

## ANALYSIS OF THE STEEL JOINT WITH FOUR BOLTS IN THE ROW

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In unbraced steel frame structures, bolted extended end-plate moment connections with four bolts in the row are very often applied. In this article a different methods are presented for the calculation of beam - to - column joints with four bolts in each row. The component method is recently the most popular method for the estimation of the moment capacity, initial stiffness, and rotation capacity of the steel and composite joints. This method has been recently largely developed, but still the resistance and stiffness of some components are not evaluated. This mainly regards the T-stubs with four bolts. An analytical model for T-stub with four bolts, which can be used for prediction of the joint resistance and initial stiffness has been elaborated.

**Introduction.** In unbraced steel frame structures, bolted end-plate moment connections are often used. These types of joints are widely applied to connect beams to columns and as beam and column splices. The most common end-plate connections utilize only two bolt rows in the tension zone with two bolts in each row. One bolt row is positioned outside the tension flange of the beam, on an extended portion of the end-plate. Second row is positioned inside the beam flanges. When the depth of the beam is large, the moment capacity of the joint with only two bolt rows in tension is not sufficient to carry the external moment. Increase of the resistance of the joint can be achieved by applying additional bolt rows; however additional bolt rows have a rather small participation in the resistance of the whole joint, because of their reduced associated lever arm. When greater resistance of joints is required, designers apply joints with four bolts in each row. Design rules given in Eurocode 3 [1] relate to bolted end-plate joints with only two bolts in each bolt row.

The resistance of such joints is determined by the resistance of the tension, compression and/or shear zones of the connection. Resistances of the shear and compression zones of joints with four bolts in each row can be predicted according to the European Standard [1] like for joints with two bolts in each row.

Resistance of tensile zone depends on the resistance of the beam flange, column flange and end-plate and is also limited to the tensile resistance of the bolts including prying forces. For joints with only two bolt rows, resistance of tension zone of end - plate can be predicted by the use of the equivalent T-stub model (Fig. 1).

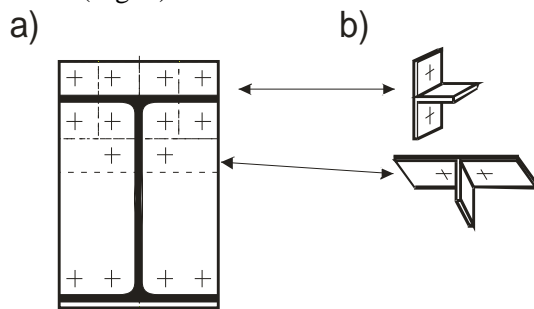


Fig. 1. Modelling of an extended end-plate joint by separate T-stubs.

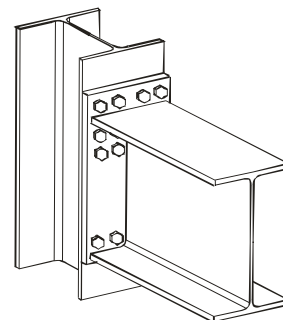


Fig. 2. Multiple rows extended end-plate joints with four bolts in each row

If inside the beam flanges there are two or more bolt rows with four bolts in each row (Fig. 2), the modeling of the end - plate by typical T-stubs cannot be applied.

The analytical models used to predict the resistance of the multiple rows end-plate joints are generally based on experimental tests. Most of the tests have been performed for beam splices. In some of these models, the distribution of the internal forces amongst the bolts have been arbitrarily established and regardless of dimensions of the joint. The same problems exist for beam - to - column connections with unstiffened column flanges.

**Reviews of the analytical models.** The analytical models of the end - plate multiple row joints with four bolts in the row are based on three main methods [2]: yield line theory, bolt force determination and mechanical models using component method.

The yield line analysis is based on the virtual work principle. Utilizing yield line patterns, the design resistance  $M_{Rd}$  of the end plate joint is given by [3]:

$$M_{Rd} = f_{y,p} \cdot t_p^2 \cdot Y \quad (1)$$

where  $f_{y,p}$  - the yield strength of the end-plate steel;  $t_p$  - the thickness of the end - plate, and  $Y$  - the geometric yield line parameter.

