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AN INVERSE PROBLEM FOR A GENERALLY DEGENERATE HEAT EQUATION

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We consider an inverse problem for determining a time-dependent coefficient for the heat equation. The coefficient at the higher-order derivative is a product of two functions which depend on time and one of them vanishes at the initial moment. It were considered two cases: weak and strong degeneration. Conditions of existence and uniqueness of solution for the problem are established.

Ключові слова: inverse problem, heat equation, strong and weak degeneration, Schauder fixed-point theorem.

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Introduction

Degenerate parabolic problems arise in a lot fields of natural and social sciences. There are some works dedicated to the inverse problems for partial differential equations degenerating with respect to a spatial variable [1]-[3]. The case of the inverse problem for a weakly degenerate parabolic equation with unknown coefficient which tends to zero as a power function $t^\beta, 0 < \beta < 1$ at the higher-order derivative was investigated in the articles [4]-[6] and when $\beta > 1$ in [7].

In the bounded domain $Q_T \equiv \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider the heat equation

$$u_t = a(t)\psi(t)u_{xx} + f(x,t), \quad (x,t) \in Q_T, \tag{1}$$

with unknown coefficient $a(t) > 0, t \in [0,T]$, initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t), \quad u(h,t) = \mu_2(t), \quad t \in [0,T], \tag{3}$$

and overdetermination condition

$$a(t)\psi(t)u_x(0,t) = \mu_3(t), \quad t \in [0,T]. \tag{4}$$

Suppose that $\psi(t) -$ given monotone increasing function, $\psi(t) > 0, t \in (0,T]$ and $\psi(0) = 0$. It means that the equation (1) is degenerate. Assuming temporally that function $a(t)$ is known, we represent the solution of direct problem (1)-(3) with the aid of Green function in the form

$$u(x,t) = \int_0^h G_1(x,t,\xi,0)\varphi(\xi)d\xi + \int_0^t G_{1\xi}(x,t,0,\tau) \times$$
$$\times a(\tau)\psi(\tau)\mu_1(\tau)d\tau - \int_0^t G_{1\xi}(x,t,h,\tau)a(\tau)\psi(\tau) \times$$
$$\times \mu_2(\tau)d\tau + \int_0^t\int_0^h G_1(x,t,\xi,\tau)f(\xi,\tau)d\xi d\tau, \tag{5}$$

where $G_1(x,t,\xi,\tau)$ is the Green function. It is known that the Green functions for the first ($k=1$) and the second ($k=2$) boundary problems for the equation (1) are defined as follows:

$$G_k(x,t,\xi,\tau) = \frac{1}{2\sqrt{\pi(\theta(t)-\theta(\tau))}} \times$$
$$\times \sum_{n=-\infty}^{\infty}\left(\exp\left(-\frac{(x-\xi+2nh)^2}{4(\theta(t)-\theta(\tau))}\right) + \right.$$
$$\left. +(-1)^k \exp\left(-\frac{(x+\xi+2nh)^2}{4(\theta(t)-\theta(\tau))}\right)\right), \quad k=1,2,$$
$$\theta(t) = \int_0^t a(\tau)\psi(\tau)d\tau. \tag{6}$$

It is easy to see that the following properties of the Green functions are correct:

$$G_{1\xi}(x,t,\xi,\tau) = -G_{2x}(x,t,\xi,\tau),$$
$$G_{2\tau}(x,t,\xi,\tau) = -a(\tau)G_{2\xi\xi}(x,t,\xi,\tau). \tag{7}$$

Suppose that given data satisfies the following conditions:

(A1) $\varphi \in C^1[0,h]$; $\mu_i \in C^1[0,T]$, $i = 1,2$; $\mu_3 \in C[0,T]$; $\psi \in C[0,T]$; $f \in C^{1,0}(\overline{Q}_T)$;

(A2) $\varphi'(x) > 0, x \in [0,h]; f(0,t) - \mu_1'(t) > 0,\ \mu_2'(t) - f(h,t) \geq 0, t \in [0,T], \mu_3(t) > 0, t \in (0,T]$; $f_x(x,t) \geq 0, (x,t) \in Q_T; \psi(t) > 0-$ *monotone increasing function on* $(0,T], \psi(0) = 0$;

(A3) *the compatibility conditions of the zero order are verified:* $\varphi(0) = \mu_1(0), \varphi(h) = \mu_2(0)$.

Integrating by parts and applying the compatibility conditions and (7), find from (5) the derivative $u_x(x,t)$:

$$u_x(x,t) = \int_0^h G_2(x,t,\xi,0)\varphi'(\xi)d\xi + \int_0^t G_2(x,t,0,\tau) \times$$
$$\times(f(0,\tau) - \mu_1'(\tau))d\tau + \int_0^t G_2(x,t,h,\tau)(\mu_2'(\tau) -$$
$$-f(h,\tau))d\tau + \int_0^t \int_0^h G_2(x,t,\xi,\tau)f_\xi(\xi,\tau)d\xi d\tau. \quad (8)$$

Put $x = 0$ in the formula (8) and substitute the result into (4). In such way we obtain the equation with respect to unknown function $a(t)$:

$$a(t) = \frac{\mu_3(t)}{\psi(t)}\left(\int_0^h G_2(0,t,\xi,0)\varphi'(\xi)d\xi +\right.$$
$$+\int_0^t G_2(0,t,0,\tau)(f(0,\tau) - \mu_1'(\tau))d\tau +$$
$$+\int_0^t G_2(0,t,h,\tau)(\mu_2'(\tau) - f(h,\tau))d\tau +$$
$$\left.+\int_0^t\int_0^h G_2(0,t,\xi,\tau)f_\xi(\xi,\tau)d\xi d\tau\right)^{-1}, \quad t \in [0,T]. \ (9)$$

