efficiency $\eta_{u\ śr}$ = 60% were the heat sources. About 40% of working equipment of this type is over 9 years old. The presented situation proves that there is unfulfilled potential of modernising the individual heat sources.

– Equipping the already working central heating boilers using natural gas with more advanced automatics will increase the efficiency of heat source adjustment and will lower the heat consumption by 5–15%, thanks to using periodicity and operation specificity of school buildings. The aforementioned activities should be accompanied by other modernisation projects.

– Taking into consideration the maximum efficiency of central heating boilers with the specified load appropriate, low-budget organisational activities aimed at creating favourable conditions should be undertaken. These activities are as follows: drawing up a schedule of operation of CH boiler group, drawing up a user manual and the current central heating boilers control of the observance of the instructions. They will enable the reduction of seasonal fuel consumption by 3 – 8%. For example, a small amount of soot and ashes from 1 to 2 mm may cause the power decrease of a hard coal boiler even by 30%. As visits at schools showed there are unfulfilled possibilities in this respect

– The conducted analyses did not prove statistically significant relationship between indexes WPz and WMK and an amount of the theoretical standard fuel necessary to produce 1 GJ of heat. It proves indirectly the lack of gross deviations from the aforementioned rules concerning the choice of central heating boilers. Frequent cases of using a different fuel than recommended by the producer in school-rooms were observed, however without a significant influence on heat production efficiency.

– The costs of producing 1 GJ of heat for heating school buildings get reduced in the buildings with their own heat sources together with the seasonal increase of heat consumption. This dependency does not exist in the case of schools supplied with heat by Heating Enterprise, which uses up the created economic effect, and does not take this fact into consideration and lower central heating charges.

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ON THE PROBLEM OF SAFETY EVALUATION IN DESIGN OF STEEL MEMBERS FOR ACCIDENTAL FIRE SITUATION

Failure probability can be applied as a basic safety measure in design of structural member under fire conditions. To reliable assess this value interacted influences of many factors should be taken into account. Some suggestions in this field are given in this paper.

Introduction. The reliable safety measure in design of steel members for fire situation is probability of failure $p_f = P(F)$. The failure in this case does not have to deal with complete decay of the opportunity to carry all external loads (including thermally generated internal forces and moments caused by fire) imposed to the structure and summed in accordance with accidental combination rule. It can be recognized also as too large member deformation, too speed increment of its values or, simply, reaching fire insulation or fire tightness limit states. However, in this paper failure is considered only as reaching classical fire resistance limit state which is generally connected with partial or even total construction collapse. If the maximum value of probability p_f, acceptable by user of the structure, is described as $p_{f,ult}$ then safety condition has the following form:

$$p_f < p_{f,ult} \tag{1}$$

Explicit determination of types of probabilities taken into consideration and compared one to another is absolutely necessary. Above all two kinds of such probabilities must be distinguished:

- probability of failure caused by fire if it is known that fire ignition has occurred and; moreover, this fire has reached the flashover point (it may be described as a fully developed fire) – in further part of this paper such a quantity is marked as p_f,
- probability of failure caused by fire which can take place; however, the designer has no information about its ignition and flashover - as opposed to the previous one is specified by authors as p_{ff}.

Relation between p_f and p_{ff} is given by *T. T. Lie* [1]:

$$p_{ff} = p_t p_f \tag{2}$$

where p_t means probability of fire occurrence (not only of fire ignition but also reaching the flashover point). In this approach quantity p_f has to be interpreted as the conditional probability of failure with the condition that fire has taken place .

At present, in professional literature, fully developed fire is differentiate from localized fire. The first one is in general characterized by uniform temperature distribution in the whole of fire compartment (like in the simplest design model, so called “simplified natural fire” [2]); whereas, the second acts only locally. Consequently, fully developed (post-flashover) fire is frequently treated as the next stage of localized fire, taking place having reached the flashover point. Probability of member failure in fire is generally obtained as the product of probabilities of events E_i [3], [4], [5], where particular events are defined as follows:

- E1 – fire ignition and localized fire,
- E2 - fire flashover and reaching the status of fully developed fire (temperature of combustion gas is uniform in the whole of fire compartment)
- E3 – failure of the member in fire.