In method of the bolt force determination, two cases of the end plate behaviour are considered, namely thick and thin end-plates [4]. In case of a thick end-plate, the end-plate is stronger than the bolts and its deformation is small. The resistance of the joint is determined by tensile resistance of the bolts loaded only by normal forces.

The design resistance of the joint with thick end- plate can be predicted as:

$$M_{Rd} = B_{t,Rd} \sum n_i \cdot h_i \quad (2)$$

where  $B_{t,Rd}$  - the bolt tensile resistance;  $n_i$  - the number of bolts in bolt row  $i$  and  $h_i$  - the distance from the centerline of the compression flange of the beam to the centerline of the bolt row  $i$ .

In case of a thin end-plate, the behaviour of the joint is characterized by end-plate yielding with large plate deformations. The resistance of the joint is determined by end-plate deformations and the tensile resistance of the bolts additionally loaded by prying forces. For joints with thin end-plates the moment capacity of the joint is given by:

$$M_{Rd} = \max \left\{ \begin{array}{l} \sum n_i \cdot h_i (B_{t,Rd} - Q_{max,i}) \\ \sum n_i \cdot h_i \cdot T_b \end{array} \right. \quad (3)$$

where  $B_{t,Rd}$ ;  $n_i$  and  $h_i$  - as above,  $T_b$  - the preload force in the bolt; and  $Q_{max,i}$  - the maximum value of the prying force in bolt row  $i$ .

The simplification of this method leads to the procedure which allows to determinate of the internal forces distributions to the individual bolts. A method presented in Polish Standard [5], based on experimental tests of splices of high beams [6], has adopted a simplified method of force determination.

In component method, the moment resistance of the joints is based on the lowest resistance of the joint components such as column web, column flange, beam flange, bolts and end-plate.

The moment resistance of the joint can be calculated as:

$$M_{j,Rd} = \sum h_r \cdot F_{r,Rd} \quad (4)$$

where  $F_{r,Rd}$  = the effective design tension resistance of bolt row  $r$  allowing resistance of all components and  $h_r$  = the distance from the centerline of the compression to the centerline of the bolt row  $r$ .

The end-plate is divided in separated T-stubs. The resistance and deformation of the equivalent T-stubs can be predicted according to different methods [7], [8]. The difference between these methods is seen mainly in the statical scheme of parts of the end-plate.

**Analytical model for joint resistance.** Based on the observed behaviour of the test specimens, a simple beam model was developed to determine the behaviour of the T-stub. The resistance of the T-stub depends on the resistance of the bolts, thickness of the end-plate and geometrical dimensions of the specimen. The possible models of failure are presented in Fig. 3.

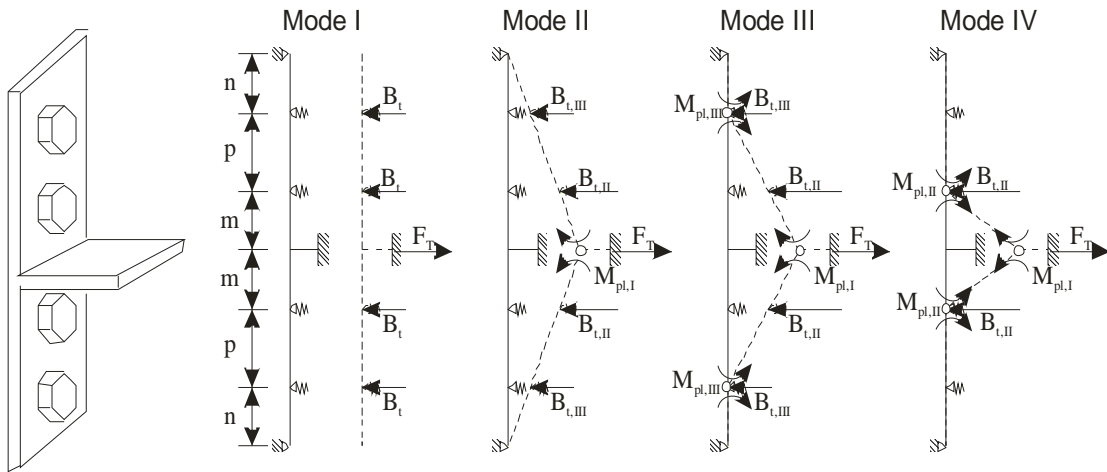


Fig. 3. T-stub model for end – plate with four bolts

Final resistance of the equivalent T-stub in tension is the minimum value of the resistances obtained for each failure modes. The bolt forces are determined by static conditions, admitting elastic distribution of internal forces to the bolts. The lowest components resistance governs the ultimate resistance of the bolt rows and the whole joint.

