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EFFECT OF SUCKER-ROD PUMPING UNIT WALKING BEAM OSCILLATION FREQUENCY ON ASYNCHRONOUS ELECTRIC DRIVE

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Abstract: The process of oil extraction requires continuous monitoring of oil wells operation. To improve the efficiency of oil wells, it is necessary to set the optimal mode of oil pumping unit operation in which the rate of liquid pumping corresponds to that of its inflow. Many oil wells are strippers and therefore are operated intermittently. Application of automated control systems requires not only reliable data on the oil-extraction equipment status, but also determination of restrictions which arise from the conditions of error-free performance of the electric drive system. This can only be done on the basis of mathematical modelling.

The method and algorithm developed to compute operating modes of sucker-rod pumping units (SRPU) allow the walking beam oscillation frequency to be determined depending on the oil formation flow rate, as well as the restrictions regarding the range of its regulation defined by torque and heat overload of the motor. The computation algorithm is underlain by the fine mathematical models of the asynchronous motor (AM) and sucker rod pump, and the method of computation of periodic dependencies of the unit operating mode coordinates by solving a boundary-value problem.

Key words: sucker-rod pumping unit, asynchronous electric drive, oscillation frequency control, steady-state mode, static characteristics.

1. Introduction

Sucker-rod pumping units (SRPU) are used in most oil fields. A sucker rod pump (Fig. 1) is powered by an asynchronous motor (AM). Ensuring efficient and reliable operation of the SRPU electric drive is one of the most challenging tasks. The specificities of its operation include a periodically variable load, whose law of variation changes within a wide range during the well operation, and a periodically variable torque. An important factor affecting the law of variation of the AM shaft torque is sucker rod pump balancing.

Adjustment of the deep-well pumping equipment and the modes of its operation must agree with the well flow rate, which decreases as the well is operated. A particular importance is attached to the optimization of stripper wells operation [1, 2, 10].

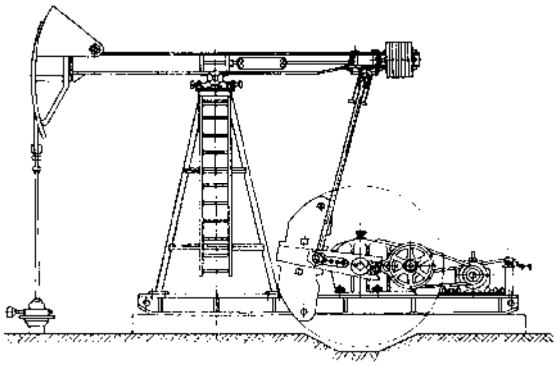

Fig. 1. A sucker-rod pumping unit.

It is important to ensure the necessary change in the mode of the sucker rod pump operation so that it can correspond to the oil inflow rate [4, 8]. As the torque from the AM rotor shaft is transferred to the crank via a reduction gear and a belt gear, the walking beam oscillation frequency can be manually changed in a discrete way by changing their reduction ratio. However, the most effective approach is using controlled electric drives [3, 5, 7, 10]. The available diagnostic facilities for monitoring the underground equipment status [3, 4, 6, 7] enable reliable SRPU load curves to be obtained, but the software for automatic control of oil pumping units does not meet the current practical needs.

For any control to be implemented it is necessary to establish the relationship between input and output variables of the control object. The complexity of this task for SRPU electric drives stems from two facts. First, the input variable is represented not by a scalar value of the torque, but by its functional periodic dependence. Second, the relation between the force acting upon the walking beam in the point of rods suspension, and the AM shaft torque is defined by a complex non-linear kinematic relationship [6, 7], which transforms the periodic law of variation of the force acting on the walking beam in the point of suspension of the rods connected to the pump into the law of variation of the AM shaft torque (Fig. 2). It should be added that the AM of the SRPU electric drive continuously operates in a dynamic mode and features a non-linear dependence between the AM load torque and electromagnetic torque.

