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MATHEMATICAL MODELLING OF THE POPULATION DYNAMICS OF HUNTING MAMMALS BASED ON RECURRENT EQUATION SYSTEM

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Abstract. Regression analysis of the population dynamics of hunting mammals based on Khmilnytskyi forestry was made. Correlation analysis of the observations results between the populations of spotted deer and wisent, spotted deer and wild boar, lepus and fox was made. Modelling of the population dynamics of hunting mammals was done using Mathcad based on the system of recurrent logistics equations and the adequacy of the model by the Pearson's criteria was proved.

Key words: mathematical modelling, population dynamics, hunting mammals, regression analysis, interspecies interaction, dynamics model.

Introduction. The use of mathematical modelling opens up new possibilities for creating a model of population dynamics and forecasting of the population, which greatly facilitates the analysis of primary data. At the same time, there are some difficulties: a) simulation data requires a fairly long period of time (as the accuracy of the model is determined by the amount of data in number); b) it is difficult to reflect the impact of certain factors and interspecies relationships in conventional models [2–5].

1. The analysis of the populations of hunting mammals in Khmilnytskyi forestry

Based on the statistics of the population dynamics of hunting mammals in Khmilnitskyi forestry, a regression analysis was made using Statistica 6.1. Program. Exponential function was used as a regression model that corresponds to modelling the dynamics of individual populations without interspecies interaction. The results of the regression analysis are shown in Fig. 1.

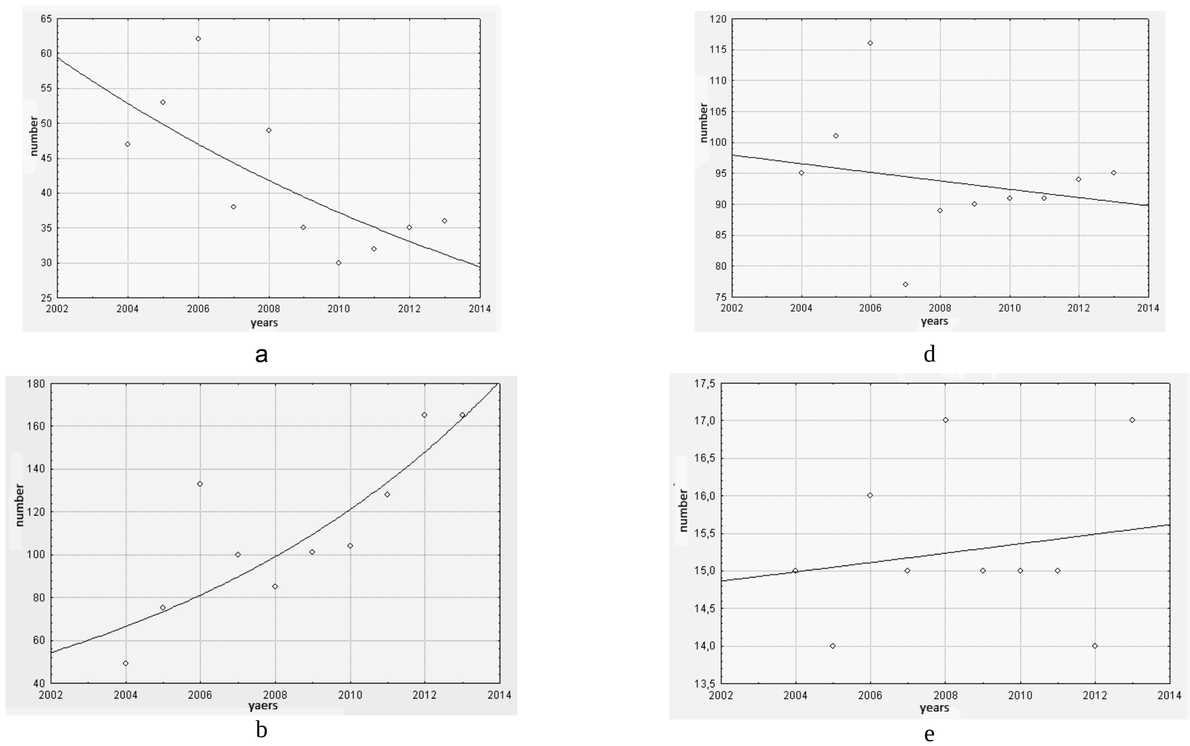
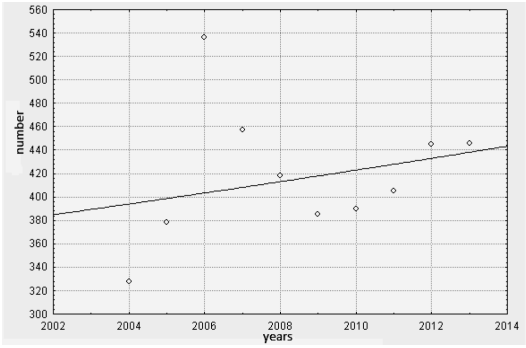


