacitance values can be also very important.

The paper deals with a range of low and medium frequency, where it is possible to treat the particular fragments of conductive layers as geometrical separate elements. In higher frequency range, the capacitive couplings should be analysed together with inductive couplings as elements of distributed-parameter system.

### 2. COUPLING CAPACITY IN SYSTEM OF PARALLEL CONDUCTING PATHS

### **2.1.** Calculation method

The Fourier integral method for determination of parasitic capacities in parallel conducting path system is here applied. In this solution method – simplified to two-dimensional problem –has been assumed:

• length, width and thickness of substrate are infinitely great;

• two parallel conductive paths of equal width and identical length are located on the same side of dielectric microcircuit substrate (Fig. 1),

- thickness of parallel conducting paths is negligibly small,
- dielectric permeability of substrate amounts to  $\varepsilon_2 = \varepsilon_0 \varepsilon_{2r}$  and of environment  $\varepsilon_1 = \varepsilon_0 \cdot \varepsilon_{1r}$



Fig. 1. Configuration of conductive paths in plane microcircuit: a - general view, b - x - z plane

Electrical potential fulfils Laplace's equation  $\nabla^2 \varphi = 0$ . The expected solution, applying the method of separation of variables, can be presented in the form of Fourier integrals [2,3]:

$$\varphi = \begin{cases} \int_{0}^{\infty} \int_{0}^{\infty} W(\alpha, \beta) \cdot e^{-rz} \cdot \sin(\alpha x) \cdot \cos(\beta y) d\alpha d\beta & \text{for } z \ge 0 \\ \int_{0}^{\infty} \int_{0}^{\infty} W(\alpha, \beta) \cdot e^{+rz} \cdot \sin(\alpha x) \cdot \cos(\beta y) d\alpha d\beta & \text{for } z < 0 \end{cases}$$
(1)

with  $r^2 = \alpha^2 + \beta^2$ . From the boundary conditions results that

• step-change of normal component of electric induction  $D_n = \varepsilon E_n$  on the region of conducting paths is equal:

$$\varepsilon_1 E_n(x, y, 0^+) - \varepsilon_2 E_n(x, y, 0^-) = \begin{cases} q(x, y) & \text{for } a \le x \le b & -c \le y \le c \\ 0 & \text{for } x \notin [a, b] & y \notin [-c, c] \end{cases},$$
(2)

whence the coefficient  $W(\alpha, \beta)$  is received as:

$$W(\alpha,\beta) = \frac{4}{\pi^2(\varepsilon_1 + \varepsilon_2)} \int_{a-c}^{b} \int_{a-c}^{c} \frac{1}{r} \cdot q(s,t) \cdot \sin(\alpha s) \cdot \cos(\beta t) \, ds \, dt \,, \tag{3}$$

where q(s,t) is the electrical charge density.

• the electrical potential  $\varphi(x,y,0)$  applied to conducting paths is equal  $\varphi = 1$  and  $\varphi = -1$ , respectively (Fig. 1,b). Hence:

$$\phi = \frac{4}{\pi^2(\varepsilon_1 + \varepsilon_2)} \int_0^\infty \int_0^\infty \left[ \int_a^b \int_{-c}^c \frac{1}{r} q(s,t) \sin(\alpha s) \cos(\beta t) \, ds \, dt \right] \sin(\alpha x) \cos(\beta y) \, d\alpha \, d\beta = 1.$$
(4)

The unknown distribution of charge density can be determined as:

$$\int_{a}^{b} \int_{-c}^{c} q(s,t) \cdot F'(s,x,t,y) \, ds \, dt = \frac{\pi^2(\varepsilon_1 + \varepsilon_2)}{4}, \tag{5}$$

where:

$$F' = \frac{1}{4} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\cos[\alpha(s-x)] \cdot \cos[\beta(t-y)]}{\sqrt{\alpha^{2} + \beta^{2}}} d\alpha d\beta + \frac{1}{4} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\cos[\alpha(s-x)] \cdot \cos[\beta(t+y)]}{\sqrt{\alpha^{2} + \beta^{2}}} d\alpha d\beta + \frac{1}{4} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\cos[\alpha(s+x)] \cdot \cos[\beta(t+y)]}{\sqrt{\alpha^{2} + \beta^{2}}} d\alpha d\beta - \frac{1}{4} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\cos[\alpha(s+x)] \cdot \cos[\beta(t+y)]}{\sqrt{\alpha^{2} + \beta^{2}}} d\alpha d\beta .$$
(6)

