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MULTI-MODULAR OPTIMIZATION OF INFORMATION TECHNOLOGIES

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Abstract: This paper involves a new conceptual methodology for improving the quality indices of vector information technologies (e.g. vector data coding) with respect to performance reliability, transmission speed, and functionality, using novel designs based on vector combinatorial configurations such as cyclic groups in extensions of Galois fields, difference sets and novel vector combinatorial constructions. Research into combinatorial structures of two- and higher dimensionality makes it possible to configure vector information systems based on the idea of an optimal placement of structural elements in the systems. These design techniques allow information to be presented, processed, transfered, and transmited as two- or multidimensional vector data flows with fewer base elementsю. The aim is to imploy the appropriate algebraic techniques to improve the technological efficiency making use of the applicable properties of interconnections of two- and multi-modular vector cyclic groups, and inter-convertible dimensionality of the vector information systems. The paper contains some examples of the minimization related to the optimal placement of structural elements in a spatially or temporarily distributed information system, including its application to the design of coded signals for communications and radar, and positioning the elements in antenna arrays.

Key words*:* V-algebra, multi-modular IRBs structure, antenna array, optimal monolithic code, optimal vector data coding, three-modular system of coordinates, optimal vector information technology.

1. Introduction

Combinatorial structures and system optimization techniques offer widespread applicability in cybernetics, computational technique, radio-communications, and related areas of science and engineering, for example, design of self-coding encode systems, data transfer, development of radio and hydro acoustic systems with high resolution, etc. Therefore, the synthesis of mathematical models of the systems is important in terms of improving the technical characteristics for the selected criteria and limitations.

The mathematical models of synthesis, and optimization of information systems reveal the main approaches to and methodology of constructing devices and systems with improved technical indicators of reliability, immunity and cryptography applying combinatorial models and methods of the systems involving the mathematical apparatus of combinatorial analysis [1], theory of algorithms, theory of numbers, matrix calculus, and elements of the algebraic theory of Galois fields [2].

2. The analysis of recent researches and publications

The general problem of system optimization relates to finding the best placement of its structural elements and events. The research into the specified mathematical area involves the appropriate algebraic structures, such as finite fields and groups in extensions of Galois fields, and difference sets [1, 2]. In modern mathematics, the theory of fields (or field theory) plays an essential role in the number theory and algebraic geometry [3–7]. Perfect fields [4] are significant because Galois theory over these fields becomes simpler, since the general drfinition of Galois field extensions being separable is automatically satisfied over these fields.

It is now accepted that a mathematical model be used to describe objects in a ***t***-dimensional space. The topology of the surface is superior to geometry for describing such a phenomenon because it deals with much more sophisticated and much deeper spatial and temporal relationships. Other scientists have also suggested that the entire universe may be shaped like a torus [7]. The major branch of geometry is the study of geometrical structures on manifolds. A manifold is a curved space of some dimension. The concept of a manifold [8] is central to many parts of geometry and modern mathematical physics because it allows more complicated structures to be described and understood in terms of the relatively well-understood properties of Euclidean space. A one-dimensional manifold includes lines and circles, but not a figure-of-eight. Two-dimensional manifolds are also called surfaces [8].

The proposed in article [9] method of adaptive data transmission in telecommunication access networks with a combined modulation type ensures the lowest possible bit error rate during data transmission at some signal-to-noise ratio. In the research paper [10] a simulation model called Verilog of the Analogue Mixed-Signal (Verilog-AMS) with the comp-drive sensing element of an integrated capacitive micro- accelerometer is developed. This model allows the reaction of the sensing elements to be simulated. They are effected by the applied force of acceleration resulting in the change in their comb-drive capacities, output voltages and currents for determining their constructive parameters and analyzing the system of a mechanical module of the integrated device, precision being very important indices for these models.

The research into various aspects of the subject are aimed at finding optimal solutions to a wide class of technological problems employing the properties of different combinatorial structures on sequences, which are based on the principle of connectedness. The research paper [11] suggests considering the models of discrete systems as mathematical objects, the elements of and operations on which are related to the topology structure of the base set. This approach follows from the nature of the formation and development of natural systems. A viazanka (*Ukr.*) or a bundle (*Eng.*) is an ordered sequence that is the base set for defining a set of operations. A viazanka-object consists of two sets (a set of elements and a set of operations), the operations on the set of elements being performed consistently over the elements. This is the effect of usual connectedness: only any directly related mathematical, physical or biological objects are subject to transactions. The concept quite adequately fits the definition of V-algebra (from “viazanka”). The narrower classes of the structures can be formed from the general definition of V-algebra by introducing additional restrictions. One-dimensional V-algebra contains elements that are one-dimensional mathematical objects (numbers, segments, 1D vectors, angular distance, etc.), and those of higher dimensionality are the vectors of a respective dimension.