Let study the behavior of the denominator. From the equality

$$\int_0^h G_2(x,t,\xi,\tau)d\xi = 1$$

we obtain the following estimates for the first and forth summands:

$$0 < M_0 = \min_{x\in[0,h]} \varphi'(x) \leq \int_0^h G_2(0,t,\xi,0)\varphi'(\xi)d\xi \leq$$
$$\leq \max_{x\in[0,h]} \varphi'(x) = M_1, \quad (10)$$

$$0 \leq t \min_{(x,t)\in\overline{Q}_T} f_x(x,t) \leq \int_0^t\int_0^h G_2(0,t,\xi,\tau) \times$$
$$\times f_\xi(\xi,\tau)d\xi d\tau \leq t \max_{(x,t)\in\overline{Q}_T} f_x(x,t). \quad (11)$$

To estimate the others summands, denote

$$a_{\max}(t) \equiv \max_{0\leq\tau\leq t} a(\tau), \quad a_{\min}(t) \equiv \min_{0\leq\tau\leq t} a(\tau). \quad (12)$$

Extracting from the series the addend which corresponds to $n = 0$, we have for the second summand from (9)

$$\int_0^t G_2(0,t,0,\tau)(f(0,\tau) - \mu_1'(\tau))d\tau =$$
$$= \int_0^t \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\pi(\theta(t)-\theta(\tau))}}d\tau + 2\int_0^t \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\pi(\theta(t)-\theta(\tau))}} \times$$
$$\times\sum_{n=1}^{\infty}\exp\left(-\frac{n^2h^2}{\theta(t)-\theta(\tau)}\right)d\tau \leq C_1\left(\frac{1}{\sqrt{\pi a_{\min}(t)}} \times\right.$$
$$\times\int_0^t\left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau + 2\int_0^t \frac{1}{\sqrt{\pi(\theta(t)-\theta(\tau))}} \times$$
$$\left.\times\sum_{n=1}^{\infty}\exp\left(-\frac{n^2h^2}{\theta(t)-\theta(\tau)}\right)d\tau\right). \quad (13)$$

For the third summand we obtain

$$\int_0^t G_2(0,t,h,\tau)(\mu_2'(\tau) - f(h,\tau))d\tau \leq$$
$$\leq C_2\int_0^t \frac{1}{\sqrt{\theta(t)-\theta(\tau)}} \times$$
$$\times\sum_{n=-\infty}^{\infty}\exp\left(-\frac{h^2(2n-1)^2}{4(\theta(t)-\theta(\tau))}\right)d\tau. \quad (14)$$

Taking into account the known inequality [8, c. 13] $\frac{1}{\sqrt{z}}\sum_{n=1}^{\infty}\exp\left(-\frac{n^2h^2}{z}\right) \leq K^*, \forall z \in [0,+\infty)$, we conclude that the last summand from (13) and expression in (14) are bounded. We will distinguish two cases of the degeneration.

Definition. The degeneration is called weak if for $t \to 0$ the expression $\int_0^t\left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau$ tends to zero, and it is called strong if the named expression tends to infinity when t tends to zero.

I. Weak degeneration

Consider the case of the weak degeneration. As a solution of the problem (1) - (4) we define a pair of function $(a(t), u(x,t))$ from the space $C[0,T] \times C^{2,1}(Q_T)\bigcap C^{1,0}(\overline{Q}_T), a(t) > 0, t \in [0,T]$, which verify the equation (1) and conditions (2) - (4).

To prove the existence of solution of the problem (1) - (4), we apply the Schauder fixed-point theorem. For this we establish apriori estimates of solution of the equation (9). We start by the estimation of function $a(t)$ from above. For this we need the estimate of $u_x(0,t)$ from below. Taking into account (11), (13), (14) we conclude that the second, third and forth summands in the expression (9) are positive and tend to zero when $t \to 0$. At the same time, the inequality (10) holds for the first summand.

Hence, we have

$$u_x(0,t) \geq M_0, \quad t \in [0,T]. \tag{15}$$

Suppose that the following condition is satistied:

(A4) *there exists the finite positive limit* $\lim\limits_{t\to+0} \dfrac{\mu_3(t)}{\psi(t)}$.

Substituting (15) into (9) and taking into account the condition (**A4**), we obtain

$$a(t) \leq \frac{\mu_3(t)}{\psi(t)M_0} \leq A_1 < \infty, \quad t \in [0,T]. \tag{16}$$

Estimate $u_x(0,t)$ from above. Using (10), (11), (13), (14) we have

$$u_x(0,t) \leq C_3 + C_4 \int\limits_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}}. \tag{17}$$

Setting (17) in the equation with respect to $a(t)$ and applying (12) and **(A4)**, we get

$$a(t) \geq \frac{\mu_3(t)}{\psi(t)\left(C_3 + C_4 \int\limits_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}}\right)} \geq$$
$$\geq \frac{C_5}{C_3 + C_4 \int\limits_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}}} \geq$$
$$\geq \frac{C_5}{C_3 + \frac{C_4}{\sqrt{a_{\min}(t)}} \int\limits_0^t \left(\int\limits_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau}.$$

Using the definition of weak degeneration, we obtain

$$a(t) \geq \frac{C_5}{C_3 + \frac{C_6}{\sqrt{a_{\min}(t)}}}$$

or

$$a_{\min}(t) \geq \left(\frac{2C_5}{\sqrt{C_6^2 + 4C_3C_5} + C_6}\right)^2 = A_0 > 0. \tag{18}$$

Write the equation (9) as operator equation $a(t) = Pa(t)$ with respect to $a(t)$ where operator P is defined by the right-hand side of the equation (9). Define the set $\mathcal{N} = \{a(t) \in C[0,T] : A_0 \leq a(t) \leq A_1\}$. According to obtained estimates (16), (18), the operator P maps the set $\mathcal{N}$ into itself. The proof of the compactness of operator P on $\mathcal{N}$ is analogous to the case of weak power degeneration for the heat equation [4].

Thus, the following existence theorem is established.

Theorem 1. *Suppose that* $\lim\limits_{t\to+0} \int\limits_0^t \left(\int\limits_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau = 0$. *If the conditions* (**A1**) *-* (**A4**) *are satisfied, then the solution of the problem (1) - (4) exists for* $x \in [0,h], t \in [0,T]$.

