ANALYSIS AND FORECAST OF THE ENVIRONMENTAL RADIOACTIVITY DYNAMICS BASED ON THE METHODS OF CHAOS THEORY: GENERAL CONCEPTIONS

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Abstract. For the first time, we present a completely new technique of analysis, processing and forecasting of any time series of the environmental radioactivity dynamics, which schematically is as follows: a) general qualitative analysis of a dynamical problem, typical environmental radioactivity dynamics (including a qualitative analysis from the viewpoint of ordinary differential equations, the "Arnold-analysis"); b) checking for the presence of chaotic (stochastic) features and regimes (the Gottwald-Melbourne's test; the correlation dimension method); c) reducing the phase space (the choice of time delay, the definition of the embedding space by the correlation dimension methods and false nearest neighbours algorithms); d) determination of the dynamic invariants of a chaotic system (computation of the global Lyapunov dimension λ_α; determination of the Kaplan-Yorke dimension d_L and average limits of predictability Pr_{max} on the basis of the advanced algorithms; e) a nonlinear prediction (forecasting) of any dynamical system evolution. The last block really includes new (in the theory of environmental radioactivity dynamics and environmental protection) methods and algorithms of nonlinear prediction such as methods of predicted trajectories, stochastic propagators and neural networks modeling, renorm-analysis with blocks of polynomial approximations, wavelet-expansions etc.

Introduction

One of the most actual and important problems of applied ecology and environment protection is associated with correct quantitative description of environmental radioactivity dynamics [1, 2]. In general, one should note the following actual problems:

• a long-term investigation of radionuclides behaviour in the environment;

• elucidation of the mechanism of radionuclides transfer in the environment by animals, through the food chain;

• elucidation of the mechanism of transformation and transportation of radioactive substances due to meteorological phenomena and other factors;

• provision of a think-tank functioning for the recovery of the environment;

• conservation of research materials and samples and archiving of research methodologies and research objects.

Key objectives of the atmospheric radionuclide dynamics include the research of radionuclides transportation in the atmospheric environment, the

dynamics of terrestrial radionuclides – research of radionuclides transfer and migration in the terrestrial environment, marine radionuclides dynamics – research of radionuclides transfer in the marine environment and radiation hydrology – research of radionuclides transfer from land by fresh water environments due to hydrological phenomena. Key radio-ecological transfers and effects include research cycles of radionuclides in forest ecosystems, the research of radionuclide transfer to biota in inland waters, the research of radionuclides transfer in soil-plant system, the research of biological effects of irradiation in microbes, algae, and plants, and biological effects of animals' radiation exposure, with an emphasis on free-range animals.

The main purposes of modeling, measurements and forecasting approach are: to evaluate and predict the transfer of radionuclides and radiation in the environment using computer simulations and other methods, to design improved technologies for monitoring and measuring radiation, to develop mechatronical systems and remote control technologies that will enable sampling and other operations in the areas out of humans reach. It is important to make analysis, to archive the research outputs and samples, obtained from IER and other institutes around the world and provide these materials to researchers around the world upon their request.

The problem of studying the dynamics of chaotic dynamical systems arises in many areas of science and technology. We are talking about classes of problems of identifying and estimating the parameters of interaction between the sources of complex (chaotic) oscillations of the time series of experimentally observed values. Such problems arise in environmental sciences, such as geophysics, chemistry, biology, medicine, neuroscience, engineering, etc. [1–10]. The problem of analysis and forecasting of the impact of anthropogenic pressure on the state of atmosphere in an industrial city and development of consistent, adequate schemes for modeling the properties of concentration fields of air pollutions has been considered in details, for example, in Ref. [3].