We can see a remarkable progress in developing innovative techniques for systems optimization, as well as combinatorial sequencing theory, namely the concept of one- and multidimensional Ideal Ring Bundles (IRBs) [11]. The concept of IRBs can be used for finding optimal solutions to a wide class of technological problems.

A new vision of this concept brings closer to unraveling its role in the laws of harmonious correlation of geometric symmetry and asymmetry, provides a better understanding of the idea of "perfect" combinatorial structures to apply this concept to the progressive vector information technologies and optimization of multi-dimensional systems based on the multi-modular combinatorial configurations theory [12].

3. Objectives

The objective of the underlying concept is the development of a new methodology in system engineering for improving the quality indices of engineering devices, systems or technologies with non-uniform structure (e.g. planar antenna arrays of radio antennas) with respect to performance reliability, transformation speed, position(al) precision and resolution. We use novel designs based on multi-modular combinatorial configurations such as two- and multi-dimensional Ideal Ring Bundles. These design techniques make it possible to configure systems with fewer elements than usual maintaining or improving resolution and other significant operating characteristics of engineering devices and technological systems, namely vector data coding of signals for communications and radars, signal processing and reconstruction, and low-side lobe antenna design.

4. The main research results

The concepts in the algebraic studies of properties of different combinatorial structures on sequences can be seen to be always arbitrary formations, which are based on the principle of connectedness. The research paper [11] suggests considering the models of discrete systems as mathematical objects, the elements of and operations on which are related to the topology structure of the base set. This approach follows from the nature of the formation and development of natural systems, including genetic structures.

Let us calculate all S_n sums of the terms in a numerical *n*-stage chain sequence of distinct positive integers $K_n = \{k_1,\ k_2,...,k_i,...,\ k_n\}$, where all the terms in each sum are required to be consecutive elements of the sequence. It is clear that the maximum sum is the sum S_{max} of all the n elements:

$$S_{max} = k_1 + k_2 + ... + k_i + ... + k_n . \qquad (1)$$

A sum of consecutive terms in a chain sequence can have any of the n terms as its starting point p_j, and finishing point q_j, and can be of any length (the number of terms) from 1 to n. Thus, each j-th numerical pair (p_j, q_j), $p_j, q_j \in \{1, 2, \dots, n\}$ corresponds to the sum $S_j = S(p_j, q_j)$, is as follows:

$$S_j = S \cdot (p_j, q_j) = \sum_{i=p_j}^{q_j} k_i ,\ p_j \le q_j , \qquad (2)$$

An ordered numerical pair (p_j, q_j) determines a sum $S(p_j, q_j)$ in a numerical *n*-stage chain sequence, and is a numerical code of the sum. All the sums of the consecutive terms of the sequence $K_n = \{k_1,\ k_2,...,k_i,...,\ k_n\}$, are shown in Table 1.

Table 1

Sums of consecutive terms in ordered-chain sequence

$K_n = \{k_1,\ k_2,...,ki,\ ...,\ k_n\}$

p_j	q_j				
	1	2	...	n-1	n
1	k_1	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n} k_i$
2		k_2	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...			...		
n-1				k_{n-1}	$\sum_{i=n-1}^{n} k_i$
n					k_n

From this Table we can see that the maximum number of distinct sums is

$$S'_{max} = 1 + 2 + ... + n = n(n+1)/2. \quad (3)$$

The "ideal" ordered-chain sequence is such a numerical n-stage sequence of distinct positive integers $k_1,\ k_2,\ ...,\ k_n$, , which exhausts the natural row of numbers wrote down into the cells of Table 1. Table of the sums of consecutive terms in an ordered-chain sequence gives a solution for finding an optimal arrangement of structural elements in a chain topology information system in terms of system resolution (e.g. linear arrays of radio astronomy antennas).

The sums of consecutive terms in the ordered-chain numerical sequence {1,3,2}, where $k_1 = 1$, $k_2 = 3$, $k_3 = 2$ are given in Table 2.

Table 2

Sums of consecutive terms in ordered-chain sequence {1, 3, 2}

p_j	q_j		
	1	2	3
1	k_1=1	$\sum_{i=1}^{2} k_i = 4$	$\sum_{i=1}^{3} k_i = 1+3+2 = 6$
2		k_2=3	$\sum_{i=2}^{3} k_i = 3+2 = 5$
3			k_3=2

If the chain sequence K_n is regarded as being cyclic so that the element k_n is followed by k_1 , it can be called a ring sequence. A sum of consecutive terms in a ring sequence can have any of the n terms as its starting point p_j, and finishing point q_j , and can be of any length (the number of terms) from 1 to n-1. In addition, there is a sum S_n of all n terms, which is the same regardless of the starting point.