The effective resistance of the equivalent T – stub can be received from:

$$F_{T,eff,Rd} = \min( F_{T.1,Rd}; F_{T.2,Rd}; F_{T.3,Rd}; F_{T.4,Rd} ) \quad (5)$$

where  $F_{T.1,Rd}$  - the resistance due to bolt failure;  $F_{T.2,Rd}$  - the resistance due to bolt failure with partial yielding of the T-stub flange.  $F_{T.3,Rd}$  - the resistance due to bolt failure with yielding of the T-stub flange; and  $F_{T.4,Rd}$  - the resistance due to the complete yielding of the T-stub flange.

The Mode I – bolt failure takes place if a very thick T-stub flange is used. The bolts are loaded by the external force without prying forces. It is a result of the large plate stiffness. The resistance of the T-stub in this case can be predicted as a sum of the tensile resistances of the bolts:

$$F_{T,1,Rd} = \sum B_{t,Rd} \quad (6)$$

where  $B_{t,Rd}$  - the tensile resistance of the bolt, considering tensile resistance of the bolt shank and punching shear resistance of the bolt head.

According to the [9], this mode of failure is not real, because the thickness of the end plate should be very thick, not used in practice.

If the thickness of the T-stub flange is reduced, the bolt failure with partial yielding of the T-stub flange occurs (Mode II). The resistance of the T-stub can be obtained from the equilibrium of the external and internal works using virtual work method for model shown in Fig. 4.

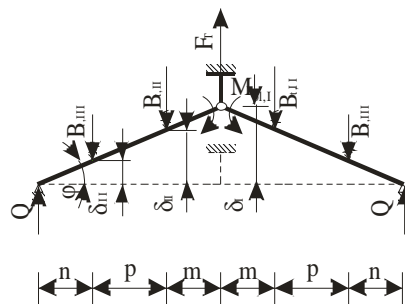


Fig. 4. Simply supported beam model of the bolt failure with partial yielding of the plate

$$F_{T,2,Rd} = \frac{2 \cdot M_{pl,II,Rd} + (4 \cdot n + 2 \cdot p) \cdot B_{t,Rd}}{m + p + n} \quad (7)$$

where  $M_{pl,I,Rd}$  - the yielding moment of T-stub flange;  $m$  - reduced distance between the bolts and the web;  $p$  - the bolt spacing;  $n$  - minimum distance between the bolt and the edge of the T-stub flange limited to  $1,25 \cdot m$ ; and  $B_{t,Rd}$  - as above.

When the thickness of the T-stub flange is smaller, plastic hinges appear close to the edge bolts. In this case a model of the bolt failure with yielding of the plate was occur (Mode III). The resistance of the T-stub can be obtained as follows:

$$F_{T,3,Rd} = \frac{2 \cdot M_{pl,I,Rd} + 2 \cdot M_{pl,III,Rd} + 2 \cdot p \cdot B_{t,Rd}}{m + p} \quad (8)$$

where  $M_{pl,III,Rd}$  - the yielding moment for T-stub flange close to the edge bolt;  $M_{pl,I,Rd}$ ;  $m$ ;  $p$ ; and  $B_{t,Rd}$  - as above.