In modern theory of the hydro-ecological systems, water resources and environmental protection, the problem of quantitative treating of pollution dynamics is also one of the most important and fundamental problems, in particular, for applied ecology and urban ecology [1–18]. Let us remind [1–3] that most of the models are currently used to

assess (as well as forecast) the state of the environment pollution by the deterministic models or their simplification, based on simple statistical regressions. The success of these models, however, is limited by their inability to describe the nonlinear characteristics of the pollutant concentration behaviour and lack of understanding of the involved physical and chemical processes. Especially serious problem occurred during the study of dynamics of the hydro-ecological systems. Though the use of chaos theory methods establishes certain fundamental limitation on the long-term predictions, however, as it has been shown in a series of our papers [2–11]), these methods can be successfully applied to a short-or medium-term forecasting. In Ref. [2–4] we presented successful examples of quantitatively correct description of temporary changes in the concentration of nitrogen dioxide (NO2) and sulfur dioxide (SO2) in several industrial cities (Odessa, Trieste, Aleppo and the cities of Gdansk region) with a discovery of low-dimensional chaos. Moreover, some elements of this technique have been successfully applied to several prediction tasks for the other environmental management system. Here we mean the prediction of the evolution ecological state (temporal or even spatial) [6–11]. As example, let us remind the research results of variations dynamics of hydro-ecological systems (nitrates and sulphates concentrations in the Small Carpathians rivers watersheds in Eastern Slovakia) in the definite region by using non-linear prediction approaches and the recurrence plots method. At first, we discovered chaotic behaviour of nitrates and sulphates in the concentration of time series in the watersheds of the Small Carpathians. Naturally, except different physical and chemical features, from the formal mathematical point of view, difference between atmospheric and hydrological environmental systems is not essential, because in both problems, we deal with time series of fundamental pollution characteristics and therefore we should develop the technique for studying pollution dynamics of the hydro-ecological system, which will only have some differences in the details.

The main purpose of this paper is formally to represent theoretical basis of new general formalism for the analysis and forecasting of the environmental radioactivity dynamics and develop a new compact general scheme for modeling of temporal fluctuations of the pollution of temporal fluctuations field concentrations, based on the

methods of chaos theory. Earlier it had been successfully realized in case of the atmosphere (air basin) of large industrial cities and some hydro-ecological systems (regions). Therefore, further we will consider the corresponding atmospheric formalism [2–4] and give the necessary comments in the case of important features of the environmental radioactivity dynamics.

New general formalism for analysis and forecasting of the dynamics of hydroecological systems pollutants

As usual, we start with the first key task on testing chaos in the time series of environmental radioactivity dynamics. Following to [2–4], one should consider scalar measurements of the system dynamical parameters, for instance:

$$(n) = s(t_0 + n\Delta t) = s(n), \qquad (1)$$

where t_0 is a start time, Δt – s time step, and n – a number of measurements. In a general case, $s(n)$ is any time series (e.g. pollutants concentration in the atmosphere). As processes resulting in chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using the information contained in $s(n)$.

Such reconstruction results in a set of d-dimensional vectors $\mathbf{y}(n)$ for each scalar measurement. The main idea is the direct use of variable lags $s(n+\tau)$, where τ is some integer to be defined, which determines the coordinate system where a structure of orbits in phase space can be restored by using a set of time lags to create a vector in d dimension,

$$\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau), .., s(n+(d-1)\tau)], \qquad (2)$$

the required coordinates are provided. In a nonlinear system, $s(n+j\tau)$ there are some unknown nonlinear combinations of the actual physical variables. The space dimension d is the embedding dimension d_E.

The choice of a proper time lag is important for the subsequent reconstruction of phase space. If τ is too small, then the coordinates $s(n+j\tau)$, $s(n+(j+1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n+j\tau)$, $s(n+(j+1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under-or overestimated. Further, it is an important task to choose some intermediate position between the above cases. The first approach is to compute the

linear autocorrelation function $C_L(\delta)$ and look for the time lag where $C_L(\delta)$ is the fastest when passing through 0. This gives a good hint of choice for τ at which $s(n+j\tau)$ and $s(n+(j+1)\tau)$ are linearly independent. It is better to use the approach of nonlinear concept of independence, e.g. of average mutual information [1–3]. The average mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any values a_i from system A and b_k from B is averaged over all possible measurements of $I_{AB}(a_i, b_k)$. In Refs. [2–4] it is suggested to choose such value of τ where the first minimum of $I(\tau)$ occurs.