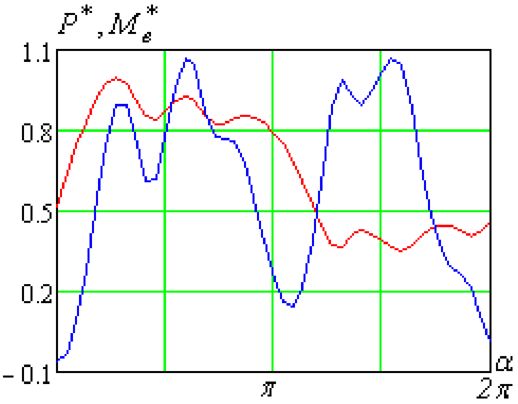

Fig. 2. Periodic dependencies of a relative value of the force $P^* = P/P_{\text{мах}}$ in the point of suspending the rods and electromagnetic torque M_e^* of the motor.

The problem of ensuring the effective operation of the SRPU, including discrepancy between the oil pumping rate and oil inflow rate, can only be solved on the basis of a complex mathematical model of the motor, sucker rod pump and control system in their interrelations. The first step is establishing relationships between a dynamogram, which determines the periodic law of variation of the force in the point of rods suspension, frequency of walking beam oscillations and reduction ratio between the AM shaft and reduction gear shaft. Solving this problem will allow the methods of SRPU control to be improved and restrictions stemming from the specificities of SRPU electric drive operation to be determined, thereby making it possible to enhance energy efficiency and reliability of the unit as well as to extend its turnaround interval.

The research aims to develop a mathematical model and algorithm enabling the determination of the AM rotor speed, which corresponds to the well flow rate and permissible range of its adjustment, based on the known periodic law of the load created by the pump.

2. Mathematical model

One revolution of the crank corresponds to a full cycle of up-and-down movement of the plunger and, respectively, to a change of the load torque $M_c(\alpha) = M_c(\alpha+2\pi)$, i.e. the torque variation period is equal to the crank rotation period. The plunger pump productivity can be determined using the expression

$$q(t) = \frac{l p D^2 S_0}{4 p_0 k_i} w(t), \qquad (1)$$

where p_0 is the number of motor pole pairs; w is the rotational speed of the rotor; D is the plunger diameter; S_0 is the plunger travel; l is the volume efficiency of the pump determined as the ratio between the theoretically possible volume of the liquid and its real one, which in practice is somewhat lower [5] due to its leak from the cylinder, and falls into the range $0{,}6 \le l \le 0{,}8$; $k_i = k_{\text{ш}} \times k_p$ is the reduction ratio between the AM shaft and crank, which is equal to the production of reduction ratios of the belt gear $k_{\text{ш}}$ and the reduction gear k_p.

Since the load torque is a periodic variable, variation of $q(t)$ will also be governed by the periodic law. Therefore, the volume of oil extracted per cycle of the SRPU operation can be calculated as a period average value

$$Q_c = \frac{1}{T} \int_0^T q(t) dt . \qquad (2)$$

Dividing the current oil inflow rate by the oil pumping rate per operation cycle gives the period average rotational speed of the AM rotor, which ensures the balance between the inflow and pumping of the liquid. The implementation of such a relationship, however, is associated with a number of restrictions on the AM operation in the SRPU system both in terms of overload capacity of the motor, and with regard to its heat load. Establishing these restrictions is a complex non-linear problem, which can be solved using mathematical modelling of the electromechanical system of the SRPU electric drive considering such factors as a saturation-caused non-linear character of electromagnetic relationships between its electric circuits, non-linear kinematic relationships of the sucker rod pump and a variable moment of inertia of the moving parts.

The rotational speed w of the rotor at the set law of variation of the AM shaft torque $M_c(t)$ is found from the differential equation (DE) of movement of the SRPU mechanical system, which, taking into account the variable moment of inertia of the unit J, has the form

$$\frac{dw}{dt} = \frac{p_0}{J}\left(M_e - M_c(t)\right) - \frac{w^2}{2J}\frac{dJ}{da}, \qquad (3)$$

where M_e is the AM electromagnetic torque; J is the moment of inertia of the moving parts corrected for the motor shaft, which is determined on the basis of the moments of inertia of the unit components performing rotational or translational movements and depends on the angle of rotation α of the crank $J(\alpha) = J(\alpha+2\pi)$, and, therefore, is a function of the angle of rotor rotation, which in turn is a function of time.