For thin T-stub flanges, the complete yielding of the T-stub flange occurs (Mode IV). The resistance of the T-stub in this case can be predicted like for T-stubs with only two bolts:

$$F_{T,4,Rd} = \frac{2 \cdot M_{pl,I,Rd} + 2 \cdot M_{pl,II,Rd}}{m} \quad (9)$$

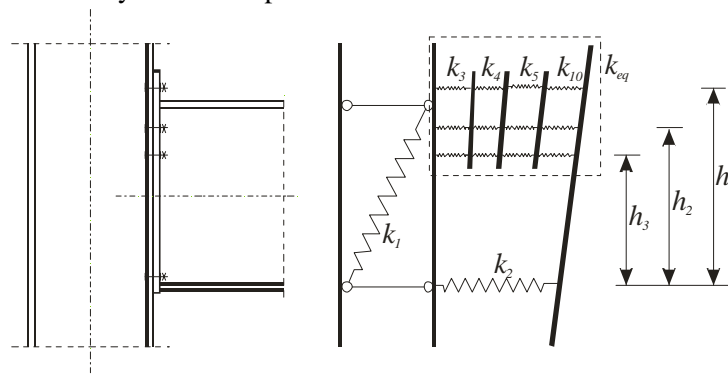
where  $M_{pl,II,Rd}$  - the yielding moment for T-stub flange in place of the bolt adjacent to the web;  $M_{pl,I,Rd}$ ; and  $m$  - as above.

The yielding moments can be determined as:

$$M_{pl,Rd} = 0,25 \cdot \sum l_{eff} \cdot t^2 \cdot f_y / \gamma_{M0} \quad (10)$$

where  $l_{eff}$  - the effective length (assumed as the width of the T-stub flange for examined T-stubs);  $t$  - the thickness of the T-stub flange;  $f_y$  - the yield resistance of T-stub;  $\gamma_{M0}$  - partial safety factor.

**Analytical model for stiffness of the joint.** Only the component method can be applied to determine the rotational stiffness of the joint. The mechanical model for calculation of the initial stiffness (Fig. 5) is based on a flexibility of the components.



- |                                   |                                  |
|-----------------------------------|----------------------------------|
| 1 - the column web in shear       | 4 - the column flange in bending |
| 2 - the column web in compression | 5 - the end-plate in bending     |
| 3 - the column web in tension     | 10-bolts in tension              |

Fig. 5. Mechanical model for end-plate joint

The stiffness of the joint can be predicted from [1], as:

$$S_j = \frac{E \cdot z^2}{\mu \sum \frac{1}{k_i}} \quad (11)$$

where  $E$  - the elastic modulus;  $z$  - the level arm;  $\mu$  - the stiffness ratio and  $k_i$  - the stiffness coefficient for basic joint component  $i$ .

The stiffness coefficients for column web in tension, compression and in shear can be taken as for joint with two bolts in the row. The flexibilities of the column flange, end-plate in bending and bolts in tension should be modelled differently. In the component method procedure, as introduced by Eurocode 3 [1], the end-plate and bolts are considered separately. Because of different loading of the particular bolts

in T-stub with four bolts, a modified model considering both bending of the end-plate and elongation of the bolts should be applied. The stiffness coefficients are obtained for each of the modes of failure as shown below.

In case of the bolt failure (Mode I), the displacement of the T-stub depends only on the elongation of the bolts. The stiffness coefficient of the T-stub with four bolts can be taken as double value of T-stub with two bolts:

$$k_{T,1} = 3,2 \cdot A_s / L_b^p \quad (12)$$

where  $A_s$  - the tensile stress area of the bolt; and  $L_b^p$  - the bolt elongation length, taken as a sum of the thickness of plate and washer, plus half of the heights of the bolt head or nut.

In case of the bolt failure with partial flange yielding (Mode II), the elastic deformation of the T-stub in tension depends on the bending of T-stub flange and bolts elongation (Fig. 6).