The goal of the embedding dimension determination is to reconstruct Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E, must be greater, or at least equal to the dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate chaos in time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. According to [2–4], one should calculate the correlation integral $C(r)$. If the time series is characterized by an attractor, the correlation integral $C(r)$ is related to the radius r as

$$d = \lim_{\substack{r \to 0 \\ N \to \infty}} \frac{\log C(r)}{\log r}, \qquad (3)$$

where d is a correlation exponent. If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension (d_2) of the attractor (see details in Refs. [3, 4].

Another method for determining d_E comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself, which arose by virtue of having projected the attractor into a too low dimensional space? [2–4]. In other words, when points in dimension d are neighbours of one another? In examining this question first in dimension one, then in dimension two, etc. by far there are no incorrect or false neighbours remaining,

so one should be able to establish a value for the necessary embedding dimension from geometrical consideration alone. Such approach was described by Kennel et al. [16, 17]. In dimension d each vector $\mathbf{y}(k)$ has the nearest neighbour $\mathbf{y}^{NN}(k)$ with nearness in the sense of some distance function. The Euclidean distance in dimension d between $\mathbf{y}(k)$ and $\mathbf{y}^{NN}(k)$ is called $R_d(k)$ [3]

$$R_d^2(k) = [s(k) - s^{NN}(k)]^2 + [s(k+t) - s^{NN}(k+t)]^2 + \\ ... + [s(k+t(d-1)) - s^{NN}(k+t(d-1))]^2. \quad (4)$$

$R_d(k)$ is presumably small when one has a lot of data, and for the dataset with N measurements, this distance is of order $1/N^{1/d}$. In dimension $d+1$ this nearest-neighbour distance is changed due to the $(d+1)$st coordinates $s(k+d\tau)$ and $s^{NN}(k+d\tau)$ to

$$R_{d+1}^2(k) = R_d^2(k) + [s(k+dt) - s^{NN}(k+dt)]^2. \quad (5)$$

We can define some threshold size R_T to decide when neighbours are false. Then if [3]

$$\frac{|s(k+dt) - s^{NN}(k+dt)|}{R_d(k)} > R_T, \quad (6)$$

(the nearest neighbours at time point k are declared false). Kennel et al. [17] showed that for values in the range $10 \le R_T \le 50$ the number of false neighbours identified by this criterion is constant. In practice, the percentage of false nearest neighbours is determined for each dimension d. The value at which the percentage is almost equal to zero can be considered as the embedding dimension.

The predictability can also be estimated by the Kolmogorov entropy, which is proportional to the sum of positive Lyapunov exponents. The spectrum of the Lyapunov exponents is one of dynamical invariants for non-linear system with chaotic behaviour. Local and global Lyapunov exponents quantify the limited predictability of chaos, which can be determined from measurements. The Lyapunov exponents are related to the eigenvalues of the linearized dynamics across the attractor. Negative values show stable behaviour while positive values show local unstable behaviour.

For chaotic systems that are both stable and unstable, the Lyapunov exponents indicate the complexity of the dynamics. Large positive values determine some average prediction limit. Since the Lyapunov exponents are defined as asymptotic average rates, they do not depend on the initial conditions, i.e. the choice of trajectory, and they do comprise an invariant measure of the attractor. An estimate of this measure is a sum of positive Lyapunov exponents. The estimate of the attractor

dimension is provided by the conjecture d_L and the Lyapunov exponents are taken in descending order. The dimension d_L gives values close to the dimension estimates discussed earlier and is preferable when estimating high dimensions. If one computes the whole spectrum of the Lyapunov exponents, other invariants of the system, such as the Kolmogorov entropy and the attractor dimension can be found. The Kolmogorov entropy measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. The estimate of the attractor dimension is provided by the Kaplan and Yorke conjecture (see details in Refs. [2–4, 16, 18]):

$$d_L = j + \frac{\sum_{a=1}^{j} l_a}{|l_{j+1}|}, \quad (7)$$

where j is such that $\sum_{a=1}^{j} l_a > 0$ and $\sum_{a=1}^{j+1} l_a < 0$, and the Lyapunov exponents are taken in descending order. The dimension d_L gives values close to the dimension estimates discussed earlier and is preferable when estimating high dimensions. To compute Lyapunov exponents, one should use linear fitted map method although maps with higher order polynomials can be used [18–23]. Another new approach was developed by Glushkov-Prepelitsa et al using the neural networks technique [25]. All calculations are performed with using "Geomath" and "Quantum Chaos" computational codes [3–11, 25–47].