Fig. 1. Regression analysis of observations of population in Khmilnitskyi forestry:
a – spotted deer, b – wild boar, d – wisent; e – fox



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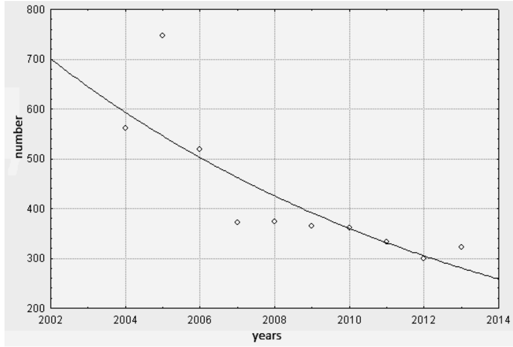

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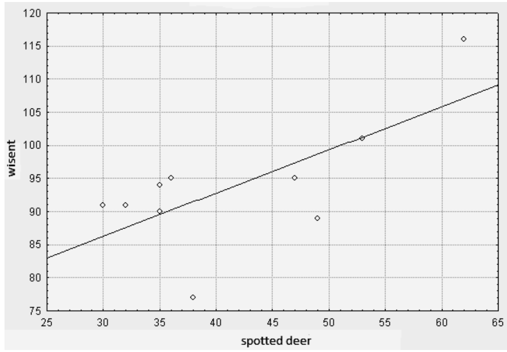
Continuation Fig. 1. Regression analysis of observations of population in Khmilnitskyi forestry:
c – European roe deer; f – lepus

As a result of the regression analysis, mathematical models of changes in the number of individual populations of hunting animals in exponential approximation were obtained. Moreover, the function index includes parameters for fertility, mortality, migration and human impacts (hunting shooting, poaching, agricultural operations, transportation systems, etc.).

The correlation between the populations of certain species of hunting mammals in Khmilnitskyi forestry was analyzed. In particular, the dependence between the populations of spotted deer and wisent, fox and lepus, spotted deer and wild boar was set Fig. 2.

As a result of the correlation analysis between the populations of spotted deer and wisent in Khmilnitskyi forestry the correlation coefficient of 0.69 was obtained, which indicates close relationship between these populations. This is because the ecological niches of wisent and spotted deer coincide. Though, the population of spotted deer is much wider because of better adaptation to the existence in small forests. Spotted deer and wisent, in terms of trophic relationships, do not compete. In this case, such a high correlation coefficient is due to the fact, that wisent competes with deer, as a factor of disturbance. In spring and summer when ungulates are calving, deer are trying to avoid encounters with other animals. Herewith, in most cases overgrown cuttings and young forests are the places of the growth of the young ones. While eating, wisents use the same overgrown cuttings and young forests. The herd disperses throughout all the territory, making a loud noise and crackle. Other animals that brought the young ones try to leave this place, what is accompanied by the death of some young animals [1–5].