Thus:

$$p_{ff} = P(F) = P(E1)P(E2)P(E3) \tag{3}$$

As it can be noticed, the assumption that the failure of structural member under fire conditions occurs only if the fire ignition has taken place, and; moreover, if this fire has developed in the whole of fire compartment and; finally, if load-bearing capacity of the element has vanished for given member temperature, is accepted. However, simplicity of formula (3) is fallacious. Let us underline that the events $E1$, $E2$, and $E3$ are not randomly independent. It is the result of the fact that the previous occurrence of event $E2$ is necessary in order to event $E3$ can occur and; consequently, occurrence of $E1$ has to go before $E2$. For this reason, probabilities applied in formula (3) are conditional probabilities [6], which means that:

$$p_{ff} = P(F) = P(E1 \cap E2 \cap E3) = P(E1)P(E2/E1)P[E3/(E2 \cap E1)] \tag{4}$$

In conclusion, formulae (3) and (4) are not equivalent to each other because of random dependence between $E1$, $E2$, and $E3$. If the form of formula (2) can be consistent with the notation applied above, its components should be rearranged as:

$$p_t = P(E1)P(E2/E1) \neq P(E1)P(E2) \tag{5}$$

$$p_f = P[E3/(E2 \cap E1)] \tag{6}$$

Not only quantitative but also qualitative distinction between probabilities p_f and p_{ff} seems to be very significant. Even if conditional probability p_f is large, probability p_{ff} is usually quite small and does not seem to be apprehensive, because in reality value of probability p_t is also slight. However, quantity p_{ff} can also be considered as a conditional probability. Both values p_f and p_{ff} allow the designer to evaluate the real safety level, but with the assumption, that she/he knows that failure will occur resolutely as a result of fire action. Meanwhile, the construction can be destroyed also in a situation where fire has not appeared at all. If the probability of such an event is described as p_{f0}, then; finally, the probability of construction collapse p_{fff} can be calculated as:

$$p_{fff} = (1-p_t)p_{f0} + p_t p_f \tag{7}$$

Formula (7) follows directly from the scheme of *Bernoulli* sampling with two samples.

Probability of fire occurrence. Estimating of probability of fire occurrence p_t may threaten the designer to fall into the trap. We are looking for the probability that fire occurs not only once but at least once during preliminary time T of construction serviceability. Fire is, by its nature, a very rare event. If it is considered as a point-in-time phenomenon then fire occurrence can be described by means of the mathematical formalism of *Poisson* process. Let us assumed that the number of fires which have taken place during time T is given by x. Then the probability of occurrence of such x fires may be calculated as follows:

$$p_x(x) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}, \quad x = 1,2,...,\infty \tag{8}$$

Parameter λ is called here the process intensity. Thus:

- the probability that fire does not occur at all in a given time T:

$$p_x(x=0) = e^{-\lambda T} \tag{9}$$

- the probability that fire occurs exactly once in a given time T:

$$p_x(x=1) = \lambda T e^{-\lambda T} \tag{10}$$

- the probability that fire occurs at least once in a given time T:

$$p_x(x \ge 1) = 1 - p_x(x=0) = 1 - e^{-\lambda T} = p_t \tag{11}$$

Estimation of value of λ parameter may be taken from *T. T. Lie* [1] suggestion compiled for buildings. He has assumed that analysed buildings are divided into fire compartments which are identical as far as their characteristics such as kind of exploitation, geometrical dimensions, fire load density, are concerned. This model leads to the formula:

$$\lambda = hA \tag{12}$$

where A is the area of fire compartment; whereas, h - probability of fire ignition (if probability $P(E1)$ is looked for) or fire flashover (if probability $p_t = P(E1)P(E2/E1)$ is evaluated), calculated for $1\,m^2$ of fire compartment per one year. In authors' opinion it would be better to call value h as the "risk" instead of "probability" because of the fact that it is a dimensional quantity. *R. H. Burros* [7] has given the generalized solution (12), possible for application in the case of different fire compartments. Then:

$$\lambda = h\overline{A} = h\frac{A_F}{N} \tag{13}$$

where A_F means the total area of building consisting of N fire compartments; whereas, $\overline{A}$ is the mean value of a single fire compartment area. From the basic properties of *Poisson* process arises that expected number of fires $\overline{x}$ during time T is equal to the variance σ_x^2, thus:

$$\overline{x} = \sigma_x^2 = \lambda T = h\overline{A}T \tag{14}$$

Usually value of $\overline{x}$ is considerably smaller than 1, then approximation (15) is acceptable:

$$p_x(x \geq 1) = 1 - e^{-\lambda T} = 1 - e^{-h\overline{A}T} \approx h\overline{A}T = p_t \tag{15}$$