Using the identity [4]:

$$\int_{0}^{\infty} \int_{0}^{\infty} \frac{\cos(AX) \cdot \cos(BY)}{\sqrt{A^{2} + B^{2}}} dA dB = \frac{\pi}{2} \cdot \frac{1}{\sqrt{X^{2} + Y^{2}}},$$
(7)

the function F'(s, x, t, y) can be written as:

$$F' = \frac{\pi}{8} \cdot \left\{ \frac{\frac{1}{\sqrt{(s-x)^2 + (t-y)^2}} + \frac{1}{\sqrt{(s-x)^2 + (t+y)^2}}}{\frac{1}{\sqrt{(s+x)^2 + (t-y)^2}} - \frac{1}{\sqrt{(s+x)^2 + (t+y)^2}}} \right\},$$
(8)

whence

$$\int_{a}^{b} \int_{-c}^{c} q(s,t) \cdot F(s,x,t,y) \, ds \, dt = 2\pi(\varepsilon_1 + \varepsilon_2) \quad \text{by} \quad F = \frac{8}{\pi} \cdot F' \,. \tag{9}$$

Capacity value will be calculated as:

$$C_M = \frac{Q}{\Delta\phi} = \frac{1}{2} \cdot \int_a^b \int_{-c}^c q(x, y) \, dx \, dy.$$
(10)

### 2.2. Numerical solution of charge distribution

The integral equation (9) can be solved with use of numerical calculation method. With this end in view the discretisation of equation has been applied (Fig. 2). It has been here assumed that equation of electrical charge distribution is fulfilled in points:

 $(x_i, y_j)$  for i = 1, ..., NH and j = 1, ..., NV, where  $x_i = a + (i - 0, 5) \cdot \Delta s$ ;  $y_j = c + (j - 0, 5) \cdot \Delta t$  and that  $q_{k,n} = \text{const.}$  for  $s_n < s < s_{n+1}, t_k < t < t_{k+1}$ .



Fig. 2. Partition of conducting layer: 
$$s_n = a + (n-1)\cdot\Delta s$$
;  $n = 1, ..., NH+1$ ;  
 $\Delta s = (b-a)/NH$ ,  $t_k = c + (k-1)\cdot\Delta t$ ;  $k = 1, ..., NV+1$ ;  $\Delta t = (d-c)/NV$ 

Taking into consideration the above assumptions and denotations the equation (9) can be written as:

$$\int_{a}^{b} \int_{c}^{d} q(s,t) \cdot F(s,x,t,y) \, ds \, dt \approx \sum_{k=1}^{NV} \sum_{n=1}^{NH} q_{k,n} \int_{t_k}^{t_{k+1}} \int_{s_n}^{s_{n+1}} F(s,t,x_i,y_j) \, ds \, dt = 2\pi(\varepsilon_1 + \varepsilon_2). \tag{11}$$

Double integral  $\int_{t_k}^{t_{k+1}} \int_{s_n}^{s_{n+1}} F(s,t,x_i,y_j) ds dt$  consisting of 4 terms can be solved analytically.

The result of integration of the expression (8) is:

$$\int_{i_{k}}^{i_{k+1}} \int_{s_{n}}^{s_{n+1}} \frac{dds}{\sqrt{(s-x_{i})^{2} + (t-y_{j})^{2}}} = \\ = wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + zl_{j,k} \cdot ln \frac{wl_{i,n} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i,n} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + \\ + w2_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + zl_{j,k} \cdot ln \frac{wl_{i,n} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + \\ + wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + zl_{j,k} \cdot ln \frac{wl_{i,n} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + \\ + wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}} + zl_{j,k} \cdot ln \frac{wl_{i,n} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} + \\ - wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} - zl_{j,k} \cdot ln \frac{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} + \\ - wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} - zl_{j,k} \cdot ln \frac{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} + \\ - wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} - zl_{j,k} \cdot ln \frac{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} + \\ - wl_{i_{n}} \cdot ln \frac{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{zl_{j,k} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} - zl_{j,k} \cdot ln \frac{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}{wl_{i_{n}} + \sqrt{(wl_{i_{n}})^{2} + (zl_{j,k})^{2}}}} + \\ - wl_{i_{n}} \cdot ln \frac{z$$