The Table of the sums of consecutive terms in the ordered-ring sequence $K_n = \{k_1,\ k_2,...,k_i,...,\ k_n\}$ is given below (Table 3).

Table 3

Sums of consecutive terms in ordered-ring sequence

$K_n = \{k_1,\ k_2,,\ ...\ ,k_i,\ ...,\ k_n\}$

p_j	q_j				
	1	2	...	n-1	n
1	k_1	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n-1} k_i$
2	$\sum_{i=1}^{n} k_i$	k_2	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...			...		
n-1	$\sum_{i=n-1}^{n} k_i + k_1$	$\sum_{i=n-1}^{n} k_i + \sum_{i=1}^{2} k_i$	...	k_{n-1}	$\sum_{i=n-1}^{n} k_i$
n	k_n+k_1	$k_n + \sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n} k_i$	k_n

A sum of consecutive terms in a ring sequence can have any of the n terms as its starting point p_j, and finishing point q_j, and can be of any length (the number of terms) from 1 to n-1. So, each j-th numerical pair (p_j,q_j), $p_j, q_j \in \{1, 2, \ldots, n\}$, corresponds to the sum $S_j = S\ (p_j, q_j)$, and can be calculated by the equation below if $p_j \le q_j$:

$$S_j = S\ (p_j, q_j) = \sum_{i=p_j}^{q_j} k_i\ . \quad (4)$$

In case $p_j > q_j$, a ring (circular) sum can be calculated by formula (5)

$$S_j = S\ (p_j, q_j) = \sum_{i=1}^{q_j} k_i + \sum_{i=p_j}^{n} k_i\ . \quad (5)$$

It is easy to see from Table 3 that the maximum number of distinct circular sums S_n of the consecutive terms of the ring sequence is

$$S_n = n(n-1) + 1. \quad (6)$$

Comparing equations (3) and (6), we see that the number of sums S_n for the consecutive terms in a ring topology is nearly twice as many as the number of sums S'_{max} in the daisy-chain topology, for the same sequence K_n of n terms.

An n-stage ring sequence $K_n = \{k_1, k_2, ... ,k_i, ..., k_n\}$ of the natural numbers for which a set of all S_n circular sums consists of the numbers from 1 to $S_n = n(n-1)+1$, that is each number occurs exactly once, is called an "Ideal Ring Bundle" (IRB).

An example of a numerical ring sequence with $n = 4$ and $S_n = n(n-1)+1 = 13$, where $k_1 = 1$, $k_2 = 3$, $k_3 = 2$, and $k_4 = 7$ is given below (Table 4).

Table 4

Circular sums for numerical ring sequence {1, 3, 2, 7}

p_j	q_j			
	1	2	3	4
1	1	4	6	13
2	13	3	5	12
3	10	13	2	9
4	8	11	13	7

Table 4 is calculated in a similar way that the one above, i.e. by using equations (4) and (5). Table 4 contains a set of all $S_n = n(n-1)+1 = 13$ sums of consecutive elements of the 4-stage ($n = 4$) ring sequence {1,3,2,7}, and each sum from 1 to n-1 occurs exactly once ($R = 1$). So, the ring sequence {1,3,2,7} is a one-dimensional Ideal Ring Bundle (1D-IRB) with the parameters $n = 4$ and $R = 1$.

Here is a graphic presentation of a one-dimensional Ideal Ring Bundle (1-D IRB) containing four (n=4) elements {1, 3, 2, 7} (Fig. 1).

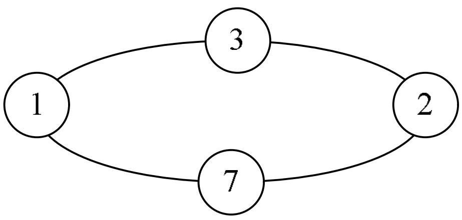

Fig. 1. Graph of one-dimensional Ideal Ring Bundle (1D-IRB) containing four (n=4) elements {1, 3, 2, 7}.

It is generally known that there exists the endless number of IRBs, and the more number of intersections n is in an IRB, the more number of IRBs are [11]. The idea of "perfect" numerical bundles provides the development and design of two- and multidimensional Ideal Ring Bundles (IRBs).