Let prove the uniqueness of solution of the problem (1) - (4). Suppose that there exist two solutions $(a_i(t), u_i(x,t)), i = 1,2$. Denote the difference of the solutions by $a(t) \equiv a_1(t) - a_2(t), u(x,t) \equiv u_1(x,t) - u_2(x,t)$. For these functions we get the following problem:

$$u_t = a_1(t)\psi(t)u_{xx} + a(t)\psi(t)u_{2xx}, \quad (x,t) \in Q_T, \tag{19}$$
$$u(x,0) = 0, \quad x \in [0,h], \tag{20}$$
$$u(0,t) = u(h,t) = 0, \quad t \in [0,T], \tag{21}$$
$$a_1(t)u_x(0,t) = -a(t)u_{2x}(0,t), \quad t \in [0,T]. \tag{22}$$

Introduce the Green functions $G_1^i(x,t,\xi,\tau)$ for the equations $u_t = a_i(t)\psi(t)u_{xx}, i = 1,2$ with boundary conditions (21). Using $G_1^1(x,t,\xi,\tau)$ we put the solution of the problem (19) - (21) as follows:

$$u(x,t) = \int\limits_0^t \int\limits_0^h G_1^1(x,t,\xi,\tau)a(\tau)\psi(\tau)u_{2\xi\xi}(\xi,\tau)d\xi d\tau. \tag{23}$$

Calculating the derivative of (23) and substituting it into (22), we obtain the integral equation with respect to $a(t)$:

$$a(t) = \int\limits_0^t K(t,\tau)a(\tau)d\tau, \tag{24}$$

where

$$K(t,\tau) = -a_1(t)a_2(t)\frac{\psi(t)}{\mu_3(t)} \int\limits_0^h G_{1x}^1(0,t,\xi,\tau) \times$$
$$\times \psi(\tau)u_{2\xi\xi}(\xi,\tau)d\xi.$$

Let prove the integrability of the kernel $K(t,\tau)$. Put the solution $u_2(x,t)$ under the form (5) and calculate the second derivative:

$$u_{2xx}(x,t) = \int\limits_0^h G_1^2(x,t,\xi,0)\varphi''(\xi)d\xi + \int\limits_0^t G_{1\xi}^2(x,t,0,\tau) \times$$
$$\times(\mu_1'(\tau) - f(0,\tau))d\tau + \int\limits_0^t G_{1\xi}^2(x,t,h,\tau)(f(h,\tau) -$$
$$-\mu_2'(\tau))d\tau - \int\limits_0^t \int\limits_0^h G_{1\xi}^2(x,t,\xi,\tau)f_\xi(\xi,\tau)d\xi d\tau. \tag{25}$$

Evaluate every summand of this expression. For the first one we have

$$\left|\int_0^h G_1^2(x,t,\xi,0)\varphi''(\xi)d\xi\right| \le \max_{x\in[0,h]} |\varphi''(x)|.$$

To estimate the second summand from (25) we use the explicit form of the Green function from (6):

$$R \equiv \left|\int_0^t G_{1\xi}^2(x,t,0,\tau)(\mu_1'(\tau) - f(0,\tau))d\tau\right| \le C_7 \times$$
$$\times\Bigg(\int_0^t \frac{x}{(\theta_2(t)-\theta_2(\tau))^{3/2}}\exp\left(-\frac{x^2}{4(\theta_2(t)-\theta_2(\tau))}\right)d\tau +$$
$$+\int_0^t \frac{1}{(\theta_2(t)-\theta_2(\tau))^{3/2}}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}|x+2nh| \times$$
$$\times\exp\left(-\frac{(x+2nh)^2}{4(\theta_2(t)-\theta_2(\tau))}\right)d\tau\Bigg) \equiv R_1 + R_2. \qquad (26)$$

Denote $\int_0^t \psi(\sigma)d\sigma = \theta_0(t)$ and consider R_1 applying the change of variable $z = \frac{\theta_0(\tau)}{\theta_0(t)}$:

$$R_1 \le C_8 \int_0^t x\left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-3/2} \times$$
$$\times\exp\left(-\frac{x^2}{C_9\int_\tau^t \psi(\sigma)d\sigma}\right)d\tau \le$$
$$\le \frac{C_8}{\theta_0^{1/2}(t)}\int_0^1 \frac{x}{z^{3/2}\psi(\theta_0^{-1}((1-z)\theta_0(t)))} \times$$
$$\times\exp\left(-\frac{x^2}{C_9\theta_0(t)z}\right)dz.$$

Realize the change of variable $\sigma = \frac{x}{\sqrt{C_9\theta_0(t)z}}$:

$$R_1 \le C_{10}\int_{\frac{x}{\sqrt{C_9\theta_0(t)}}}^{\infty} \frac{e^{-\sigma^2}d\sigma}{\psi\left(\theta_0^{-1}\left(\theta_0(t) - \frac{x^2}{C_9\sigma^2}\right)\right)}.$$

In the obtained integral we decompose the interval of integration on the parts $\left[\frac{x}{\sqrt{C_9\theta_0(t)}}, \frac{2x}{\sqrt{C_9\theta_0(t)}}\right]$ and $\left[\frac{2x}{\sqrt{C_9\theta_0(t)}}, \infty\right)$. Evaluate the following summand:

$$R_{11} \equiv C_{10}\int_{\frac{2x}{\sqrt{C_9\theta_0(t)}}}^{\infty} \frac{e^{-\sigma^2}d\sigma}{\psi\left(\theta_0^{-1}\left(\theta_0(t) - \frac{x^2}{C_9\sigma^2}\right)\right)} \le$$
$$\le \frac{C_{10}}{\psi(\theta_0^{-1}(\frac{3}{4}\theta_0(t)))}\int_{\frac{2x}{\sqrt{C_9\theta_0(t)}}}^{\infty} e^{-\sigma^2}d\sigma \le$$
$$\le \frac{C_{11}}{\psi(\theta_0^{-1}(\frac{3}{4}\theta_0(t)))}. \qquad (27)$$