where:  $wI_{i,n} = s_{n+1} - x_i = \Delta s \cdot (n - i + 0,5),$   $w2_{i,n} = s_n - x_i = \Delta s \cdot (n - i - 0,5),$   $w3_{i,n} = s_{n+1} + x_i = 2a + \Delta s \cdot (n + i - 0,5),$   $w4_{i,n} = s_n + x_i = 2a + \Delta s \cdot (n + i - 1,5),$   $z1_{j,k} = t_{k+1} - y_j = \Delta t \cdot (k - j + 0,5),$   $z2_{j,k} = t_k - y_j = \Delta t \cdot (k - j - 0,5),$  $z3_{j,k} = t_{k+1} + y_j = 2c + \Delta t \cdot (k + j - 0,5),$   $z4_{j,k} = t_k + y_j = 2c + \Delta t \cdot (k + j - 1,5).$ 

The numerical calculation of charge density  $q_{k,n}$  resolves itself into the solution of the algebraical equation system (11).

Denoting:

$$\int_{t_k}^{t_{k+1}} \int_{s_n}^{s_{n+1}} F(s,t,x_i,y_j) \, ds \, dt = D_{t_k,s_n}^{y_j,x_i} = A_{p,r} \, ,$$

where  $p = (j - 1) \cdot \text{NH} + i$  and  $r = (k - 1) \cdot \text{NH} + n$ , the equation system (11) can be presented in matrix form:

$$\mathbf{A} \cdot \mathbf{Y} = \mathbf{B} \tag{13}$$

where

$$A = \begin{bmatrix} A_{I,I} & \cdots & A_{I,NH \cdot NV} \\ \vdots & A_{p,r} & \vdots \\ A_{NH \cdot NV,I} & \cdots & A_{NH \cdot NV,NH \cdot NV} \end{bmatrix},$$

Y – vector of the unknown charge density,

$$Y = [q_1 \quad \cdots \quad q_m \quad \cdots \quad q_{NH \cdot NV}],$$

and

$$B^T = [B_1 \cdots B_m \cdots B_{NH \cdot NV}]$$
 with  $B_m = 2\pi \cdot (\varepsilon_1 + \varepsilon_2)$ .

Therefore, the capacitance C<sub>M</sub> between conducting paths is given by

$$C_M = 0.5 \cdot \Delta s \cdot \Delta t \cdot \sum_{m=1}^{NV \cdot NH} q_m \,. \tag{14}$$

### 2.3. Calculation results

Calculations of electrical charge distribution have been realised using MathCad program, and the exemplary result is shown in Fig. 3.



Fig. 3. Numerical calculated distribution of electrical charge density

On this basis the capacity values (equation (14)) for two parallel conducting paths of determined width and length have been estimated. The computations were made for following data: distance between paths equal 0,4 mm, the width of two paths equal 0.6 mm and their length – variable, dielectric permeability  $\varepsilon_0 = 8,85 \cdot 10^{-12}$  F/m, dielectric constant of the air  $\varepsilon_{2r} = 1$  and of ceramic substrate  $\varepsilon_{2r} = 9,08$ .

The results of calculations on relation (14) have been compared with the capacity values computed by the assumption that the conducting layers are of infinite length and the capacitance between these conductors is determined per unit length [5] (Fig. 4).

This comparison shows that the values of capacitance obtained on the base of equation (14), especially for small length of conducting paths are considerably greater. It results from taking into account the surface density of electrical charge, which becomes infinitely great on the edges of paths.



Fig. 4. Variations of capacity  $C_{M}$ : 1 – calculated in according to relation (14; 2 – on the basis of [5]; L – length of conducting paths

## 3. CAPACITANCE CALCULATION IN DEFLECTED CONDUCTING PATH SYSTEM

An analysed problem deals with the system of two rectangular, symmetrically deflected for each other conducting paths (Fig. 5).



Fig. 5. Configuration of deflected path system in plane microcircuit: DL – length, SZ – width of each path

The solution of capacitance determination between these paths consists here of three parts.