Let us consider an n-stage ring sequence $K_{2D} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), ..., (k_{i1}, k_{i2}), ..., (k_{n1}, k_{n2})\}$, where all terms in each circular vector sum are required to be consequtive 2-stage sequences as the elements of this sequence. A circular vector-sum of the consequtive terms in this ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to n-1. The n-stage ring sequence K_{2D}, for which the set of all

$$S_{2D} = n(n-1), \tag{7}$$

two-modular vector-sums (mod m_1, mod m_2) forms a two-dimensional grid over a torus $m_1 \times m_2$, where each node of the grid occurs exactly R-times, is called a two-dimensional Ideal Ring Bundle (2D–IRB) with parameters n, R, and m_1, m_2.

Next, we consider a two-dimensional IRB with four ($n = 4$) terms of ring topology, where $k_1 = (0,1)$, $k_2 = (1,3)$, $k_3 = (0,2)$, $k_4 = (2,3)$, the graph of which is depicted below (Fig. 2).

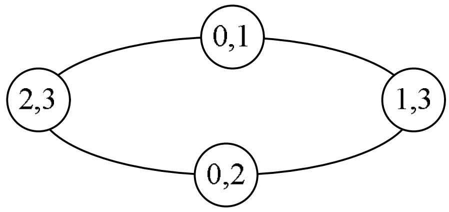

Fig. 2. Graph of 2D-IRB {(0,1), (1,3), (0,2), (2,3)}; $n = 4$, $R = 1$, $m_1 = 3$, $m_2 = 4$.

We can easily calculate all circular two-dimensional vector-sums, modulo $m_1 = 3$ for the first components of the vector-sums and modulo $m_2 = 4$ for the second ones:

(0,0)≡(1,3)+(0,2)+(2,3);
(1,0)≡(0,1)+(1,3)+(1,1)+(1,2);
(0,1); (1,1)≡(1,3)+(0,2);
(0,2);
(1,2)≡(0,1)+(1,3)+(0,2);
(0,3)≡(2,3)+(0,1)+(1,3); (1,3);

(2,0)≡(2,3)+(0,1);
(2,1)≡(0,2)+(2,3);
(2,2)≡(0,2)+(2,3)+(0,1);
(2,3).

As the elements (0,1), (1,3), (0,2), (2,3) of a ring sequence, by themselves, are also circular vector-sums, the circular vector-sums set is as follows:

(0,0)	(0,1)	(0,2)	(0,3)
(1,0)	(1,1)	(1,2)	(1,3)
(2,0)	(2,1)	(2,2)	(2,3)

The result of the calculation forms a two-dimensional grid over the torus 3 × 4, where 2D modular coordinates of each node of the grid occurs exactly once (R=1). So, the ring sequence of the 2D vectors {(0,1), (1,3), (0,2), (2,3) is a two-dimensional Ideal Ring Bundle (2D-IRB) with $n = 4$, $R = 1$, $m_1 = 3$, $m_2 = 4$.

The problem of structural optimization of radar or sonar systems relates to finding the best placement of its structural elements in spatially distributed systems, as well as a better understanding of the role of geometric structure in the behavior of the systems. The research into this specified mathematical area involves the appropriate algebraic constructions based on finite groups in extensions of Galois fields [2].

Classical theory of combinatorial configurations can hardly be expected effective for constructing 2D or 3D antenna arrays of high resolution for radar or sonar systems, as well as for finding the optimal solutions to other problems related to constructing such systems with low side lobe antenna. So, for such problems to be optimally solved, there is a need for an advanced theory and regular method.

The application of algebraic constructions and modern combinatorial analysis provides the optimal solution to many problems of high-resolution interferometry for radar, data communications, and signal design [11, 12]. The regular methods for constructing the non-redundant two- and three-dimensional *n*-element antenna arrays, based on 2D or 3D IRBs are proposed in [11], while the optimal 2D and 3D coding design is suggested in [12].

Here is an example of constructing a planar antenna array configuration based on a two-dimensional IRB with the parameters $n = 13, R = 4, m_1 = 5, m_2 = 8$, where $S = m_1 \times m_2 = 5 \times 8 = 40$, which can be reconstructed into an antenna array over $5 \times 6 = 30$ grids (Fig. 3).

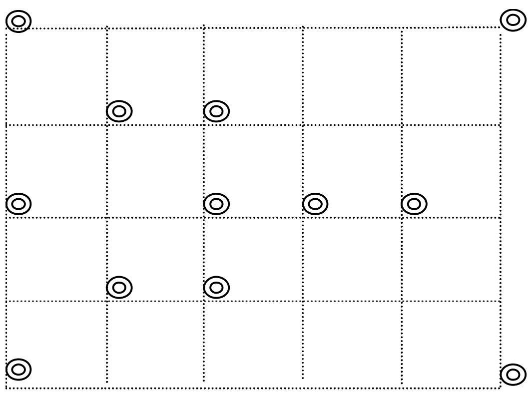

Fig. 3. Antenna array over 5 ´ 6 grids reconstructed from the array over 5 ´ 8 grids based on the 2D IRB.