For the second summand we use the change of variable $z = \theta_0(t) - \frac{x^2}{C_9\sigma^2}$:

$$R_{12} \equiv C_{10}\int_{\frac{x}{\sqrt{C_9\theta_0(t)}}}^{\frac{2x}{\sqrt{C_9\theta_0(t)}}} \frac{e^{-\sigma^2}d\sigma}{\psi\left(\theta_0^{-1}\left(\theta_0(t) - \frac{x^2}{C_9\sigma^2}\right)\right)} \le$$
$$\le C_{10}\exp\left(-\frac{x^2}{C_9\theta_0(t)}\right) \times$$
$$\times\int_{\frac{x}{\sqrt{C_9\theta_0(t)}}}^{\frac{2x}{\sqrt{C_9\theta_0(t)}}} \frac{d\sigma}{\psi\left(\theta_0^{-1}\left(\theta_0(t) - \frac{x^2}{C_9\sigma^2}\right)\right)} \le$$
$$\le C_{12}x\exp\left(-\frac{x^2}{C_9\theta_0(t)}\right) \times$$
$$\times\int_0^{\frac{3}{4}\theta_0(t)} \frac{dz}{(\theta_0(t)-z)^{3/2}\psi(\theta_0^{-1}(z))}.$$

Let $z = \theta_0(\sigma)$ and estimate the denominator:

$$R_{12} \le C_{13}x\exp\left(-\frac{x^2}{C_9\theta_0(t)}\right) \times$$
$$\times\int_0^{\theta_0^{-1}(3/4\theta_0(t))} \frac{\psi(\sigma)d\sigma}{(\theta_0(t)-\theta_0(\sigma))^{3/2}\psi(\theta_0^{-1}(\theta_0(\sigma)))} \le$$
$$\le C_{13}x\exp\left(-\frac{x^2}{C_9\theta_0(t)}\right) \times$$
$$\times\int_0^{\theta_0^{-1}(3/4\theta_0(t))} \frac{d\sigma}{(\theta_0(t)-\frac{3}{4}\theta_0(t))^{3/2}} \le$$
$$\le \frac{C_{14}\theta_0^{-1}(3/4\theta_0(t))}{\theta_0(t)} \le C_{14}t\left(\int_0^t \psi(\sigma)d\sigma\right)^{-1}.$$

We obtained this estimate with the aid of the inequality $x^p \exp(-qx^2) \le M_{p,q} < \infty, x \in [0,\infty), p \ge 0, q > 0$.

Finally, we have for R the following estimation:

$$R \le C_{14} t \left(\int_0^t \psi(\sigma)d\sigma\right)^{-1} + \frac{C_{11}}{\psi(\theta_0^{-1}(\frac{3}{4}\theta_0(t)))} + C_{15}. \quad (28)$$

The others summands in $u_{2xx}(x,t)$ are evaluated by the same way. Hence, we find

$$|u_{2xx}(x,t)| \le C_{16} t \left(\int_0^t \psi(\sigma)d\sigma\right)^{-1} + \\ + \frac{C_{17}}{\psi(\theta_0^{-1}(\frac{3}{4}\theta_0(t)))} + C_{18}. \quad (29)$$

Substituting (29) into the kernel $K(t,\tau)$, we come to the inequality

$$|K(t,\tau)| \le C_{19} \left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2}. \quad (30)$$

From this, it follows that the singularity of the kernel of the equation (24) is integrable. Hence, the Volterra integral equation of the second kind (24) has only trivial solution $a(t) \equiv 0$, and therefore $u(x,t) \equiv 0, (x,t) \in \overline{Q}_T$. Thus, the following uniqueness theorem is proved.

Theorem 2. *Suppose that the conditions* **(A4)** *and*

(A5) $\varphi \in C^2[0,h]; \mu_i \in C^1[0,T], i = 1,2; \mu_3 \in C[0,T]; \psi \in C[0,T]; f \in C^{1,0}(\overline{Q}_T); \mu_3(t) > 0, \psi(t) > 0, t \in (0,T], \psi(0) = 0; \lim_{t\to+0} \int_0^t \left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau = 0.$

are fulfilled.

Then the solution of the problem (1)-(4) is unique.

II. Strong degeneration

Consider the strong degeneration case. As a solution of the problem (1) - (4) we define a pair of functions $(a(t), u(x,t))$ from the space $C[0,T] \times C^{2,1}(Q_T) \bigcap C(\overline{Q}_T), u_x(0,t) \in C(0,T], a(t) > 0, t \in [0,T]$, which verify the equation (1) and the conditions (2) - (4). Taking into account the definition, from (10), (11), (13), (14) we conclude that all summands of the derivative $u_x(0,t)$, except one, are bounded. Integral $\int_0^t \frac{f(0,\tau)-\mu_1'(\tau)}{\sqrt{\pi(\theta(t)-\theta(\tau))}} d\tau$ tends to infinity when $t \to +0$.

Then the estimate of $u_x(0,t)$ from below takes form

$$u_x(0,t) \ge \frac{1}{\sqrt{\pi}} \int_0^t \frac{f(0,\tau)-\mu_1'(\tau)}{\sqrt{\theta(t)-\theta(\tau)}} d\tau. \quad (31)$$

Substitute (31) into (9) and use (12), after what we obtain

$$a(t) \le \frac{\sqrt{\pi a_{\max}(t)}\mu_3(t)}{\psi(t) \int_0^t (f(0,\tau)-\mu_1'(\tau)) \left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau}. \quad (32)$$

Denote

$$H(t) \equiv \frac{\sqrt{\pi}\mu_3(t)}{\psi(t) \int_0^t (f(0,\tau)-\mu_1'(\tau)) \left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau}. \quad (33)$$

From the conditions **(A1)**, **(A2)**, it follows that the function $H(t)$ is continuous and positive on the segment $(0,T]$. Assume that the following condition is fulfilled:

(A6) *there exists the finite positive limit* $\lim_{t\to+0} \frac{\mu_3(t)}{\sqrt{\psi(t)t}} = M.$

Prove that the function $H(t)$ tends to a finite positive limit when $t \to +0$. Applying the mean value theorem and the condition **(A6)**, we have

$$\lim_{t\to+0} H(t) = \lim_{t\to+0} \frac{\sqrt{\pi\psi(t^*)}\mu_3(t)}{\psi(t)(f(0,t^*)-\mu_1'(t^*)) \int_0^t \frac{d\tau}{\sqrt{t-\tau}}} = \\ = \frac{\sqrt{\pi}M}{2(f(0,0)-\mu_1'(0))},$$

where t^* is some point from the segment $[0,T]$.