The lack of significant correlation relationship between the populations of fox and lepus in Khmilnitskyi forestry is explained by strong anthropogenic interference into ecological niches of animals and substantive hunting compared to interspecies interaction of “predator-prey”. No evident correlation relationship between the populations of deer and wild boar was found, although their ecological niches coincide. This is because their number is small and they are not disturbing factors for each other. In terms of trophic relationships, they also do not significantly compete. Wild boar is able to live in any forestlands as well as wetlands. It is omnivorous, which allows repeated changing of food base throughout the year.



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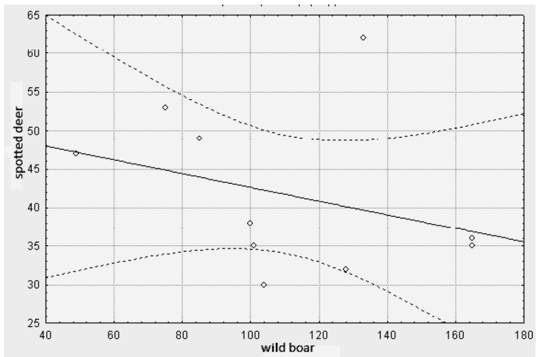

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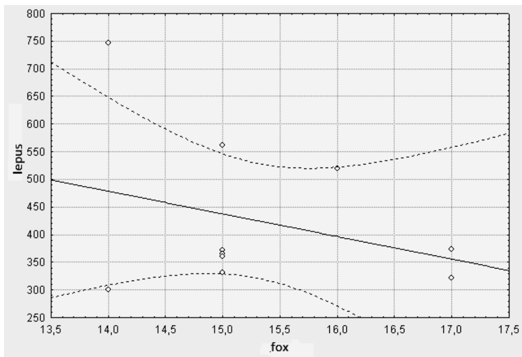

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Fig. 2. Correlation analysis of observations between the populations in Khmelnytskiy forestry:
a – spotted deer and wisent, b – spotted deer and wild boar, c – lepus and fox

2. Modelling of the population dynamics of hunting mammals in Khmilnitskyi forestry using Mathcad

Mathematical model of dynamics of the amount of separate population under conditions with sufficient food, lack of overcrowding and enemies is described by the following equation:

$$N(t) = N_0 e^{r(t-t_0)}; \qquad (1)$$

where N_0 – population amount at the initial time; r – specific rate of reproduction.

Equation (1) is received by solving Malthus differential equation $\dfrac{dN}{dt} = rN$.

Under adverse conditions, specific mortality rate d exceeds specific birth rate b, while the specific rate of reproduction $r = d - b$ is negative.

When taking crowding into account, the population dynamics of a separate population is described by the following equation:

$$N(t) = \frac{K}{1 + e^{\ln\left(\frac{K-N_0}{N_0}\right) - r(t-t_0)}}; \qquad (2)$$

where K – the maximum possible amount of the population.

Equation (2) is received by solving logistics differential equation $\dfrac{dN}{dt} = rN - \dfrac{r}{K}N^2$.

In consideration of interspecies interactions, it is necessary to solve a system of differential equations:

$$\begin{cases} \dfrac{dN_1}{dt} = r_1 N_1 - \dfrac{r_1}{K_1} N_1^2 + g_1 N_1 N_2; \\ \dfrac{dN_2}{dt} = r_2 N_2 - \dfrac{r_2}{K_2} N_2^2 + g_2 N_2 N_1. \end{cases} \qquad (3)$$

where γ_1 and γ_2 – factors that take into account the interaction of species.

A more accurate study of the dynamics of interspecies interaction is possible with the use of systems of nonlinear Lotka-Volterra differential equations, which take into account food digestion of the predator, predator's hunting strategy, defensive reaction of the victim, the presence of hiding places for the victim, etc.