On the x and y axes, the electromagnetic torque of AM is determined using the formula

$$M_e = 1{,}5 p_0 \left(y_x i_y - y_y i_x\right).$$

To determine the variables in this formula, the system of DE of AM electromagnetic balance is solved:

$$\frac{d\vec{y}}{dt}=\vec{u}-\Omega_0\vec{y}-R\vec{i}\,, \tag{4}$$

where $\vec{u}=\left(u_{sx},u_{sy},0,0\right)^*$; $\vec{y}=\left(y_{\,sx},y_{\,sy},y_{\,rx},y_{\,ry}\right)^*$;

$\vec{i}=\left(i_{sx},i_{sy},i_{rx},i_{ry}\right)^*$ are the vectors of voltage, current and flux linkage of the circuits (upper index * means transposition);

$$R=\begin{array}{|c|c|c|c|}\hline r_s & & & \\ \hline & r_s & & \\ \hline & & r_r & \\ \hline & & & r_r \\ \hline\end{array}$$

is the diagonal matrix in which r_s , r_r are the resistances of the rotor and stator windings, respectively;

$$\Omega_0=\begin{array}{|c|c|c|c|}\hline & -w_0 & & \\ \hline w_0 & & & \\ \hline & & & w-w_0 \\ \hline & & w_0-w & \\ \hline\end{array}\;;$$

w_0, w are the angular frequencies of the AM supply voltage and movement of the rotor, respectively.

System of DE (4) makes it possible to determine the vectors $\vec{y}$ and $\vec{i}$ for a set value of the vector $\vec{u}$ and a set angular speed w of the rotor movement, taking into consideration the AM magnetic path saturation. To do this, computation of flux linkage of the circuits and matrix of electromagnetic parameters relies on using magnetization curves for the main magnetic path and the paths of leakage fluxes of the stator and rotor circuits [9].

3. Algorithm of computing a steady-state mode

In a steady-state mode, due to a cyclic character of the unit operation, processes in the electromechanical system of the SRPU electric drive change according to the periodic law determined by a complete cycle of the sucker rod pump operation. However, its time period is unknown, and its angular period is determined by a complete revolution of the crank and is equal to $T=2p$ irrespective of the crank shaft rotational speed. Thus, the solution to the system of DE (3), (4) of electromagnetic balance is a totality of periodic dependencies of the coordinates within a period determined by one revolution of the crank. Determination of these dependencies by integrating the system of DE (3), (4) in the time domain before the process reaches the steady state is unreasonable, and the most effective way of their finding is to solve a boundary-value problem, which enables these dependencies to be obtained using a minimum amount of computations. To do this, let us replace the time coordinate t in equations (3), (4) with the angle of crank rotation a,

$$\frac{d\vec{y}}{da}=\frac{\vec{u}-\Omega_0\vec{y}-R\vec{i}}{w_{kr}}\,, \tag{5a}$$

$$\frac{dw}{da}=\frac{p_0^2k_i}{Jw}\left(M_e-M_c\right)-\frac{w}{2J}\frac{dJ}{da}\,, \tag{5b}$$

where w_{kr} is the angular speed of the crank.

To shorten the description of the algorithm of solving the problem as a boundary-value one, DE system (5) is presented by one vector equation

$$\frac{\partial\vec{y}(\vec{x},a)}{\partial a}=\vec{f}\left(\vec{u},\vec{y},\vec{x},a\right) \tag{6}$$

where

$$\vec{y}=\begin{array}{|c|}\hline \vec{y} \\ \hline w \\ \hline\end{array}\;;\quad \vec{x}=\begin{array}{|c|}\hline \vec{i} \\ \hline w \\ \hline\end{array}\;;$$

$$\vec{f}=\begin{array}{|c|}\hline \left(\vec{u}-\Omega_0\vec{y}-R\vec{i}\right)/\,w_{kr} \\ \hline \dfrac{p_0^2k_i}{Jw}\left(M_e-M_c\right)-\dfrac{w}{2J}J' \\ \hline\end{array}\,.$$

To solve the problem as a boundary-value one, DE are to be transformed into algebraic ones. According to [6], one can use approximation of state variables by the 3rd order splines, which provide high accuracy for a small number of nodal points on the period. The spline approximation of the coordinates on the grid of n nodes of the period results in an algebraic counterpart of system (6) in the form of a non-linear algebraic equation of the $(m=5n)^{th}$ order

$$H\vec{Y}\left(\vec{X}\right)-\vec{F}\left(\vec{Y},\vec{X}\right)=\vec{0}\,, \tag{7}$$

where H is a $m´m$ sized matrix of transition from the continuous change in the $\vec{y}$-vector coordinates to their nodal values, whose elements are determined by a grid of nodes of the period (6); $\vec{X}=\left(\vec{x}_1,\vec{x}_n\right)^*$, $\vec{F}=\left(\vec{f}_1,\vec{f}_n\right)^*$ are the vectors consisting of nodal values of the respective vectors coordinates.

