COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 6, No. 1, 2016

APPROXIMATION OF EQUIVALENT SOURCE PARAMETERS CHANGE FUNCTION BY FRANKLIN FUNCTIONS

Valery Zhuikov, Kateryna Osypenko

"Igor Sikorsky Kyiv Polytechnic Institute" National Technical University of Ukraine, Kyiv, Ukraine

valery_zhuikov@yahoo.com, ekateryna.osypenko@gmail.com

Abstract: A problem of approximation of the primary energy flow parameters change function by linear approximations to ensure a minimal error is considered in the paper. A representation of the parameters of voltage and internal resistance of equivalent generators of renewable sources as linear functions allows an increase in the level of energy transmitted to the load. To solve the problem set, it is proposed to approximate the primary energy flow parameters change function by the system of Franklin orthonormal functions. It is shown that applying the *m*-shift operation to the Franklin orthogonal functions minimizes the error and the number of approximating functions.

Key words: distributed generation, renewable energy sources, Heisenberg's uncertainty principle, approximation, Franklin functions.

1. Introduction

The proliferation of renewable energy systems, increase of their installed capacity with the trend of power growth, first, have led to the increase in electricity generation [1], and, second, have impacted on the intensification of autonomous power systems [2], which in turn has evoked great interest in the assurance of maximum energy efficiency [3]. Indeed, from 2010 until 2015, the share of renewable energy grew from 16.7 % to 28 %, and is (for 2015 year) 1,849 GW [4] that at an efficiency increase by 1 % would give additional 184.9 MW per hour. In addition to improving the efficiency of distributed generation systems in the static mode, the attention should be payed to the efficiency in terms of the dynamic change of primary energy flow parameters, such as changing the value and direction of wind flow or value of insolation. With this approach, a time change of equivalent energy sources voltage and internal resistance should be considered. One of the methods of such consideration involves the use of the hypothesis of linear changes of these parameters in a distributed generation system [5], with a simplified equivalent circuit shown in Fig. 1.

In the circuit, the renewable energy source is represented by the equivalent voltage source *E(t)* with the internal resistance *Ri(t)*; *L* is the lines inductance; $R_L(t)$ is the load resistance.

The condition of maximum energy selection is performed at equality of the load resistance $R_L(t)$ and the internal resistance *Ri(t)*, $R_L(t) = Ri(t)$.

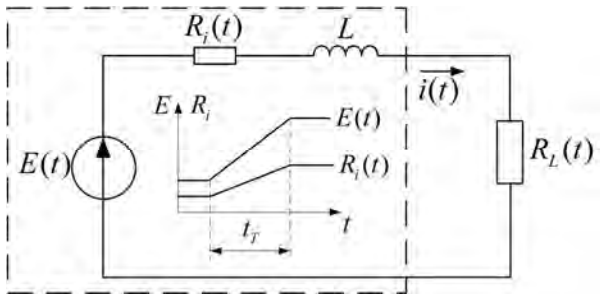


Fig. 1. Simplified equivalent circuit of a distributed generation system.

In particular, with the direction of wind flow changing and the wind wheel turning in the plane perpendicular to the wind speed vector, the current of the sources should change according to the equation:

$$i(t)=t-\sqrt{p}\ \mathrm{erf}\left(\sqrt{b}\,t\right)\Big/2\sqrt{b}\ , \qquad (1)$$

where erf is the error function; β is the parameter that characterizes the circuit with Gaussian transient processes. The formation of current according to equation (1) provides additional 7 % of the energy transmitted to the load [6].

To ensure the effective operation of distributed generation systems in addition to selecting maximum energy at a given point time, it is also necessary to calculate the amount of energy that exceeds the average value of the energy generated, as well as the amount of energy that is lower than the average value at a certain time interval. Moreover, for the base interval, for example a day, it is necessary to balance these energies, that allows an acceptable operating mode of energy storage to be provided [7]. According to Heisenberg's uncertainty principle in regard to the distributed generation systems, it is principally impossible to provide the operation of renewable sources in the mode of maximum energy selection and to determine an average value of the energy for the efficient operation of the energy storage on the selected interval of observation with the same arbitrary precision. The indicated contradiction can be solved to some extent by applying the two-channel control. One channel ensures the control of maximum energy selection with the minimization of an observation interval and generation of a source current according to equation (1). The other channel controls the charge-discharge processes of the energy storage. The control is provided by deviations from the average value of the energy generated. The deviations are calculated by interpolation characteristics on the selected base interval.