With some loss of accuracy in the analysis and replacement of dt by Δt, the system of nonlinear differential equations may be replaced by the system of recurrent equations. This can significantly simplify the calculations and make them transparent and visible. For example, system (3) is converted into the following system:

$$\begin{cases} N_{i+1} = N_i + \left(r_n N_i - \dfrac{r_n}{K_n} N_i^2 + g_n N_i M_i \right); \\ M_{i+1} = M_i + \left(r_m M_i - \dfrac{r_m}{K_m} M_i^2 + g_m M_i N_i \right). \end{cases} \qquad (4)$$

System (4) uses discrete time i which corresponds to the step of real time (average breeding season $T = \min(T_n, T_m)$).

Modelling of the population dynamics in the ecosystem with six most common populations of hunting mammals was carried out. Initial data for the simulation are shown in Table 1.

Table 1

Initial data for the simulation of the population dynamics

Species	Initial amount	Specific rate of change in population	Maximum population size	
Spotted deer	564	-0,0585	200	$N1_i$
Wild boar	49	0,1003	160	$N2_i$
European roe deer	384	0,0262	800	$N3_i$
Wisent	96	-0,0073	100	$N4_i$
Lepus	584	-0,0932	600	$N5_i$
Fox	15	0,0041	17	$N6_i$

Logistics system of recurrent equations for modelling the population dynamics:

$$\begin{pmatrix} N1_{i+1} \\ N2_{i+1} \\ N3_{i+1} \\ N4_{i+1} \\ N5_{i+1} \\ N6_{i+1} \\ N7_{i+1} \end{pmatrix} := \begin{bmatrix} N1_i + m1 \cdot N1_i - \frac{m1}{K1} \cdot (N1_i)^2 + \gamma12 \cdot N1_i \cdot N2_i + \gamma13 \cdot N1_i \cdot N3_i + \gamma14 \cdot N1_i \cdot N4_i + \gamma15 \cdot N1_i \cdot N5_i + \gamma16 \cdot N1_i \cdot N6_i + \gamma17 \cdot N1_i \cdot N7_i \\ N2_i + m2 \cdot N2_i - \frac{m2}{K2} \cdot (N2_i)^2 + \gamma21 \cdot N2_i \cdot N1_i + \gamma23 \cdot N2_i \cdot N3_i + \gamma24 \cdot N2_i \cdot N4_i + \gamma25 \cdot N2_i \cdot N5_i + \gamma26 \cdot N2_i \cdot N6_i + \gamma27 \cdot N2_i \cdot N7_i \\ N3_i + m3 \cdot N3_i - \frac{m3}{K3} \cdot (N3_i)^2 + \gamma31 \cdot N3_i \cdot N1_i + \gamma32 \cdot N3_i \cdot N2_i + \gamma34 \cdot N3_i \cdot N4_i + \gamma35 \cdot N3_i \cdot N5_i + \gamma36 \cdot N3_i \cdot N6_i + \gamma37 \cdot N3_i \cdot N7_i \\ N4_i + m4 \cdot N4_i - \frac{m4}{K4} \cdot (N4_i)^2 + \gamma41 \cdot N4_i \cdot N1_i + \gamma42 \cdot N4_i \cdot N2_i + \gamma43 \cdot N4_i \cdot N3_i + \gamma45 \cdot N4_i \cdot N5_i + \gamma46 \cdot N4_i \cdot N6_i + \gamma47 \cdot N4_i \cdot N7_i \\ N5_i + m5 \cdot N5_i - \frac{m5}{K5} \cdot (N5_i)^2 + \gamma51 \cdot N5_i \cdot N1_i + \gamma52 \cdot N5_i \cdot N2_i + \gamma53 \cdot N5_i \cdot N3_i + \gamma54 \cdot N5_i \cdot N4_i + \gamma56 \cdot N5_i \cdot N6_i + \gamma57 \cdot N5_i \cdot N7_i \\ N6_i + m6 \cdot N6_i - \frac{m6}{K6} \cdot (N6_i)^2 + \gamma61 \cdot N6_i \cdot N1_i + \gamma62 \cdot N6_i \cdot N2_i + \gamma63 \cdot N6_i \cdot N3_i + \gamma64 \cdot N6_i \cdot N4_i + \gamma65 \cdot N6_i \cdot N5_i + \gamma67 \cdot N6_i \cdot N7_i \\ N7_i + m7 \cdot N7_i - \frac{m7}{K7} \cdot (N7_i)^2 + \gamma71 \cdot N7_i \cdot N1_i + \gamma72 \cdot N7_i \cdot N2_i + \gamma73 \cdot N7_i \cdot N3_i + \gamma74 \cdot N7_i \cdot N4_i + \gamma75 \cdot N7_i \cdot N5_i + \gamma76 \cdot N7_i \cdot N6_i \end{bmatrix} \qquad (5)$$