System (7) of non-linear equations is a discrete reflection of non-linear DE system (6) approximating it on the period T. The unknown variable is the vector $\vec{X}$ made up of the vectors of nodal values of AM circuit currents and rotational speed of the AM rotor, and the Jacobi matrix is

$$W=\left(H-\frac{\partial\vec{F}}{\partial\vec{Y}}\right)\frac{\partial\vec{Y}}{\partial\vec{X}}-\frac{\partial\vec{F}}{\partial\vec{X}}\,.$$

4. Algorithm of computing the characteristics.

System (7) of the mn^{th} order finite equations allows us not only to compute a steady state, but also to study the effect of variation of any coordinate c characterizing this state, for instance, the amplitude or frequency of the supply voltage, moment of inertia, etc. To achieve this, the system is differentiated with respect to this parameter [11], which results in the DE system

$$W\frac{d\vec{X}_c}{dc} = \frac{\partial \vec{F}}{\partial c}\,. \qquad (8)$$

For different coordinates c, DE systems (8) will differ only in the vector of the right parts, which enables solution of the problems of computation of different static characteristics using the same algorithm. Integration of DE system (8) with respect to the variable c as a parameter using the numerical method gives a multi-dimensional static characteristic of the periodic process as a totality of the steady-state modes (vectors $\vec{X}$) corresponding to different values of c in the range from c_1 to c_2. Using the known dynamogram, the algorithm developed to compute characteristics makes it possible to determine steady-state dynamic operation modes of the SRPU permissible with respect to certain indicators.

Changing the oil flow rate requires adjustment of the oil-pumping rate, which is achieved by setting up appropriate speeds of the walking beam oscillation. However, as other calculations show, the crank rotational speed cannot always be changed within the required range, as this can cause the AM overload with respect to both heat and maximum load torque. In order to determine the permissible range of adjustment of the oil pumping rate and formulate the restrictions, it is necessary to carry out a preliminary study, which can only be done by means of mathematical modelling.

As an example, Fig. 3 presents periodic dependencies of the active power, and Fig. 4 demonstrates an effective value of the current computed according to the above-discussed algorithm for different values of the crank rotation frequency, which correspond to the dynamogram for the 4AP160S4У3 (P_n =15kW, U_n =380/220V) motor setting in motion the 7CK8-3,5-4000 sucker rod pump (Fig. 2).

Fig. 5 and Fig. 6 show the static characteristics obtained by processing the computed periodic dependencies of the respective coordinates, which allow determination of the utmost permissible conditions of regulation resulting from the limitations of the motor operation according to the maximum period average value of power (Fig. 5) and current (Fig. 6), as well as the overload capacity with respect to the torque (Fig. 2). As shown in Fig. 5 and Fig. 6, for the dynamogram in Fig. 2, the permissible rotational speed of the AM rotor is the one at which the crank rotational speed does not exceed 10 rpm.

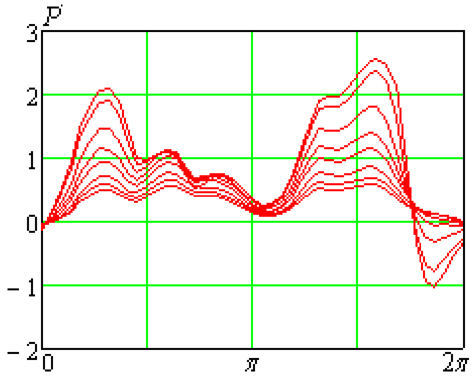

Fig. 3. Relative value of the active power P consumed by the motor versus the angle a for various values of the crank rotational frequency.