• in the first one the Fourier integral method for determination of electrical charge distribution in parallel conducting path system (described above in p.2.1) is applied,

• in the second step the primary coordinate system is rotated at an  $\alpha$ -angle. As a result of this transformation the modified integral equation of charge distribution is obtained.

• the last one consists in the numerical solution of unknown charge distribution and finally, the capacitance value is calculated.

### 3.1. Rotation of coordinate system

To simplify the solution way, the coordinate system was rotated – by taking into account symmetry of path system – at an  $\alpha$ -angle (Fig.3). Relations between primary (s, t) and new (u, v) coordinates amount to:

$$s(u,v) = u \cos(\alpha) + v \sin(\alpha), \text{ and } t(u,v) = -u \sin(\alpha) + v \cos(\alpha). \tag{15}$$

A consequence of coordinate transformation is the conversion of variables in double integral according to formula [5]:

$$\iint_{\Omega} f(s,t,) ds dt = \iint_{\Delta} f(s(u,v), t(u,v)) \cdot \left| J(u,v) \right| du dv$$
(16)

where  $\Omega$  and  $\Delta$  are the closed areas in planes (s,t) and (u,v) respectively, and Jacobian J(u,v) is received as

$$|J| = \begin{vmatrix} \frac{\partial s}{\partial u} & \frac{\partial s}{\partial v} \\ \frac{\partial t}{\partial u} & \frac{\partial t}{\partial v} \end{vmatrix} = \cos^2(\alpha) + \sin^2(\alpha) = 1$$

Hence, the distribution of charge density is equal

$$\iint_{\Delta} q[s(u,v),t(u,v)] \cdot F[s(u,v),t(u,v)] \, du \, dv = 2\pi(\varepsilon_1 + \varepsilon_2). \tag{17}$$

### 3.2. Numerical solution of charge distribution

The integral equation (17) can be solved with use of numerical calculation method, similarly as in p. 2.2. The discretisation of this equation has been applied (Fig. 3).



Fig. 6. Partition of conducting layer: NH, NV – numbers of width and length partition  $u_n = a + (n - 1) \cdot \Delta u; n = 1, ..., NH+1; v_k = c + (k - 1) \cdot \Delta v; k = 1, ..., NV+1;$  $\Delta u = SZ/NH, \Delta v = DL/NV;$ 

Besides, it was here assumed that equation of electrical charge distribution is fulfilled in points ( $\eta_i$ ,  $\xi_j$ ): for i = 1, ..., NH and j = 1, ..., NV, where  $\eta_i = a + (i - 0,5) \cdot \Delta u$ ;  $\xi_j = c + (j - 0,5) \cdot \Delta v$  and that  $q_{k,n} = \text{const.}$  for  $u_n < u < u_{n+1}, v_k < v < v_{k+1}$ .

As a common point for both coordinate systems there is accepted the point A with coordinates:

 $A(s, t): \Rightarrow s_A, t_A \text{ and } A(u, v): \Rightarrow a = s_A \cos(\alpha) - t_A \sin(\alpha), c = s_A \sin(\alpha) + t_A \cos(\alpha).$ Taking into consideration the above assumptions and denotations the equation (17) can be written as:

$$\iint_{\Delta} q(u,v) \cdot F(u,v,\eta_i,\xi_j) du dv \approx \sum_{k=1}^{NV} \sum_{n=1}^{NH} q_{k,n} \int_{v_k}^{v_{k+1}} \int_{u_n}^{u_{n+1}} F(u,v,\eta_i,\xi_j) du dv = 2\pi(\varepsilon_1 + \varepsilon_2).$$
(18)

The subintegral function  $F(u, v, \eta, \xi)$  is a result of variable conversion of function F(s, t, x, y).

Double integral  $\int_{v_k}^{v_{k+1}} \int_{u_n}^{u_{n+1}} F(u, v, \eta_i, \xi_j) du dv$  consists also of 4 terms S1 – S4, and each of them

can be transformed according to formula (16).