Given the fact that with this approach, the parameters of voltage and internal resistance in the equivalent generators of renewable sources are modelled by linear functions, there apprears a task of linear approximation of primary energy flow parameters change function. Such approximation should ensure the minimal approximation error both on the observation interval and on the base interval.

2. The approximation by Franklin fuctions

From the theory of approximation, it is know that with the approximation functions being appropriately chosen, the orthogonal functions provide the minimal error [8]. Since it is necessary to provide the linear approximation of the primary energy flow parameters change function, it is expedient to choose the orthogonal functions that provide such approximation. Such functions, in particular, are the Franklin orthonormal functions that are built in accordance with the following equations:

$$f_0 = 1\,,$$

$$f_{n+1} = \frac{u_{n+1} - \sum\limits_{i=1}^{n} f_i \int\limits_0^1 f_i(t)\cdot u_{n+1}(t)dt}{\pm\sqrt{\int\limits_0^1 f_{n+1}^2(t)dt}}\,, \qquad (2)$$

where i is the Franklin function numer; u_i is the set of functions defined on the variable change interval $0 \le t \le 1$:

$$\begin{cases} u_0 = 1 \\ u_1 = t \\ u_2 = \begin{cases} 0, t \le \frac{1}{2} \\ t - \frac{1}{2}, t \ge \frac{1}{2} \end{cases} \\ \mathbf{L\ L\ L\ L\ L\ L\ L\ L} \\ u_i = \begin{cases} 0, t \le a_i \\ t - a_i, t \ge a_i \end{cases} \end{cases}, \qquad (3)$$

where $a_i = \left(2i - 1 - 2^k\right)\Big/2^k$, k is the maximum power of 2 in $2i - 1$. Thus a_i is the i-th term of the sequence $0\,,\ \frac{1}{2}\,,\ \frac{1}{4}\,,\ \frac{3}{4}\,,\ \frac{1}{8}\,,\ \frac{3}{8}\,,\ \frac{5}{8}\,,\ \frac{7}{8}\,,\ \frac{1}{16}\ldots$

The expressions describing the first, fourth and eighth Franklin functions and their graphical representations are shown in Table 1.

Since the output range of the primary energy flow parameters change function is usually represented by discrete data, let us proceed to the discrete representation of Franklin functions, which has the following form:

$$f_0(n) = 1\,,$$

$$f_{n+1}(n) = \frac{u_{n+1}(n) - \sum\limits_{i=1}^{n} f_i(n) \sum\limits_{j=0}^{N-1} f_i(n)\cdot u_{n+1}(n)}{\pm\sqrt{\sum\limits_{j=0}^{N-1} f_{n+1}^2(n)}}\,, \quad (4)$$

where j is the count number; $u_i(n)$ is the set of discrete functions defined on the variable change interval $0 \le n \le N-1$, similary to the continuous functions in equation (3).

The approximation by the Franklin discrete functions is carried out by the following equation:

$$\Psi(n) = \sum_{i=0}^{\infty} C_i f_i(n)\,, \qquad (5)$$

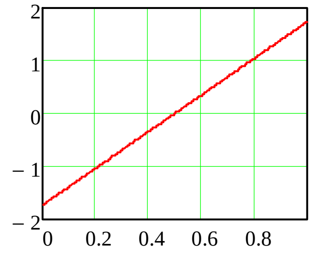
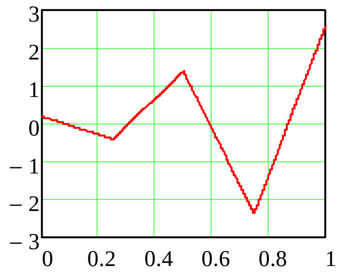
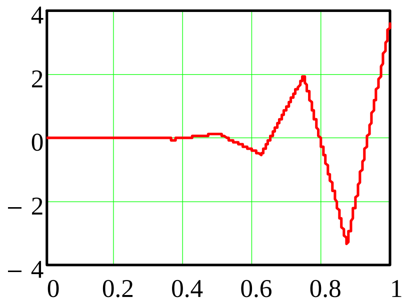
where $C_i = \frac{1}{N}\sum\limits_{n=0}^{N-1} Y(n) f_i(n)$ are the coefficients of the Franklin discrete series; n is the number of observation intervals on the basic interval N. It should be noted that the discrete values of Franklin functions may differ from the break points of continuous functions.

The obtained values of the coefficients for the corresponding functions contribute to the transition to continuous functions both for the observation interval and for the entire base interval.

For example, for the nine values of wind speed $y(n)$ [m/s], obtained from the data that are taken every 30 minutes from the meteorological station located at Boryspil International Airport, Kyiv, $y(n) = \{5, 4, 4, 3, 3, 4, 4, 3, 4\}$ (value 1), the result of expansion in a Franklin discrete series by 9 functions (value 2) is shown in Fig. 2. The approximation coefficients are given in Table 2.

Table 1

Expressions and graphical representation of Franklin functions

Expression	Graphical representation
$\sqrt{3}(2t-1), 0 \le t \le 1$	
$\begin{cases} \sqrt{\frac{3}{77}}(1-12t), 0 \le t < \frac{1}{4} \\ \sqrt{\frac{3}{77}}(36t-11), \frac{1}{4} \le t < \frac{1}{2} \\ \sqrt{\frac{3}{77}}(45-76t), \frac{1}{2} \le t < \frac{3}{4} \\ \sqrt{\frac{3}{77}}(100t-87), \frac{3}{4} \le t \le 1 \end{cases}$	
$\begin{cases} \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(1-24t), 0 \le t < \frac{1}{8} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(72t-11), \frac{1}{8} \le t < \frac{1}{4} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(73-264t), \frac{1}{4} \le t < \frac{3}{8} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(984t-395), \frac{3}{8} \le t < \frac{1}{2} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(1933-2672t), \frac{1}{2} \le t < \frac{5}{8} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(13704t-8927), \frac{5}{8} \le t < \frac{3}{4} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(23413-29416t), \frac{3}{4} \le t < \frac{7}{8} \\ \sqrt{\frac{3}{2131 \cdot 97 \cdot 7}}(38776t-36255), \frac{7}{8} \le t \le 1 \end{cases}$	

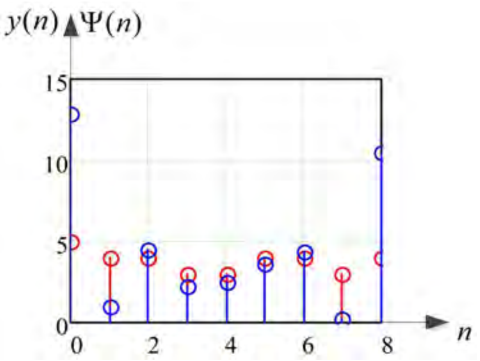


Fig. 2. Result of expansion of the function $y(n)$ in a Franklin discrete series.

Table 2

Coefficients of the series

Coefficient	C_0	C_1	C_2	C_3	C_4
Value	3,778	-0,289	1,155	0,725	0,416
Coefficient	C_5	C_6	C_7	C_8	
Value	1,207	0,36	-0,386	1,168	

The above graph shows that at the ends of the interval, the deviations of approximating values are maximum. This is similar to the Gibbs phenomenon [9].

With all the values taken into account, the approximating error is:

$$d_1 = \sqrt{\frac{1}{9}\sum_{n=0}^{8}\left(y(n)-\Psi(n)\right)^2} = 122,6\,\%, \qquad (6)$$

and given the values from the 2nd to 6th , the error is significantly reduced and is:

$$d_2 = \sqrt{\frac{1}{5}\sum_{n=2}^{6}\left(y(n)-\Psi(n)\right)^2} = 23,4\,\%. \qquad (7)$$

It should be noted that with the number of approximating functions increasing, the irregularity of approximation and reduction of the approximation error is observed. Despite these features, using Franklin functions allow the necessary linear approximation to be ontained.