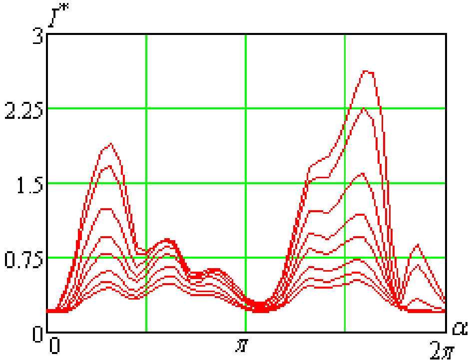

Fig. 4. Relative value of the AM current versus the angle a for various values of the crank rotational frequency.

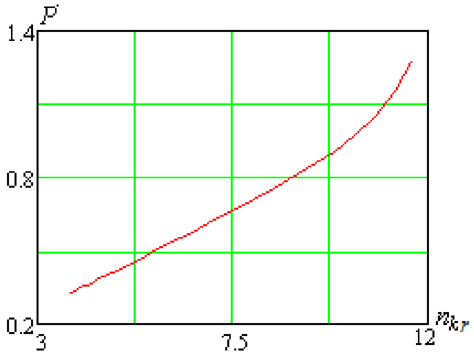

Fig. 5. Relative value of the active power versus the crank rotational frequency.

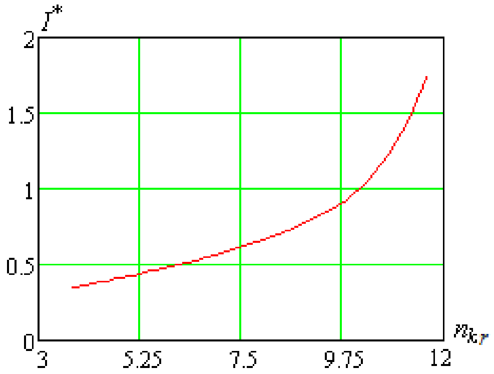

Fig. 6. Relative value of the current versus the crank rotational frequency.

4. Conclusions

The developed computation algorithm enables determination of the walking beam oscillation frequency which corresponds to the continuous operation of SRPU depending on the oil formation flow rate, as well as makes it possible to establish restrictions regarding the range of its regulation determined by the torque and heat overload of the motor. The algorithm relies on the fine mathematical models of the asynchronous motor and sucker rod pump and the method of computation of periodic dependencies of SRPU mode coordinates based on solving a boundary-value problem.

The high-speed software programs designed on the basis of the developed algorithm can be applied for real-time analysis of the SRPU operation, which can be used for creating effective intellectual control systems.

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ВПЛИВ ЧАСТОТИ ГОЙДАНЬ БАЛАНСИРА ШТАНГОВОЇ НАФТОВИДОБУВНОЇ УСТАНОВКИ НА РОБОТУ АСИНХРОННОГО ЕЛЕКТРОПРИВОДУ

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Процес нафтовидобутку нафти потребує постійного моніторингу роботи обладнання свердловин. Для підвищення ефективності роботи нафтових свердловин необхідно встановити оптимальний режим роботи нафтовидобувної установки, за якого відбір рідини відповідає її притоку. Значна кількість свердловин мають невеликий дебіт, а тому працюють у режимі періодичної експлуатації. Для застосування автоматизованих систем керування необхідно мати достовірну інформацію не тільки про стан нафтовидобувного обладнання, але й визначати обмеження, які витікають з умов надійної роботи системи електроприводу. Це можливо здійснити лише на основі математичного моделювання.

Розроблений метод і алгоритм розрахунку режимів роботи штангових глибиннопомпомпових нафтовидобувних установок дає змогу визначати частоту гойдань балансира залежно від зміни дебіту пласта, а також обмеження щодо меж її регулювання, які визначаються перевантаженням двигуна за моментом та нагріванням. В основу алгоритму розрахунку покладено математичні моделі асинхронного двигуна і верстата-гойдалки високого рівня адекватності і метод розрахунку періодичних залежностей координат режиму роботи установки на основі розв’язування крайової задачі.

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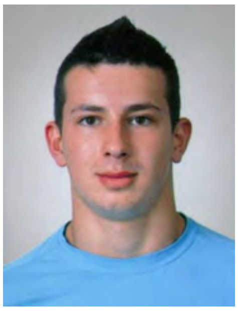

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