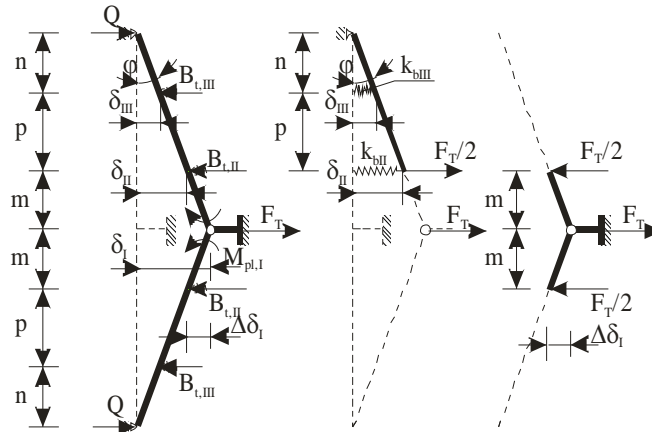


Fig. 6. The model for calculation of the end-plate deflection

The stiffness coefficient with the consideration of the prying forces according to Eurocode 3 [1], is obtained from:

$$k_{T,2} = \frac{1}{\frac{(n+p) \cdot L_b^p}{1,6 \cdot (2 \cdot n + p) \cdot A_s} + \frac{m^3}{0,9 \cdot l_{eff} \cdot t^3}} \quad (13)$$

where  $A_s$ ;  $L_b^p$ ;  $l_{eff}$ ;  $n$ ;  $p$ ;  $m$ ; and  $t$  - as above.

In case of the bolt failure with flange yielding (Mode III), the stiffness coefficient can be received as:

$$k_{T,3} = \frac{1}{\frac{L_b^p}{1,6 \cdot A_s} + \frac{m^3}{0,9 \cdot l_{eff} \cdot t^3}} \quad (14)$$

where  $A_s$ ;  $L_b^p$ ;  $l_{eff}$ ;  $m$ ; and  $t$  - as above.

When the mode IV was occurring, the elastic deformation of the T-stub in tension depends only on bending of the T-stub flange. The stiffness coefficient can be obtained like for T-stub with only two bolts, from:

$$k_{T,4} = \frac{0,9 \cdot l_{eff} \cdot t^3}{m^3} \quad (15)$$

where  $l_{eff}$ ;  $m$ ; and  $t$  - as above.

**Parametric study.** On the basis of the developed model, a wide parametric study of the resistance and stiffness of the simply T-stub with four bolts in the row has been conducted.

Constant values of the following parameters were established:

- steel grade S235,
- bolts M20,

As a variable parameters, the following joint properties were analyzed:

- thickness of the flange plate of T-stub  $t_p$ , in the range 20-60 mm,
- distance between center of inner bolts and web  $m$ , in the range 15 –30 mm,
- distance between center of end bolts and end of the T stub flange  $n$ , in the range 15 –30 mm,
- bolt spacing  $p$ , in the range 25 –80 mm,
- T-stub flange width  $b_p$ , in the range 100 –850 mm,
- bolts grade 3.6 to 10.9.

Influence of each component on the resistance and stiffness of the T-stub are presented in Fig. 7 to 14. In these pictures, ranges of failure modes were indicated by numbers I, II, III or IV.

Main conclusions from parametric study are as follows:

- thickness of the T-stub flange  $t_p$  and its width  $b_p$  have a big influence on the resistance of the T-stub,
- web spacing  $m$  influences the resistance less than end plate thickness,
- bolt grade have a influence on resistance and stiffness of the joint when the failure is govern by II, III and IV mode,
- bigger thickness of the end plate  $t_p$  and web distance  $m$  decrease the stiffness of the T-stub,
- stiffness of the joint is depended on the failure mode (leaps on the diagram).

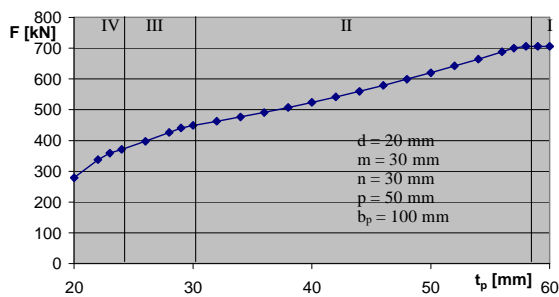


Fig. 7. Influence of the thickness of T-stub flange on the resistance of the T-stub