The solution we need can be found after constructing the 2D matrix of all circular two-dimensional vector-sums on the 2D IRB, with each of them being considered with regard to finding minimum of the sum by using crossing out as depicted in Fig. 3. These methods make it possible to obtain planar antenna array configurations having lower peak sidelobe levels than the existing ones and thus maintain or improve their resolution.

For any two-dimensional phased antenna array configurations, the antenna or sensor elements are positioned in a manner as prescribed by the appropriate 2D Ideal Ring Bundle. The considerable collection of the 2D IRB sets found also contributes to the obtaining of optimized planar antenna arrays with much more elements than the currently existing ones, having the low peak sidelobe levels.

The antenna or sensor elements positioned in a manner as prescribed by the underlying combinatorial technique make it possible to configure systems, using the appropriate variant of 2D IRB for constructing the radar or sonar planar antenna arrays. The search algorithm is employed to find the optimal solution in the simplest way based on the appropriate matrix of circular two-dimensional vector-sums on the suitable 2D IRB, as well as on the crossing out operations. Clearly, we keep to the known relationships between the grid sizes and parameters of a working range to configure a radar or sonar system.

A three-dimensional Ideal Ring Bundle (3D IRB) is an ***n***-stage ring sequence of 3-type sub-sequences of the sequence, which forms the "perfect" 3- axis vector-space coordinate system of a finite 3-modular toroidal manifold over the $m_1 \times m_2 \times m_3$ grid. The solution needed consists in the construction of a 3D matrix based on the appropriate 3D IKB, for which a set of all 3-modular (mod m_1, mod m_2, mod m_3) circular sums enumerate a set of node coordinates over a 3-modular manifold exactly ***R***-times. The ring sequence $K_{3D} = \{(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), ..., (k_{n1}, k_{n2}, k_{n3})\}$ of 3D terms is cyclic, so that (k_{n1}, k_{n2}, k_{n3}) is followed by (k_{11}, k_{12}, k_{13}). A sum of consecutive terms in the 3D ring sequence can have any of the *n* terms as its starting point, and can be of any length, with the condition being kept to:

$$(m_1, m_2, m_3) = 1 . \qquad (8)$$

Such a model makes it possible to configure a 3-modular (mod m_1, mod m_2, mod m_3) toroidal manifold over the $m_1 \times m_2 \times m_3$ grid, each nodal coordinate of the grid occuring exactly ***R***-times.

For example, a ring ordered 6-stage (n = 6) sequence of 3-stage (3D) terms {(0,1,4), (0,2,4), (1,1,1), (1,1,2), (1,0,3), (0,2,2)} allows a 3-modular (mod2, mod3,mod5) manifold to be configured over the $2 \times 3 \times 5$ grid, based on this 3D IRB. The set of all circular 3-modular vector sums $m_1 = 2$, $m_2 = 3$, $m_3 = 5$ enumerates the set of the grid nodes exactly once (***R***=1):

(0,0,1) ≡ (0,2,2) + (0,1,4)
(0,0,2) ≡ (1,1,2) + (1,0,3)
(0,0,3) ≡ (0,1,4) + (0,2,4)
(0,0,4) ≡ (1,0,3) + (0,2,4) + (0,1,4) + (0,2,4) + (1,1,1)
………………………………………………
....

Finally,

(0,0,0) ≡ (0,1,4) + (0,2,4) + (1,1,1) + (1,1,2) + + (1,0,3) + (0,2,2).

A more general type of the “perfect” combinatorial configurations are *t*-dimensional Ideal Ring Bundles as an *n*-stage ring sequence {(k_{11}, k_{21}, ... k_{t1}), (k_{12}, k_{22}, ..., k_{t2}), ... (k_{1n} , k_{2n}, ..., k_{tn})}. The *t*-D IRB allows a set of the nodal coordinates of circular *t*-modular (mod m_1, mod m_2, …, mod m_t) vector sums on the $m_1 \times m_2 \times ... \times m_t$ grid of a toroidal manifold to be enumerated exactly *R*- times.

The graphical model of a *t*-D IRB is given below (Fig. 4).