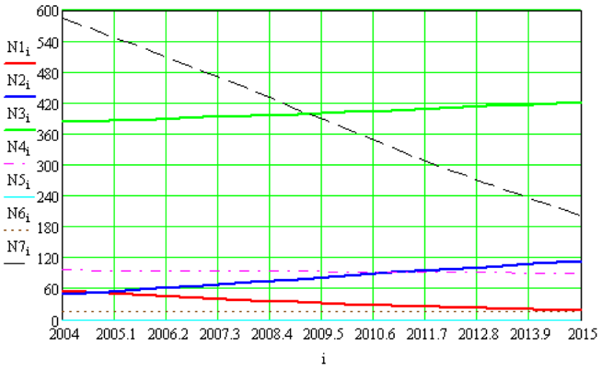


Fig. 3. The dynamics of the population amount of hunting mammals in Khmilnitskyi forestry

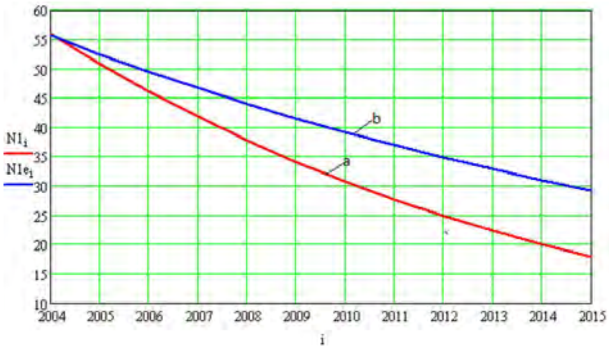
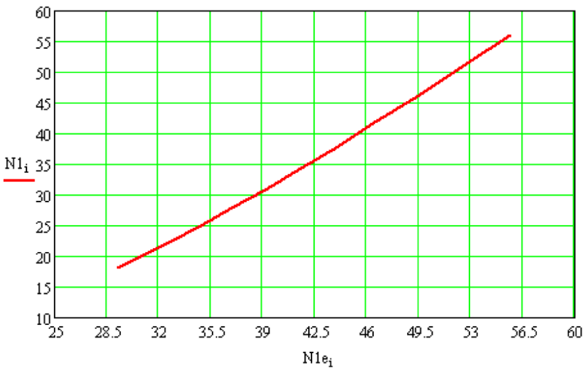


Fig. 4. Changes in the number of deer populations in Khmilnitskyi forestry (a – simulation results of equations (5), b – exponential regression of observations)



Simulation results are shown in Fig. 3.

Compare the results of modelling the dynamics of populations of spotted deer in Khmilnitskyi forestry from the system of recurrent equations with regression model based on the processing of observations using the exponential function. For comparison, we use the Pearson criteria. Since χ^2=0,998, it indicates the adequacy of the model based on the system of recurrent equations to the results of observations (Fig. 4).

The analysis of the adequacy of the data model of observations on the example of the dynamics of the populations of spotted deer (the Pearson criteria χ^2=0,998).

4. Conclusions

The obtained model is somewhat arbitrary and cannot be used as a completely accurate prediction of the populations of hunting mammals. However, the scenarios of the population dynamics are recommended for the development of measures for the integrated management of the groups of hunting mammals both for Khmilnitskyi forestry hunting, and all hunting lands of Podillia region.

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