Using the definition of $H(t)$, from (32) we obtain the estimate

$$a_{\max}(t) \le H_{\max}(t)\sqrt{a_{\max}(t)} \quad \text{or} \quad a_{\max}(t) \le H_{\max}^2(t), \quad (34)$$

where $H_{\max}(t) \equiv \max_{0\le\tau\le t} H(\tau)$. This means that we have the estimate of $a(t)$ from above

$$a(t) \le A_1 < \infty, \quad t \in [0,T]. \quad (35)$$

To evaluate $u_x(0,t)$ from above we use (10), (11), (13), (14). Then

$$u_x(0,t) \le C_{20} + \frac{1}{\sqrt{\pi}} \int_0^t \frac{f(0,\tau)-\mu_1'(\tau)}{\sqrt{\theta(t)-\theta(\tau)}} d\tau. \quad (36)$$

Substituting (36) into (9) and applying (12), we find

$$a(t) \ge \frac{\mu_3(t)}{\psi(t)} \sqrt{\pi a_{\min}(t)} \Bigg(C_{21} + \int_0^t (f(0,\tau)-\mu_1'(\tau)) \times \\ \times \left(\int_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau \Bigg)^{-1} \ge \frac{\sqrt{a_{\min}(t)}}{\frac{C_{21}\psi(t)}{\sqrt{\pi}\mu_3(t)} + \frac{1}{H(t)}} \ge \\ \ge \frac{\sqrt{a_{\min}(t)}H(t)}{\frac{C_{21}\psi(t)H(t)}{\sqrt{\pi}\mu_3(t)} + 1}. \quad (37)$$

Consider the fraction in the denominator from (37). Applying the mean value theorem we obtain $\int\limits_0^t \left(\int\limits_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau = 2\sqrt{\frac{t}{\psi(t^*)}}$, where $t^* \in [0,T]$. It follows from the strong degeneration definition that the expression $\sqrt{\frac{\psi(t)}{t}}$ tends to zero when $t \to +0$. Then from **(A6)** we have $\frac{C_{21}\psi(t)H(t)}{\sqrt{\pi}\mu_3(t)} \le C_{22}\sqrt{\frac{\psi(t)}{t}}$. Applying this in (37), we obtain

$$a_{\min}(t) \ge \frac{\sqrt{a_{\min}(t)}H_{\min}(t)}{C_{22}\sqrt{\frac{\psi(t)}{t}}+1} \quad \text{or}$$

$$a_{\min}(t) \ge \frac{H^2_{\min}(t)}{\left(C_{22}\sqrt{\frac{\psi(t)}{t}}+1\right)^2}, \quad t \in [0,T], \quad (38)$$

where $H_{\min}(t) \equiv \min\limits_{0\le\tau\le t} H(\tau)$. Consequently, we find the estimation of $a(t)$ from below

$$a(t) \ge A_0 > 0, \quad t \in [0,T]. \quad (39)$$

Hence, we have established the apriori estimates of solutions of the equation (9).

Put the equation with respect to $a(t)$ into the form

$$a(t) = \frac{\widetilde{\mu}_3(t)}{\widetilde{v}(0,t)} \quad \text{or} \quad a(t) = Pa(t), \quad t \in [0,T], \ (40)$$

where $\widetilde{\mu}_3(t) = \frac{\mu_3(t)}{\sqrt{t\psi(t)}}$, $\widetilde{v}(0,t) = u_x(0,t)\sqrt{\frac{\psi(t)}{t}}$. Define the set $\mathcal{N} = \{a(t) \in C[0,T] : A_0 \le a(t) \le A_1\}$. According to obtained estimates (35), (39), the operator P maps the set $\mathcal{N}$ into itself. Let show that P is compact on $\mathcal{N}$. Following the Ascolli-Arcella theorem, it is necessary to establish that for all $\varepsilon > 0$ there exists such $\delta > 0$, that

$$|P(t_2) - P(t_1)| < \varepsilon, \quad \forall\, a(t) \in \mathcal{N},$$
$$\text{when} \quad |t_2 - t_1| < \delta, \quad t_1, t_2 \in [0,T].$$

We will show how to verify this inequality, on the example of the following expression which enters to the integral operator P :

$$K \equiv \left| \sqrt{\frac{\psi(t_2)}{t_2}} \int\limits_0^{t_2} (f(0,\tau) - \mu_1'(\tau))G_2(0,t_2,0,\tau)d\tau - \right.$$
$$\left. -\sqrt{\frac{\psi(t_1)}{t_1}} \int\limits_0^{t_1} (f(0,\tau) - \mu_1'(\tau))G_2(0,t_1,0,\tau)d\tau \right|.$$

Suppose that $t_i, i = 1,2$ are sufficiently small. Consider the integral

$$\widehat{K} \equiv \sqrt{\frac{\psi(t)}{t}} \int\limits_0^t (f(0,\tau) - \mu_1'(\tau))G_2(0,t,0,\tau)d\tau =$$
$$= \left(\int\limits_0^t \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\theta(t)-\theta(\tau)}} d\tau + 2\int\limits_0^t \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\theta(t)-\theta(\tau)}} \times \right.$$
$$\left. \times \sum_{n=1}^{\infty} \exp\left(-\frac{n^2h^2}{\theta(t)-\theta(\tau)}\right) d\tau \right) \sqrt{\frac{\psi(t)}{\pi t}} \equiv \widehat{K}_1 + \widehat{K}_2.$$