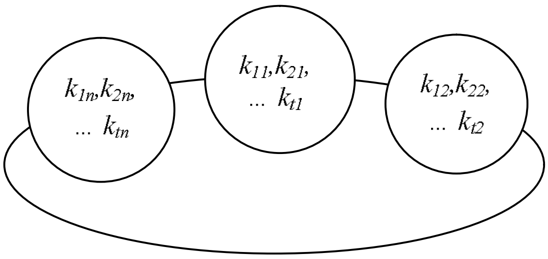

Fig. 4. Graphical model of t-D IRB.

The traditional methods for vector data coding are not always applicable because the optimization technique that could be used to solve such a problem requires revision of many options. To solve the problem, the property of *t*-dimensional IRB can be assumed as a basis of the design of an optimal *t*-dimensional binary monolithic IRB-code [12]. The underlying code forms a set of combinations from the same consecutive symbols in each of them. Here are the examples of four-stage (n = 4) code words of the Monolithic binary ring code: 0000,

0001, 0010, 0100, 1000,
0011, 0110, 1100, 1001,
0111, 1110, 1101, 1011

For example, the two-dimensional IRB {(0,1), (1,3), (0,2), (2,3)} provides a possibility of configuring the optimal *t*-dimensional Monolithic IRB-code as follows (Table 5).

Table 5

2D-IRB Monolithic Code {(0,1),(1,3), (0,2), (2,3)}

Vector	Cyclic binary digits			
	(0,1)	(1,3)	(0,2)	(2,3)
(0,0)	0	1	1	1
(0,1)	1	0	0	0
(0,2)	0	0	1	0
(0,3)	1	1	0	1
(1,0)	1	1	0	0
(1,1)	0	1	1	0
(1,2)	1	1	1	0
(1,3)	0	1	0	0
(2,0)	1	0	0	1
(2,1)	0	0	1	1
(2,2)	1	0	1	1
(2,3)	1	0	0	0

Table 5 contains all binary code combinations of 2D IRB Monolithic Code {(0,1), (1,3), (0,2), (2,3)} from (0,0) to (2,3) obtained in only one way.

5. Perfect multi-modular coordinate system

Let us regard a chain *n*-sequence of non-negative integer 3-stage sub-sequences of the sequence {(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), ... , (k_{i1}, k_{i2}, k_{i3}), ... , (k_{n1}, k_{n2},k_{n3})} as being cyclic, so that (k_{n1}, k_{n2}, k_{n3}) is followed by (k_{11}, k_{12}, k_{13}), we call this a three-dimensional ring *n*-sequence.

Here we obtain next 3-modular sums of the connected sub-sequences of a three-dimensional ring *n*-sequence, and modulo m_1, m_2, and m_3 summations are as follows:

(k_{11}, k_{12}, k_{13}) + (k_{21}, k_{22}, k_{23}) ≡ ((k_{11} + k_{21})(mod m_1), (k_{12} + k_{22})(mod m_2), (k_{13} + k_{23})(mod m_3);

(k_{21},, k_{22} ,k_{23}) + (k_{31},, k_{32} , k_{33}) ≡ ((k_{21}+k_{31})(mod m_1), (k_{22}+ k_{32})(mod m_2),(k_{23} +k_{33})(mod m_3));

. . . .
. . . .

(k_{11}, k_{12}, k_{13}) + (k_{21}, k_{22}, k_{23}) + (k_{31}, k_{32}, k_{33}) ≡ ((k_{11} + k_{21} + + k_{31}) (mod m_1), (k_{12}+ k_{22}+ k_{32})(mod m_2), (k_{13} + k_{21} + + k_{33})(mod m_3));

. . . .
. . . .

(k_{11}, k_{12}, k_{13}) + (k_{21}, k_{22}, k_{23})+ ... + (k_{i1}, k_{i2}, k_{i3}) + ... + +(k_{n1}, k_{n2}, k_{n3}) ≡ ((k_{11} + k_{21} + …+ k_{i1} +…+ k_{n1})(mod m_1), (k_{12} + k_{22} + …+ k_{i2} +…+ k_{n2})(mod m_2), (k_{13} + k_{23} + …+ k_{i3} + +…+ k_{n3})(mod m_3)).

So, we configure a 3-modular $m_1 \times m_2 \times m_3$ cyclic matrix as a coordinate axis system based on the ring *n*-sequence {(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), ... , (k_{i1}, k_{i2}, k_{i3}), ... , (k_{n1}, k_{n2},k_{n3})}.

The three-modular (mod m_1, mod m_2, mod m_3) system of coordinates being the product of three circles is useful in visualizing (Fig. 5).