The first one, by  $x_i = \eta_i cos(\alpha) + \xi_j sin(\alpha)$  and  $y_j = -\eta_i sin(\alpha) + \xi_j cos(\alpha)$ , is equal

$$S1 = \int_{t_k}^{t_{k+1}} \int_{s_n}^{s_{n+1}} \frac{dsdt}{\sqrt{(s-x_i)^2 + (t-y_j)^2}} = \int_{v_k}^{v_{k+1}} \int_{u_n}^{u_{n+1}} \frac{|J(u,v)| dudv}{\sqrt{[s(u,v) - x_i(\eta_i,\xi_j)]^2 + [t(u,v) - y_j(\eta_i,\xi_j)]^2}} = \int_{v_k}^{v_{k+1}} \int_{u_n}^{u_{n+1}} \frac{dudv}{\sqrt{(u-\eta_i)^2 + (v-\xi_j)^2}}.$$

Analogically

$$S2 = \int_{t_{k}}^{t_{k+1}} \int_{s_{n}}^{s_{n+1}} \frac{dsdt}{\sqrt{(s-x_{i})^{2} + (t+y_{j})^{2}}} = \int_{v_{k}}^{v_{k+1}} \int_{u_{n}}^{u_{n+1}} \frac{dudv}{\sqrt{(u-D_{i,j})^{2} + (v+E_{i,j})^{2}}},$$

$$S3 = \int_{t_{k}}^{t_{k+1}} \int_{s_{n}}^{s_{n+1}} \frac{dsdt}{\sqrt{(s+x_{i})^{2} + (t-y_{j})^{2}}} = \int_{v_{k}}^{v_{k+1}} \int_{u_{n}}^{u_{n+1}} \frac{dudv}{\sqrt{(u+D_{i,j})^{2} + (v-E_{i,j})^{2}}},$$

$$S4 = \int_{t_{k}}^{t_{k+1}} \int_{s_{n}}^{s_{n+1}} \frac{dsdt}{\sqrt{(s+x_{i})^{2} + (t+y_{j})^{2}}} = \int_{v_{k}}^{v_{k+1}} \int_{u_{n}}^{u_{n+1}} \frac{dudv}{\sqrt{(u+\eta_{i})^{2} + (v+\xi_{j})^{2}}},$$

where  $D_{ij} = \eta_i \cos(2\alpha) + \xi_j \sin(2\alpha)$  and  $E_{ij} = \xi_j \cos(2\alpha) - \eta_i \sin(2\alpha)$ . All these integrals can be solved analytically.

The numerical calculation of charge density  $q_{k,n}$  resolves itself into the solution of the algebraical equation system (18) identically as in p.2.2, where

$$A_{p,r} = G_{v_k,u_n}^{\xi_j,\eta_i} = \int_{v_k}^{v_{k+1}} \int_{u_n}^{u_{n+1}} F(u,tv,\eta_i,\xi_j) \, du \, dv,$$

and

$$C_M = 0.5 \cdot \Delta u \cdot \Delta v \cdot \sum_{m=1}^{NV \cdot NH} q_m \,. \tag{18}$$

### **3. CALCULATION RESULTS**

Calculations of electrical charge distribution have been realised using MathCad program. On this basis the capacity values (equation (18)) for two non-parallel conducting paths of determined

width and length as a function of  $\alpha$ -angle have been estimated. The exemplary computations (Fig. 4) were made for following data: distance of unchangeable point A to y-axe equal 0.2 mm, the width and length of two paths equal 0.5 mm, dielectric permeability $\varepsilon_0 = 8.5 \cdot 10^{-12}$  F/m, dielectric constant of the air  $\varepsilon_{2r} = 1$  and of ceramic substrate  $\varepsilon_{2r} = 9.08$ .



*Fig.* 7. *Capacity as a function of*  $\alpha$ *-angle:* a – *path configuration,* b – *capacitance value* 

### 4. CONCLUSIONS

The obtained results of presented solution method of capacity calculation confirm the nature of capacity changes conditioned by geometrical parameter of layers and the usefulness of this method for determining of capacity values especially for layers of small dimensions.

From the waveform of the curve in Fig. 7,b it can be concluded about the correctness of the elaborated calculation method. For the exemplary configuration of the square conducting paths the capacitance value both for angle  $\alpha$  and 90 –  $\alpha$  is the same. For  $\alpha = 0$ , the result is identical with result for parallel conducting path system (p.2)

The experimental verification – in case of thick-film circuits – requires to take into consideration the finite thickness of ceramic substrate (it is a subject of further studies) and applying of a high accuracy measuring equipment.

The Fourier's integral equation approach, with the help of effective computers, enables to treat rather complex and finite geometric configurations of conducting paths that are useful in the design of modern film circuits.

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