Using the notation (12), boundedness of integrand in $\widehat{K}_2$, and tendency of $\sqrt{\frac{\psi(t)}{t}}$ to zero when $t \to +0$, we obtain that $\widehat{K}_2$ tends to zero as $t \to +0$. Consider $\widehat{K}_1$ applying the mean value theorem:

$$\widehat{K}_1 = C_{23}\sqrt{\frac{\psi(t)}{t}} \int\limits_0^t (f(0,\tau) - \mu_1'(\tau)) \times$$
$$\times \left(\int\limits_\tau^t \psi(\sigma)d\sigma\right)^{-1/2} d\tau = C_{23}\sqrt{\frac{\psi(t)}{t}} \frac{(f(0,\widetilde{t}) - \mu_1'(\widetilde{t}))}{\sqrt{\psi(\widetilde{t})}} \times$$
$$\times \int\limits_0^t \frac{d\tau}{\sqrt{t-\tau}} = 2C_{23}\sqrt{\frac{\psi(t)}{\psi(\widetilde{t})}}(f(0,\widetilde{t}) - \mu_1'(\widetilde{t})),$$

where $\widetilde{t} \in [0,T]$. Denote $\lim\limits_{t\to+0} \widehat{K}_1 = \varkappa_0$. Then, returning to K, we get

$$K \le$$
$$\le \left| \sqrt{\frac{\psi(t_2)}{t_2}} \int\limits_0^{t_2} (f(0,\tau) - \mu_1'(\tau))G_2(0,t_2,0,\tau)d\tau - \varkappa_0 \right| +$$
$$+ \left| \sqrt{\frac{\psi(t_1)}{t_1}} \int\limits_0^{t_1} (f(0,\tau) - \mu_1'(\tau))G_2(0,t_1,0,\tau)d\tau - \varkappa_0 \right|.$$

There exists such value t_*, that for $0 < t_i < t_* \le T, i = 1,2$, the following inequalities are verified:

$$\left| \sqrt{\frac{\psi(t_i)}{t_i}} \int\limits_0^{t_i} (f(0,\tau) - \mu_1'(\tau))G_2(0,t_i,0,\tau)d\tau - \varkappa_0 \right| < \frac{\varepsilon}{2}.$$

Hence, $K < \varepsilon$ when $0 < t_i < t_*, i = 1,2$.

Consider the expression K in the case when $t_* <$

$t_1 < t_2$:

$$K \le \left| \sqrt{\frac{\psi(t_2)}{t_2}} - \sqrt{\frac{\psi(t_1)}{t_1}} \right| \int_0^{t_2} \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\pi(\theta(t_2) - \theta(\tau))}} \times$$
$$\times \sum_{n=-\infty}^{\infty} \exp\left(-\frac{n^2h^2}{\theta(t_2) - \theta(\tau)}\right) d\tau + \sqrt{\frac{\psi(t_1)}{\pi t_1}} \times$$
$$\times \int_0^{t_1} (f(0,\tau) - \mu_1'(\tau)) \sum_{n=-\infty}^{\infty} \left| \frac{\exp\left(-\frac{n^2h^2}{\theta(t_2)-\theta(\tau)}\right)}{\sqrt{\theta(t_2) - \theta(\tau)}} - \right.$$
$$\left. - \frac{\exp\left(-\frac{n^2h^2}{\theta(t_1)-\theta(\tau)}\right)}{\sqrt{\theta(t_1) - \theta(\tau)}} \right| d\tau + \int_{t_1}^{t_2} \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{\theta(t_2) - \theta(\tau)}} \times$$
$$\times \sum_{n=-\infty}^{\infty} \exp\left(-\frac{n^2h^2}{\theta(t_2) - \theta(\tau)}\right) d\tau \sqrt{\frac{\psi(t_1)}{\pi t_1}} \equiv$$
$$\equiv K_1 + K_2 + K_3. \qquad (41)$$

The integrand of K_3 has integrable singularity, thus $K_3 \le C_{24}\sqrt{t_1 - t_2}$. For K_1 we have

$$K_1 \le C_{25} \left| \sqrt{\frac{\psi(t_2)}{t_2}} - \sqrt{\frac{\psi(t_1)}{t_1}} \right| \left(\int_0^{t_2} \frac{d\tau}{\sqrt{\theta(t_2) - \theta(\tau)}} + \right.$$
$$\left. +2 \int_0^{t_2} \frac{1}{\sqrt{\theta(t_2) - \theta(\tau)}} \sum_{n=1}^{\infty} \exp\left(-\frac{n^2h^2}{\theta(t_2) - \theta(\tau)}\right) d\tau \right) \le$$
$$\le C_{26} \left| 1 - \sqrt{\frac{\psi(t_1)t_2}{\psi(t_2)t_1}} \right| \left(\sqrt{\frac{\psi(t_2)}{A_0\psi(t^*)}} + \sqrt{\psi(t_2)t_2} \right).$$

For all $\varepsilon > 0$ there exists $\delta > 0$ that $K_1 < \varepsilon$ when $|t_2 - t_1| < \delta$. Detaching from the series in K_2 the summand which corresponds to $n = 0$, we obtain

$$K_2 \le C_{27} \left(\int_0^{t_1} \left| \frac{1}{\sqrt{\theta(t_2) - \theta(\tau)}} - \frac{1}{\sqrt{\theta(t_1) - \theta(\tau)}} \right| d\tau + \right.$$
$$+2 \int_0^{t_1} \sum_{n=1}^{\infty} \left| \frac{\exp\left(-\frac{n^2h^2}{\theta(t_2)-\theta(\tau)}\right)}{\sqrt{\theta(t_2) - \theta(\tau)}} - \right.$$
$$\left. \left. - \frac{\exp\left(-\frac{n^2h^2}{\theta(t_1)-\theta(\tau)}\right)}{\sqrt{\theta(t_1) - \theta(\tau)}} \right| d\tau \right) \sqrt{\frac{\psi(t_1)}{\pi t_1}} \equiv K_{21} + K_{22}.$$