For example, if the wind flow speed changes from the value n_1 to the value n_2, the equivalent source voltage changes from the value $E_1 = R_{WW} n_1^{3/2}\sqrt{\frac{p}{2} r C_P R_{i1}}$ to the value $E_2 = R_{WW} n_2^{3/2}\sqrt{\frac{p}{2} r C_P R_{i2}}$, where R_{i1} and R_{i2} are the values of internal resistance of the equivalent generators that correspond to the wind flow speed n_1 and n_2 respectively; r is the air density; R_{WW} stands for the radius of the wind wheel; C_P denotes the power factor. The transition from the wind speed data to the change functions *E(t)* and $R_i(t)$ is carried out according to the equations given. This makes it possible to track the equivalent source current change according to equation (1).

Improving the accuracy of approximation with the simultaneous decrease in the number of approximating functions is possible by using the ***m***-shift operation.

3. The approximation by Franklin fuctions with *m*-shift

The representation of the function argument and operation with it in finite fields [10] allowed the concept of *m*-shift to be introduced. The application of *m*-shift to a particular function significantly changes its appearance. This makes it possible to choose such a function that best suits a series being approximated. For example, the given 9 values of the argument from the previous example, are represented with number 3^2, and the argument values are divided into 3 blocks with values 0, 1, 2. With the ***m***-shift by 0, 1, 2 samples, the shift is made within each block. With the ***m***-shift by 4 samples, not only the values of the function within the block are shifted, but also the blocks themselves are.

The illustration of how the first Franklin function appearance changes at the ***m***-shift by 4 samples in Fig. 3 shows that the appearance of the resulting function differs significantly from the original one, shown in Table 1.

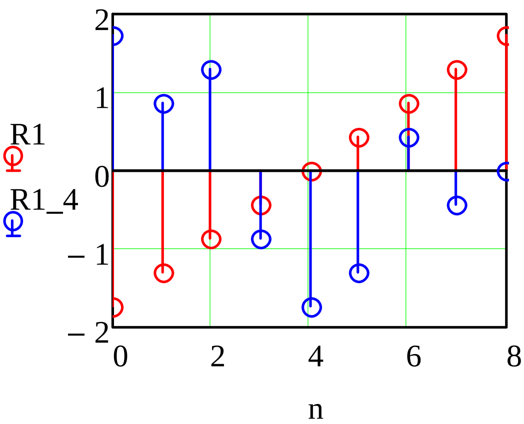


Fig. 3. Change of the first Franklin function appearance at m-shift.

This feature allows the functions that implement the appoximation with a less error to be chosen. Table 3 shows the minimal values of the approximation error for different Franklin functions with different ***m***-shift values. The second column of Table 3 shows the argument change procedure at the ***m***-shift operation.