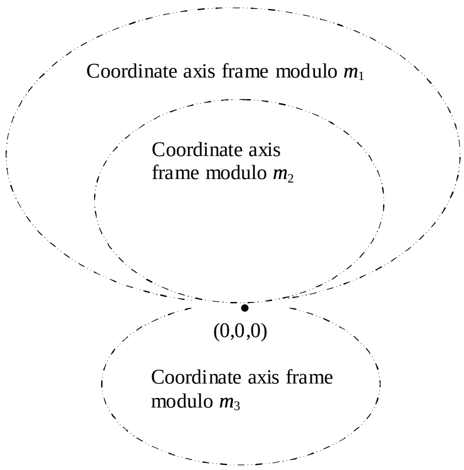

Fig. 5. Three-modular (mod m_1, mod m_2, mod m_3) system of coordinates with ground coordinate (0,0,0).

The ring *n*-sequence $\{(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), ..., (k_{i1}, k_{i2}, k_{i3}), ..., (k_{n1}, k_{n2}, k_{n3})\}$, for which a set of all 3-modular sums occurs in an enumeration set of node points space coordinate system $m_1 \times m_2 \times m_3$ exactly R times, is called a perfect 3-modular vector ring. The three-modular coordinate systems of *n*-stage perfect rings are based on their useful property to create a non-redundant three-modular cyclic system of coordinates with a ground coordinate (0,0,0) over the torus $(n-1) \times \times n = m_1 \times m_2 \times m_3$.

Here is an example of the 3-modular sequence with $n = 6$, $m_1 = 2$, $m_2 = 3$, $m_3 = 5$, and $R = 1$, which contains six (n = 6) 3-stage sub-sequences of the sequences: {(0,2,3), (1,1,2), (0,2,2), (1,0,3), (1,1,1), (0,1,0)}.

The set of all circular sums over the 6-stage sequence, 3-tuple modulo (mod2, mod3, mod5) gives the following model of the 3-modular manifold topology system:

(0,0,0) ≡{(0,2,3) + (1,1,2)+ (0,2,2)+(1,0,3)+ (0,1,0)}
(0,0,1) ≡{(0,2,2) + (1,0,3) + (1,1,1)}
(0,0,2) ≡{(1,1,2) + (0,2,2) + (1,0,3)}
(0,0,3) ≡ {(0,2,3) + (0,1,0)}
(0,0,4) ≡ {(0,2,2) + (1,0,3) + (1,1,1) + (0,1,0) + (0,2,3)}
(0,1,1) ≡ {(0,2,2)+ (1,0,3) + (1,1,1) + (0,1,0)}
(0,1,2) ≡ ((1,0,3)+ (1,1,1) + (0,1,0) + (0,2,3)),
(0,1,3) ≡ {(1,1,1)+ (0,1,0)+ (0,2,3)+(1,1,2)+ (0,2,2)}
(0,1,4) ≡ {(0,1,3) + (1,1,1)}
(0,2,0) ≡ {(0,2,3)+ (1,1,2) + (0,2,2) + (1,0,3)}
(0,2,1) ≡ {(1,1,1)+ (0,1,0) + (0,2,3) + (1,1,2)}
. . . .
. . . .

Finally, (1,2,4) ≡ {(0,2,3) +(1,1,2) +(1,1,1)+ + (1,0,3)+ (0,1,0)}.

The maximum number $S_{max} = m_1 \cdot m_2 \cdot m_3 = 30$ of such sums is fixed exactly once (R=1), and the set of all 3-modular (2, 3, 5) sums enumerates a set of node points space coordinate system $m_1 \times m_2 \times m_3$.

The optimum *t*-modular relationship is the *n*-stage ring sequence $C_{n3} = \{(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), ..., (k_{i1}, k_{i2}, k_{i3}), ... , (k_{n1}, k_{n2}, k_{n3})\}$ of non-negative integer *t*-stage sub-sequences (*t*D vectors) of the sequences as being a cyclic proportion of the *n* three-dimensional vectors. Note, the optimal *t*-modular relationship is a non-redundant system with respect to partitioning *t*D space with the smallest possible number of intersections.

An example of an optimum 3-modular (t=3) relationship follows from the perfect ring {(0,2,3), (1,1,2), (0,2,2), (1,0,3), (1,1,1), (0,1,0)}. This optimal 3-modular relationship (0,2,3) : (1,1,2) : (0,2,2) : (1,0,3) : (1,1,1) : (0,1,0) contains six (n = 6) 3D vectors as a cyclic group in a finite field, and forms a complete set of 3D vectors over the 3-modular manifold grid.

The applications profiting from the code are compression, signal reconstruction, operation speed, and security.