Put K_{22} into the form

$$K_{22} = 2C_{27} \sqrt{\frac{\psi(t_1)}{t_1}} \times$$
$$\times \int_0^{t_1} \sum_{n=1}^{\infty} \left| \int_{\theta(t_1)-\theta(\tau)}^{\theta(t_2)-\theta(\tau)} \frac{d}{dz} \left(\frac{1}{\sqrt{z}} \exp\left(-\frac{n^2h^2}{z}\right) \right) dz \right| d\tau \le$$
$$\le C_{28} \sqrt{\psi(t_1)t_1} \int_{t_1}^{t_2} \psi(\sigma) d\sigma.$$

There exists such $\delta > 0$ that $K_{22} < \varepsilon$ when $|t_2 - t_1| < \delta$. Consider the expression

$$\frac{1}{\sqrt{\theta(t_1) - \theta(\tau)}} - \frac{1}{\sqrt{\theta(t_2) - \theta(\tau)}} = \frac{\theta(t_2) - \theta(t_1)}{\sqrt{\theta(t_2) - \theta(\tau)}} \times$$
$$\times \frac{1}{\sqrt{\theta(t_1) - \theta(\tau)}(\sqrt{\theta(t_1) - \theta(\tau)} + \sqrt{\theta(t_2) - \theta(\tau)})} =$$
$$= \frac{\theta(t_2) - \theta(t_1)}{\theta(t_1)\sqrt{\theta(t_2)\left(1 - \frac{\theta(\tau)}{\theta(t_1)}\right)\left(1 - \frac{\theta(\tau)}{\theta(t_2)}\right)}} \times$$
$$\times \frac{1}{\left(\sqrt{1 - \frac{\theta(\tau)}{\theta(t_1)}} + \sqrt{\frac{\theta(t_2)}{\theta(t_1)}\left(1 - \frac{\theta(\tau)}{\theta(t_2)}\right)}\right)}.$$

Taking into account that the function $\frac{1}{t}\theta(t)$ is increasing and $\frac{\theta(\tau)t_i}{\theta(t_i)} \le \tau, \tau \le t_i, i = 1, 2$, we can write for K_{21}

$$K_{21} \le C_{27} \sqrt{\frac{\psi(t_1)}{t_1}} \frac{(\theta(t_2) - \theta(t_1))\sqrt{t_1t_2}}{\theta(t_1)\sqrt{\theta(t_2)}} \times$$
$$\times \int_0^{t_1} \frac{d\tau}{\sqrt{(t_1 - \tau)(t_2 - \tau)}\left(\sqrt{\frac{t_1 - \tau}{t_1}} + \sqrt{\frac{t_2 - \tau}{t_2}}\right)} \le$$
$$\le \frac{C_{27}\sqrt{\psi(t_1)}t_2}{\theta(t_1)\sqrt{\theta(t_2)}} \int_0^{t_1} \left(\frac{1}{\sqrt{t_1 - \tau}} - \frac{1}{\sqrt{t_2 - \tau}} \right) d\tau =$$
$$= \frac{C_{29}t_2\sqrt{\psi(t_1)}}{\theta(t_1)\sqrt{\theta(t_2)}} (\sqrt{t_1} - \sqrt{t_2} + \sqrt{t_2 - t_1}).$$

From this, it is easy to see that $\lim\limits_{t_1 \to t_2} K_{21} = 0$.

The proof of compactness of the others summands of the integral operator P is realized by the analogous way. Thus, the operator P is compact on the set $\mathcal{N}$. According to Schauder fixed-point theorem there exists a solution of the problem (1) - (4) with appropriate smoothness. Hence, the existence of solution for the problem (1) - (4) in the case of strong degeneration is proved.

Let prove the uniqueness of solution for the problem (1) - (4). Supposing the existence of two solutions for the problem (1) - (4), we get the problem (19) - (22) for its differences. Write the equation (22) under the form

$$a(t) = -a_1(t)a_2(t)\frac{u_x(0,t)\psi(t)}{\mu_3(t)}, \quad t \in [0, T]. \qquad (42)$$

We will realize the proof of uniqueness by evaluating $a(t)$ from the equation (42). Consider for example one of the summands of $u_x(0,t) \equiv u_{2x}(0,t) - u_{1x}(0,t)$. Denote

$$I \equiv \frac{1}{\sqrt{\pi}} \int_0^t (f(0,\tau) - \mu_1'(\tau)) \left(\frac{1}{\sqrt{\theta_2(t)-\theta_2(\tau)}} - \right.$$

$$\left. - \frac{1}{\sqrt{\theta_1(t)-\theta_1(\tau)}} \right) d\tau + \frac{2}{\sqrt{\pi}} \int_0^t (f(0,\tau) - \mu_1'(\tau)) \times$$

$$\times \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{\theta_2(t)-\theta_2(\tau)}} \exp\left(-\frac{n^2h^2}{\theta_2(t)-\theta_2(\tau)} \right) - \right.$$

$$\left. - \frac{1}{\sqrt{\theta_1(t)-\theta_1(\tau)}} \exp\left(-\frac{n^2h^2}{\theta_1(t)-\theta_1(\tau)} \right) \right) d\tau \equiv I_1 + I_2.$$