Conclusions

Summing up the above said and the results of the work done [1–3], it is useful to summarize the key points into the system sequence for investigating chaos availability and wording the forecast (evolution) model for the environmental radioactivity dynamics. It should be noted that the overall difference between the modeling of environmental radioactivity dynamics and the radioactivity dynamics of usual chemical pollution of atmospheric and hydrological systems is not essential and is connected only with treating the dynamics of these systems from the viewpoint of the evolutionary theory of differential equations. The above methods are just a part of a large set of approaches (see our versions in [1–11]), which is used in the identification and analysis of chaotic regimes of the time series in the environmental radioactivity dynamics. Shortly speaking, the whole technique of analysis,

processing and forecasting of any time series of the radioactive pollutants in different geospheres will look as follows (see figure 1 below):
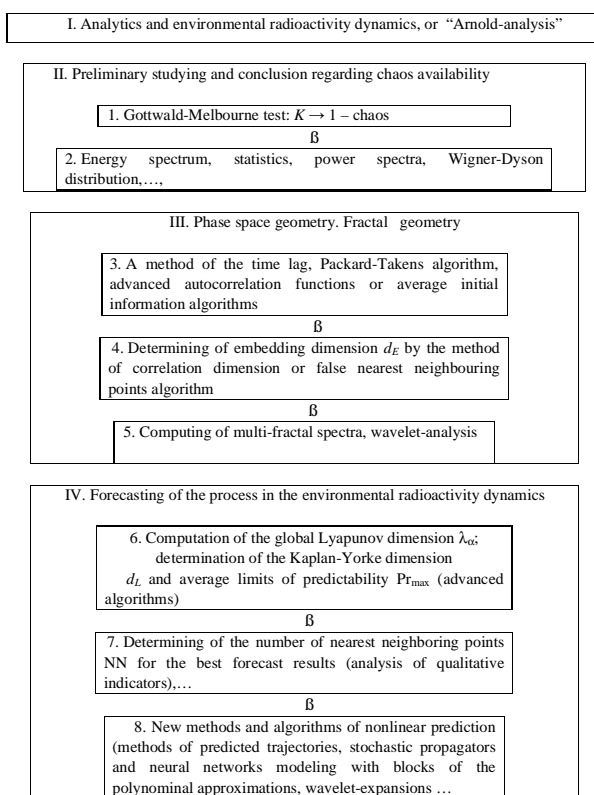

Fig. 1. General compact scheme for computation of the characteristics of the environment, radioactivity dynamics of time series and a non-linear analysis, modeling and prediction

A) General qualitative analysis of dynamical problem of typical hydro-ecological systems (including qualitative analysis from the viewpoint of ordinary differential equations, the "Arnold-analysis");

B) Checking for the presence of chaotic (stochastic) features and regimes (the Gottwald-Melbourne test; the method of correlation dimension);

C) Reducing of the phase space (choice of time delay, the definition of the embedding space by the methods of correlation dimension and false nearest neighbors algorithms);

D) Determination of the dynamic invariants of a chaotic system (computation of the Lyapunov exponent λ_α; determination of the Kaplan-Yorke dimension d_L and average limits of predictability Pr_{max} on the basis of advanced algorithms;

E) A non-linear prediction (forecasting) of dynamical evolution of the system.

The last block includes new methods and algorithms of nonlinear prediction, such as methods of predicted trajectories, stochastic propagators and neural networks modeling, renorm-analysis with blocks of polynomial approximations, wavelet-expansions [10, 11, 25]. Indeed, one should use a few algorithms at any step of studying. Naturally, if aggregate and dynamic topological invariants [1–11] are identical for two chosen systems, then evolutions of these systems are also subject to the same laws, including the same or analogous systems of differential equations. This fact is very useful especially while using such new methods and algorithms of nonlinear prediction as methods of predicted trajectories, stochastic propagators and neural modeling networks with blocks of the polynomial approximations, wavelet-expansions [1, 10, 11, 25].