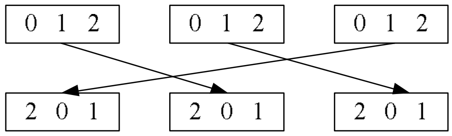
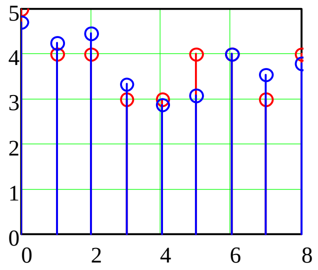
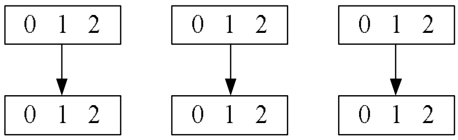
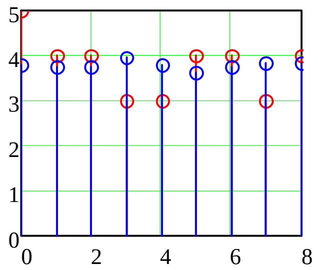
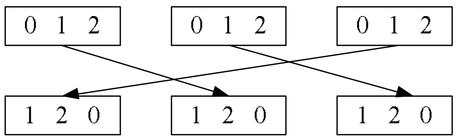
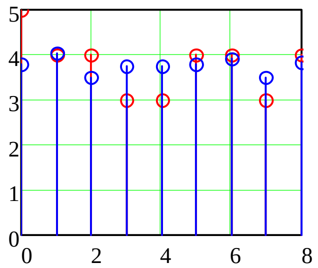
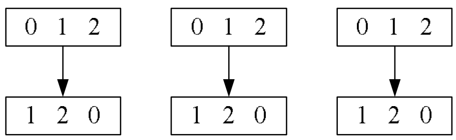
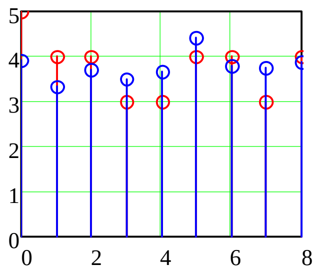
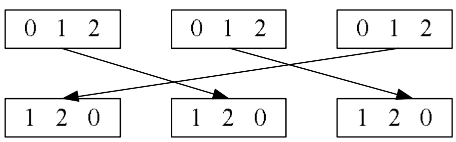
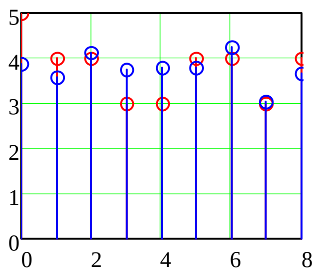
It is evident that the minimal approximation error corresponds to the first Franklin function with the ***m***-shift value equal to 4. That is the first Franklin function ***m***-shifted by 4 samples can be the only one used for linear approximation.

4. Conclusions

Thus, the linear approximation of the function of changing the parameters of equivalent generators of renewable sources results in increasing the level of energy transmitted to the load. In addition, the approximation by the Franklin orthogonal functions using the ***m***-shift minimizes the error and the number of approximating functions for implementing the control ensuring a rise in the level of energy that can be received from renewable sources.

Table 3

Minimal values of approximation error

Franklin function number	***M*-shift value**	**Error value** d	**Expansion result**
1	4 	14,3 %	
3	3 	22,5 %	
4	5 	19,8 %	
6	2 	19,8 %	
7	5 	19,1 %	

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АПРОКСИМАЦІЯ ФУНКЦІЇ ЗМІНИ ПАРАМЕТРІВ ЕКВІВАЛЕНТНОГО ДЖЕРЕЛА ФУНКЦІЯМИ ФРАНКЛІНА

Валерій Жуйков, Катерина Осипенко

Розглянуто проблему апроксимації функції зміни параметрів потоку первинної енергії лінійними наближеннями із забезпеченням мінімальної похибки наближення. Подано параметри напруги та внутрішнього опору еквівалентних генераторів відновлюваних джерел лінійними функціями, яке дає змогу підвищити рівень енергії, що передається в навантаження. Запропоновано вирішення сформульованої проблеми через наближення функції зміни параметрів потоку первинної енергії системою ортонормованих функцій Франкліна. Показано, що застосування операції *m*-зсуву до ортогональних функцій Франкліна дає можливість мінімізувати похибку та кількість апроксимувальних функцій.

Valery Zhuikov – Professor, D.Sc., Dean of the Faculty of Electronics, "Igor Sikorsky Kyiv Polytechnic Institute" National Technical University of Ukraine, Kyiv, Ukraine. Scientific interests: calculation theory of transient and steady-state processes in power electronic devices; analysis, synthesis and optimization of electronic systems; deterministic chaotic processes in converters; intelligent control of power consumption.

Kateryna Osypenko – PhD, senior lecturer of Department of Industrial Electronics, "Igor Sikorsky Kyiv Polytechnic Institute" National Technical University of Ukraine, Kyiv, Ukraine. Scientific interests: analysis and calculation of the electronic circuits using the method of structural numbers; converters in Smart Grid; maximum power point tracking for renewable energy sources; approximation of the primary energy flow change function.