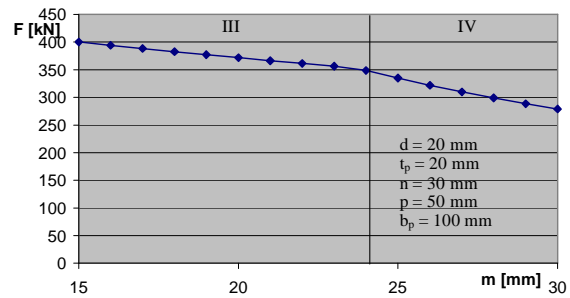


Fig. 8. Influence of the web distance  $m$  on the resistance of the T-stub

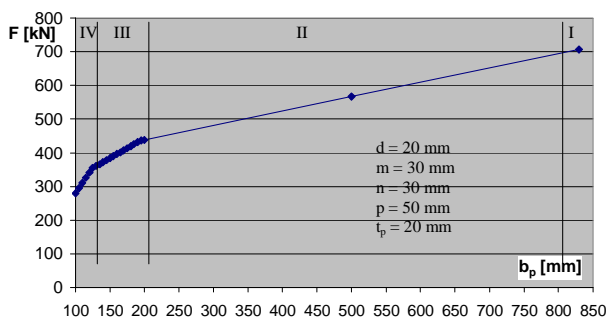


Fig. 9. Influence of the plate width  $b_p$  on the resistance of the T-stub

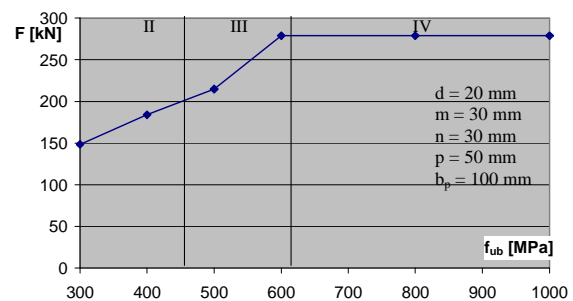


Fig. 10. Influence of the strength of the bolt on the resistance of the T-stub

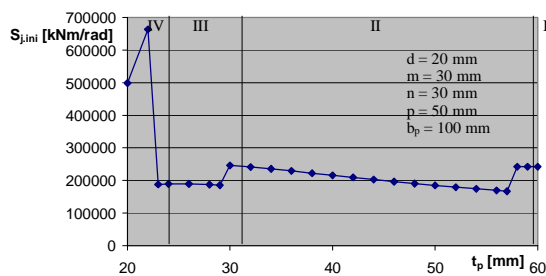


Fig. 11. Influence of the thickness of end plate on the stiffness of the T-stub

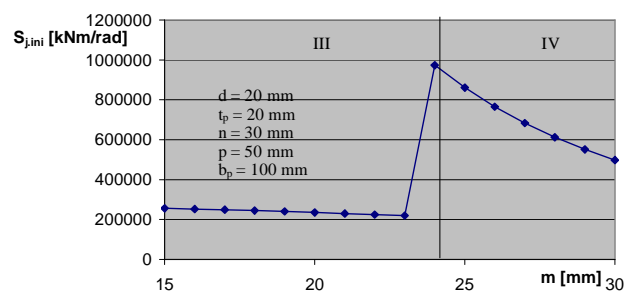


Fig. 12. Influence of the web distance  $m$  on the stiffness of the T-stub

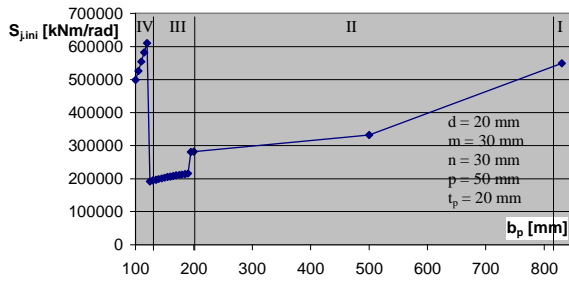


Fig. 13. Influence of the end plate width on the stiffness of the T-stub

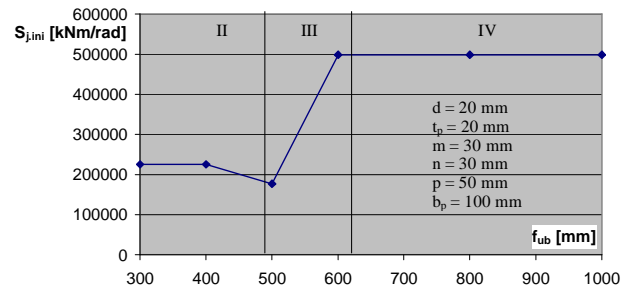


Fig. 14. Influence of the strength of the bolt on the stiffness of the T-stub