Applying the estimates (38), we have

$$|\theta_1(t) - \theta_1(\tau) - \theta_2(t) + \theta_2(\tau)| \le$$

$$\le \left| \int_\tau^t (a_1(\sigma) - a_2(\sigma))\psi(\sigma)d\sigma \right| \le \widetilde{a}_{\max}(t) \int_\tau^t \psi(\sigma)d\sigma,$$

$$\theta_i(t) - \theta_i(\tau) = \int_\tau^t a_i(\sigma)\psi(\sigma)d\sigma \ge$$

$$\ge \frac{H^2_{\min}(t)}{\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^2} \int_\tau^t \psi(\sigma)d\sigma, \quad i = 1, 2, \quad (43)$$

where $\widetilde{a}_{\max}(t) \equiv \max\limits_{0 \le \tau \le t} |a_1(\tau) - a_2(\tau)|$. Then write for I_2

$$|I_2| \le \frac{2}{\sqrt{\pi}} \int_0^t (f(0,\tau) - \mu_1'(\tau)) \times$$

$$\times \left| \int_{\theta_1(t)-\theta_1(\tau)}^{\theta_2(t)-\theta_2(\tau)} \frac{d}{dz} \left(\frac{1}{\sqrt{z}} \sum_{n=1}^{\infty} \exp\left(-\frac{n^2h^2}{z} \right) \right) dz \right| d\tau.$$

Taking into account the boundedness of integrand and inequality (43), we obtain the estimate

$$|I_2| \le \quad C_{30} \int_0^t |\theta_1(t) - \theta_1(\tau) - \theta_2(t) + \theta_2(\tau)| d\tau \le$$

$$\le F(t)\widetilde{a}_{\max}(t),$$

where $F(t) = \int_0^t d\tau \int_\tau^t \psi(\sigma)d\sigma$. Put I_1 under the form

$$I_1 = \frac{1}{\sqrt{\pi}} \int_0^t \frac{f(0,\tau) - \mu_1'(\tau)}{\sqrt{(\theta_2(t)-\theta_2(\tau))(\theta_1(t)-\theta_1(\tau))}} \times$$

$$\times \frac{(\theta_1(t) - \theta_1(\tau) - \theta_2(t) + \theta_2(\tau))d\tau}{\sqrt{\theta_2(t)-\theta_2(\tau)} + \sqrt{\theta_1(t)-\theta_1(\tau)}}.$$

Using (43) and definition of the function $H(t)$, we get for I_1

$$|I_1| \le \frac{\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^3}{2\sqrt{\pi} H^3_{\min}(t)} \widetilde{a}_{\max}(t) \times$$

$$\times \int_0^t (f(0,\tau) - \mu_1'(\tau)) \left(\int_\tau^t \psi(\sigma d\sigma) \right)^{-1/2} d\tau \le$$

$$\le \frac{\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^3 \mu_3(t)}{2H^4_{\min}(t)\psi(t)} \widetilde{a}_{\max}(t).$$

Others summands in the expression $u_x(0,t)$ are evaluated as I. Then we have from (42)

$$\widetilde{a}_{\max}(t) \le \frac{H^4_{\max}(t)\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^3}{2H^4_{\min}(t)} \widetilde{a}_{\max}(t) +$$

$$+ F^*(t)\widetilde{a}_{\max}(t), \quad (44)$$

where the function $F^*(t) > 0$ vanishes at $t = 0$. From the existence of limit $\lim\limits_{t \to +0} H(t) > 0$ it follows

$$\lim_{t \to +0} \frac{H^4_{\max}(t)\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^3}{2H^4_{\min}(t)} = \frac{1}{2}.$$

Hence, there exists such value $t_1 : 0 < t_1 \le T$, for which the inequality holds

$$\frac{H^4_{\max}(t)\left(C_{22}\sqrt{\frac{\psi(t)}{t}} + 1 \right)^3}{2H^4_{\min}(t)} \le \frac{3}{4}, \quad t \in [0, t_1]. \quad (45)$$

Then we rewrite the inequality (44) under the form

$$\tfrac{1}{4}\widetilde{a}_{\max}(t) - F^*(t)\widetilde{a}_{\max}(t) \le 0 \quad \text{or}$$
$$\widetilde{a}_{\max}(t)(\tfrac{1}{4} - F^*(t)) \le 0.$$

It may be indicated such value $t_2 : 0 < t_2 \le T$, for which $\frac{1}{4} - F^*(t) > 0$ as $t \in [0, t_2]$. Then $\widetilde{a}_{\max}(t) \le 0$ on the segment $[0, t_2]$, what is impossible. Consequently, $a_1(t) \equiv a_2(t)$ on the segment $[0, t^*]$, where $t^* = \min(t_1, t_2)$. In the case $t > t^*$ the theorem is proved analogously as in the case of weak degeneration. Thus, the following theorem is proved.

Theorem 3. *Suppose that* $\lim\limits_{t \to +0} \int_0^t \left(\int_\tau^t \psi(\sigma)d\sigma \right)^{-1/2} d\tau = \infty$. *Let the conditions* (**A1**) *-* (**A3**)*,* (**A6**) *are satisfied. Then there exists the unique solution of the problem (1) - (4) defined for* $x \in [0, h], t \in [0, T]$.

Remark. The conditions **(A2)** may be weakened. In the case of weak degeneration instead of condition $f(0,t) - \mu(t) > 0$ one can suppose $f(0,t) - \mu(t) \ge 0$. Analogously, in the case of strong degeneration it may be supposed the condition $\varphi'(x) \ge 0$.

As it may be seen from above, the weak degeneration is provided by the behavior only of the function $\mu_3(t)$ which tends to zero when $t \to +0$ by the same law as the function $a(t)$. In the case of the strong degeneration this dependence between given data is more complicated.

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ОБЕРНЕНА ЗАДАЧА ДЛЯ РІВНЯННЯ ТЕПЛОПРОВІДНОСТІ З ВИРОДЖЕННЯМ ЗАГАЛЬНОГО ТИПУ

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Розглянуто обернену задачу визначення невідомого коефіцієнта для рівняння теплопровідності. Коефіцієнт за старшої похідної представлений у вигляді добутку двох функцій, залежних від часу, одна з яких перетворюється в нуль в початковий момент часу. Розглянуто випадки сильного та слабкого виродження. З'ясовано умови існування та єдиності розв'язку задачі.

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