The underlying multidimensional models make it possible to apply the concept of Ideal Ring Bundles to the configuration of high-performance vector data information technologies and communication systems, based on the combinatorial techniques, with the direct applications to elements positioning in an antenna array, and design of coded signals for communications.

6. Conclusions

The multi-modular Ideal Ring Bundles (IRB) provides a new conceptual methodology for improving the quality indices of vector information technologies (e.g. vector data coding) with respect to performance reliability, transmission speed, and functionality, using novel designs based on the IRB. The applicable property and structural perfection of one- and multidimensional IRBs provides the new mathematical principles relating to the optimal placement of structural elements in spatially or temporally distributed systems. This property makes the underlying methodology useful for finding the optimal solutions to a wide class of problems in the field of information technologies, including applications in the design of coded signals for communications and radars, positioning of elements in an antenna array, and vector data processing. The useful applications of the multi-modular IRBs theory are for example, non-redundant aperture, low-side lobe antenna arrays, and self-correcting vector data coding. The two- and multidimensional Ideal Ring Bundles can be well used for configuring the high performance vector data information technologies and spatial control systems. Spatial perfection and structural harmony exist not only in the abstract models but in the real world as well.

References

[1] M. Hall, *Combinatorial Theory*. Waltham, MA, USA: Blaisdell Publishing Company, 1967.

[2] J. Rotman, *Galois Theory*, 2nd ed. Berlin, Germany: Springer, 1998.

[3] T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms*, 2nd ed. The MIT Press and McGraw-Hill Book Company, 2001.

[4] N. Bourbaki, *Algebra II*. Berlin, Germany: Springer, 2003.

[5] M. Greenberg, *Lectures on Forms in Many Variables*. New York-Amsterdam: W.A. Benjamin, Inc, 1969.

[6] A. Fröhlich and M. Taylor, *Algebraic number theory* (*Cambridge studies in advanced mathematics. 27).* Cambridge, UK: Cambridge University Press, 1991.

[7] P. McMullen and E. Schulte, *Abstract Regular Polytopes.* Cambridge, UK: Cambridge University Press, 2002.

[8] Manifold, *https://en.wikipedia.org/wiki/Manifold*

[9] Yu. Shapovalov, B. Mandziy, and D. Bachyk, “Optimization of linear parametric circuits in the frequency domain”, *Econtechmod: an international quarterly journal on economics in technology*, *new technologies and modelling processes*, vol. 4, no. 2, pp. 73–77, 2013.

[10] A. Holovatyy, V. Teslyuk, and M. Lobur, “Verilog-ams model of comb-drive sensing element of integrated capacitive microaccelerometer for behavioural level of computer aid design”, *Econtechmod: an international quarterly journal on economics in technology, new technologies and modelling processes,* vol. 3, no.1, pp. 49–53, 2014.

[11] V. V. Riznyk, *Synthesis of the optimum combinatorial systems*, Lviv, Ukraine: Vyshcha shkola, 1989. (Ukrainian)

[12] V. Riznyk “Applications of the combinatorial configurations for optimization of technological systems”, *Econtechmod: an international quarterly journal on economics in technology*, *new technologies and modelling processes*, vol. 5, no.1, pp. 52–57, 2016.

МУЛЬТИМОДУЛЯРНА ОПТИМІЗАЦІЯ ІНФОРМАЦІЙНИХ ТЕХНОЛОГІЙ

Володимир Різник

Розглянуто нову концепцію в методології покращення якісних показників векторних інформаційних технологій (наприклад, кодування векторних даних) стосовно надійності, швидкості пересилання даних та функціональних можливостей, використовуючи інноваційні розробки на основі векторних комбінаторних конфігурацій, таких як циклічні групи в розширених полях Ґалуа, різницеві множини та новітні векторні комбінаторні конструкції. Дослідження двовимірних та комбінаторних структур вищої розмірності дає змогу створювати векторні інформаційні системи на основі ідеї вигідного розміщення структурних елементів у цих системах. Цей метод проектування дає змогу представляти, опрацьовувати, перетворювати та пересилати інформацію у вигляді дво- або багатовимірних векторних потоків даних з меншою кількістю базових елементів, ніж тепер. Мета полягає у використанні відповідних алгебричних методів для покращення технологічної ефективності, використовуючи корисні властивості взаємозв’язків дво- й багатомодулярних векторних циклічних груп та здатність конвертування розмірності векторних інформаційних систем. Стаття містить кілька прикладів мінімізації, пов’язаних з оптимальним розміщенням структурних елементів у просторовій або часово розподіленій інформаційній системі, зокрема застосування для розроблення кодованих сигналів для зв’язку і радіолокаційних сигналів та розміщення елементів в антенних решітках.

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