**Effective length for one bolt and group bolt row.** During calculations of the joint resistance, when an II, III and IV mode of failure is occurring, the main influence plays an effective length. According to Eurocode 3 [1], the effective length should be determined for one bolt and a group bolts. Based on the yield line pattern, the effective length, for T-stub with two bolts, can be obtained according to [10]. Based on these assumptions, and on the experimental tests for simply T-stubs, the effective length was predicted [9] for two bolts.

For circular yield line pattern (Fig. 15) the effective length can be obtain from:

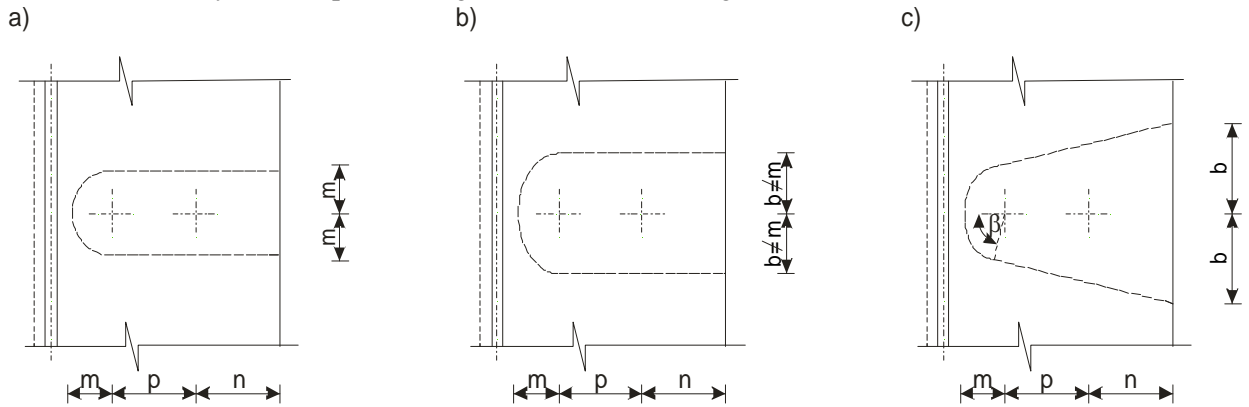


Fig. 15. Circular yield line pattern for two bolts

$$l_{eff} = \pi \cdot m + 2 \cdot (p + n) \quad (16a)$$

$$l_{eff} = \sqrt{\pi \cdot m \cdot (\pi \cdot m + 4 \cdot p + 4 \cdot n)} \quad (16b)$$

$$l_{eff} = 2 \cdot (m \cdot \beta + m \cdot ctg\beta + (p + n) / \sin\beta) \quad (16c)$$

where  $l_{eff}$ ;  $m$ ;  $n$ ,  $p$ , - as above and  $\beta$ -in [rad].

For a non-circular yield line pattern (Fig. 16) the effective length can be obtain from:

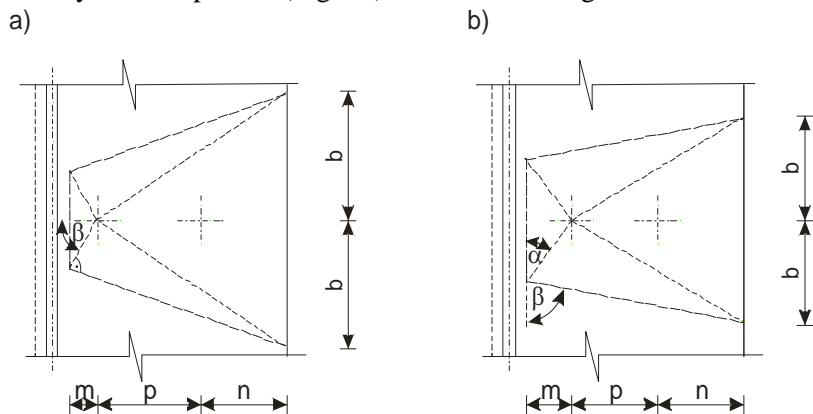


Fig. 16. Non-circular yield line pattern for two bolts

$$l_{eff} = 4 \cdot \sqrt{m^2 / 2 + p \cdot m + n \cdot m} \quad (17a)$$

$$l_{eff} = 2 \cdot \sqrt{3 \cdot m^2 + 4 \cdot p \cdot m + 4 \cdot n \cdot m} \quad (17b)$$

where  $l_{eff}$ ,  $m$ ,  $n$ ,  $p$ , - as above.

In case bolt row adjacent to the stiffener, the effective length can be obtained similar, with consider distances between bolt and perpendicular stiffeners. But this is not clear for consider second bolt row. Therefore, a research program for modelling of end-plate connections with four bolts in each row will be planed in this way.

**Conclusions.** The design methods existing up to now for determination of the resistance of the joint with four bolts in each row give results which differ very much. Design rules given in Eurocode 3 [1] relate to bolted end-plate joints with only two bolts in each bolt row.

The analytical model for assessment of the resistance and stiffness of T-stub with four bolts presented in the paper can be used in the component method. Comparison of the resistance of the T-stub from proposed method and test results shows that analytical models rather well agree with experimental results.

Parametric study allows to write the following conclusions: thickness of the T-stub flange  $t_p$  and its width  $b_p$  have a big influence on the resistance of the T-stub, web spacing  $m$  influences the resistance less than end plate thickness, bolt grade have a influence on resistance and stiffness of the joint when the failure is govern by II, III and IV mode, bigger thickness of the end plate  $t_p$  and web distance  $m$  decrease the stiffness of the T-stub, stiffness of the joint is depended on the failure mode.

The effective length for one bolt and a group bolts mainly depend on the joint configuration. As far as in case perpendicular unstiffened T-stubs can be adopted yield pattern line for one bolt, then for second bolt row under beam flange it is not clear. This mainly regards the interaction between perpendicular T-stubs and determination of the effective length of T-stub not adjacent to the web.

1. EN 1993-1-8. Eurocode 3. Design of Steel Structures. Part 1.8: Design of joints. CEN. 2005. 2. Kozłowski A., Pisarek Z. Characteristics of bolted end-plate joints with four bolts in the row. Proc. of 10<sup>th</sup> Scient. Conf. Rzeszow-Lviv-Kosice, State of Art, Trends of Development and Challenges in Civil Engineering, 11-13 September, 2005, Kosice, Slovakia. 2005. 3. Sumner, E.A and Murray, T.M Behavior and design of multi-row extended end-plate moment connections. Proceedings of the International Conference on Advances in structures. (ASCCA '03). Sydney 2003. 4. Kennedy, N.A., Vinnakota, S., and Sherbourne, A. The split-tee analogy in bolted splices and beam-column connections. Joints in Structural Steelwork. John Wiley and Sons, New York 1981., pp. 2.138-2.157. 5. PN-90/B-03200. Polish Standard. Steel Structures. Design Rules. PKN Warsaw 1990, (in Polish). 6. Łaguna, J. Resistance of the end-plate moment connections preloaded by high grade bolts. PhD Thesis, ITB, Warsaw 1984, (in Polish). 7. Krumm, R. Calculation of Rigid Face Plate Connections According to the DSTV/DASt Guidelines. Stahlbau Vol. 60/3. Berlin 1991. 8. Ungermann, D., Schmidt B. Moment Resistance of Bolted Beam to Column Connections with four Bolts in each Row. Proceedings of IV European Conference on Steel and Composite Structures. Eurosteel 2005. Maastricht 2005. 9. Sedlacek G. Plastics Calculation of the End - Plate Connections with four Bolts in the Row. DASt Guidelines 5/2000. Düsseldorf 2000. (in German). 10. Zoetemeijer P. A design method for the tension side of statically loaded bolted beam to column connections. Heron, Vol. 20, No 1, Delft 1974.