References

[1] Wada T., Tomiya A., Enomoto M. et al: Journ. of Env. Radioactivity, 2016, 151, 144.

[2] Bunyakova Yu. Ya., Glushkov A. V.: Analysis and forecast of the impact of anthropogenic factors on air basein of an industrial city. Odessa, Ecology, 2010.

[3] Glushkov A. V.: Ukranian hydrometeorological journal, 2014, N15, 32.

[4] Glushkov A. V., Khokhlov V. N., Bunyakova Yu. Ya., Prepelitsa G. P., Tsenenko I. A.: Sensor Electr. and Microsyst.Techn., 2005, 4, 27.

[5] Glushkov A. V., Khokhlov V. N., Prepelitsa G. P., Tsenenko I. A. : Optics of atmosphere and ocean, 2004, 14, 219.

[6] Glushkov A. V., Khetselius O. Yu., Kuzakon V. M., Prepelitsa G. P., Solyanikova E. P., Svinarenko A. A.: Dynamical Systems - Theory and Applications, 2011, BIF110.

[7] Glushkov A. V., Kuzakon' V. M., Khetselius O. Yu. etal: Proc. of Internat. Geometry Center, 2013, 6, N3, 6.

[8] Rusov V. N., Glushkov A. V., Khetselius O. Yu. et al: Adv. in Space Research (Elsevier), 2008, 42(9), 1614.

[9] Bunyakova Yu. Ya., Khetselius O. Yu.: Proc. of the 8th International Carbon Dioxide Conference (Germany), 2009, T2-098.

[10] Glushkov A. V., Khetselius O. Yu., Brusentseva S. V. *et al:* Adv. in Neural Networks, Fuzzy Systems and Artificial Intelligence. Series: Recent Adv. in Computer Engineering (Gdansk), 2014, 21, 69.

[11] Glushkov A. V., Svinarenko A. A., Buyadzhi V. V. et al : Adv. in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Adv. in Computer Engineering (Gdansk), 2014, 21, 143.

[12] Rusov V. D., Glushkov A. V., Vaschenko V. N. *et al:* Journ. of Atm. and Solar-Terrestrial Phys., 2010, 72, 498.

[13] Sivakumar B.: Chaos, Solitons & Fractals, 2004, 19, No. 2, 441.

[14] Chelani A. B.: Int. J. Environ. Stud, 2005, 62, 181.

[15] Gottwald G. A., Melbourne I.: Proc. Roy. Soc. London. Ser. A. Math. Phys. Sci., 2004, 460, 603.

[16] Packard N. H., Crutchfield J. P., Farmer J. D., Shaw R. S.: Phys. Rev. Lett., 1980, 45, 712.

[17] Kennel M., Brown R., Abarbanel H.: Phys. Rev. A., 1992, 45, 3403.

[18] Abarbanel H. D. I., Brown R., Sidorowich J. J., Tsimring L. Sh.: Rev. Mod. Phys, 1993, 65, 1331.

[19] Schreiber T.: Phys. Rep., 1999, vol. 308, 1.

[20] Fraser A. M., Swinney H.: Phys. Rev. A., 1986, 33, 1134.

[21] Grassberger P., Procaccia I. ,Physica D, 1983, 9, 189.

[22] Gallager R. G.: Information theory and reliable communication. NY: Wiley, 1968.

[23] Mañé R.: Lecture Notes in Mathematics (Berlin: Springer), 1981, 898, 230.

[24] Takens F.: Lecture Notes in Mathematics (Berlin: Springer), 1981, 898, 336.

[25] Prepelitsa G. P., Glushkov A. V., Lepikh Ya. I. et al: Sensor Electr. and Microsystems Techn., 2014, 11, No. 4, 43.

[26] Glushkov A. V., Khetselius O. Yu., Prepelitsa G. P., Svinarenko A. A.: Proc. of Internat. Geometry Center, 2013, 6, No. 1, 43.

[27] Khetselius O. Yu., Glushkov A. V., Gurnitskaya E. P. etal: Spectral Line Shapes, Vol.15-19th Internat. Conf. on Spectral Line Shapes (AIP), 2008, 15, 231.

[28] Khetselius O. Yu.: Spectral Line Shapes, Volume 15-19th Internat. Conf. on Spectral Line Shapes (AIP), 2008, 15, 363.

[29] Rusov V. N., Glushkov A. V., Khetselius O. Yu. et al: Adv. in Space Research (Elsevier), 2008, 42(9), 1614.

[30] Glushkov A. V., Khetselius O. Yu., Loboda A. V., Svinarenko A. A.: Frontiers in Quantum Systems in Chemistry and Physics, Ser.: Progress in Theor. Chem. and Phys., Eds. S.Wilson, P.J.Grout, J. Maruani, G.Delgado-Barrio, P. Piecuch (Springer), 2008, 18, 541.

[31] Glushkov A. V., Khetselius O. Yu., Lovett L.: Advances in the Theory of Atomic and Molecular Systems: Dynamics, Spectroscopy, Clusters, and Nanostructures. Ser.: Progress in Theor. Chem. and Phys., Eds. Piecuch P., Maruani J., Delgado-Barrio G., Wilson S. (Springer), 2009, 20, 125.

[32] Khetselius O. Yu.: Sensor Electronics and Microsystem Techn., 2008, 3, 28.

[33] Khetselius O. Yu.: Photoelectronics, 2005, 14, 83

[34] Khetselius O. Yu.: Journ. of Physics: Conf. Series (IOP, UK), 2012, 397, 012012.

[35] Khetselius O. Yu., Florko T. A., Svinarenko A. A., Tkach T. B.: Phys.Scripta (IOP), 2013, T153, 014037.

[36] Glushkov A. V., Rusov V. D., Ambrosov S., Loboda A.: New projects and new lines of research in nuclear physics, Eds. G.Fazio, F.Hanappe (World Scientific), 2003, 126

[37] Glushkov A. V., Malinovskaya S. V.: New projects and new lines of research in nuclear physics. Eds. G.Fazio and F. Hanappe (World Scientific), 2003, 242.

[38] Glushkov A. V., Malinovskaya S. V., Gurnitskaya E. P., Khetselius O. Yu., Dubrovskaya Yu. V.: Journ.of Physics: Conf. Series (IOP), 2006, 35, 425.

[39] Malinovskaya S. V., Glushkov A. V., Khetselius O. Yu., Lopatkin Yu. M., Loboda A. V., Svinarenko A. A., Nikola L. V., Perelygina T. B.: Int.Journ. Quant.Chem., 2011, 111(2), 288.

[40] Malinovskaya S. V., Glushkov A. V., Khetselius O. Yu., Svinarenko A. A., Mischenko E. V., Florko T. A.: Int.Journ. Quant.Chem., 2009, 109(14), 3325.

[41] Glushkov A. V., Khetselius O. Yu., Malinovskaya S. V.: Molec. Phys. , 2008, 106 (9-10), 1257

[42] Glushkov A. V., Khetselius O. Yu., Svinarenko A. A.: Coherence and Ultrashort Pulse Laser Emission, Ed. by F. J. Duarte (InTech), 2010, 159.

[43] Abarbanel H. D. I., Brown R., Sidorowich J. J., Tsimring L. Sh.: Rev. Mod. Phys, 1993, 65, 1331.

[44] Schreiber T.: Phys. Rep., 1999, vol. 308, 1.

[45] Malinovskaya S. V., Glushkov A. V., Dubrovskaya Yu. V., Vitavetskaya L. A., Recent Advances in the Theory of Chemical and Physical Systems (Springer), 2006, 15, 301.

[46] Glushkov A. V., Khetselius O. Yu., Svinarenko A. A.: Advances in the Theory of Quantum Systems in Chem. and Phys., Ser.: Progress in Theor. Chem. and Phys., ed. by P.Hoggan, E.Brandas,J.Maruani, G. Delgado-Barrio, P. Piecuch (Springer), 2012, 22, 51

[47] Glushkov A. V., Malinovskaya S. V., Sukharev D. E., Khetselius O.Yu., Loboda A.V., Lovett L.: Int. Journ. Quantum Chem., 2009, 109 (8), 1717.