Estimation of failure probability by means of the complete probability concept. Procedure of looking for the value $P(F) = p_{fff}$ is more clear if it is presented as a logical tree (Fig. 1) scheme. It should be the fault tree concept because probability of failure is the unknown quantity. Let us assumed that events $\overline{E1}$, $\overline{E2}$ and $\overline{E3}$ are the contrary events to $E1$, $E2$ and $E3$, respectively. Obviously always equation $P(E_i) \cup P(\overline{E_i}) = 1$ has to be true. Application of complementary events E_i and $\overline{E_i}$ allows to describe the probability $P(F)$ as a complete probability value:

$$P(F) = \sum_{i=1}^{n} P(F/E_i)P(E_i) \tag{16}$$

Then

$$P(F) = P(F/E1)P(E1) + P(F/\overline{E1})P(\overline{E1}) \tag{17}$$

where

$$P(F/E1) = P(F/E2)P(E2/E1) + P(F/\overline{E2})P(\overline{E2}/E1) \tag{18}$$

$$P(F/E2) = P(F/E3)P[E3/(E2 \cap E1)] + P(F/\overline{E3})P[\overline{E3}/(E2 \cap E1)] \tag{19}$$

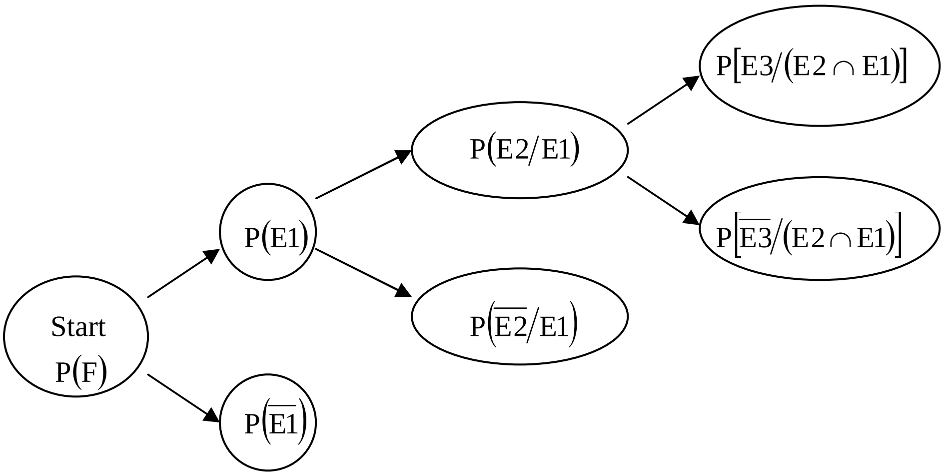

Fig. 1. Logical tree to calculate probability of failure $P(F)$ of structural member in fire according to the complete probability concept

The quantity $P(F)$ means here the probability of failure due to any reason, $P(F/E1)$ - probability of failure due to the fire occurrence, but when the designer knows that fire ignition has occurred, $P(F/\overline{E1})$ - probability of failure due to any reason, but except for fire. However, value of the probability $P(F/E1)$ is different in the case of fire flashover has taken place then when it has not taken place. On the other hand, the probability $P(F/E2)$ refers the case that fire has been initiated, its flashover has taken place and; furthermore, it has led to the collapse of the structure. Slightly different approach to the estimation of values of such probabilities, according to the complete probability concept, is presented by *M. Holicky* and *J.-B. Schleich* [8].

Application of the network diagram. Value of the probability $p_f \neq p_{ff}$ can be estimated by applying of some type of the network diagram, proposed by *W. Fitzgerald* [9]. Its scheme is presented in Fig. 2. Let us notice that it has not the logical tree structure. Events E_i are now the junctions of the network. Respective contrary events $\overline{E_i}$ always have to accompany them; therefore, they are localized on the same level of the network. In this paper, we are looking for the probability $p_f = P(F)$, where F is considered as an event that "fire has not been extinguished at all". Such an event in this case may be treated as an equivalent of failure. The assumption that fire extinguishing depends on three, and only three, factors $E1$, $E2$ and $E3$, such as:

- $E1$ – fully developed fire (which means that fire flashover has previously occurred) has burned out spontaneously,
- $E2$ – fire has been extinguished by sprinklers or by other active fire protection measures,
- $E3$ – fire has been extinguished by fire brigade,

is accepted. However, such a partition must be considered only as a simplified description of the reality. Let us notice that no cases when the interaction between these factors takes place can be analysed in this way. For instance simultaneous activity of fire brigade ($E3$) and sprinkler system ($E2$) is frequently observed during firefighting action. In our approach all above given factors, taken into account in the safety analysis, are discussed in further study as the complete disjoint sets, according to *Venn* interpretation. Such a limitation is necessary to assume that events $\overline{E1}$, $\overline{E2}$, $\overline{E3}$ are randomly independent.

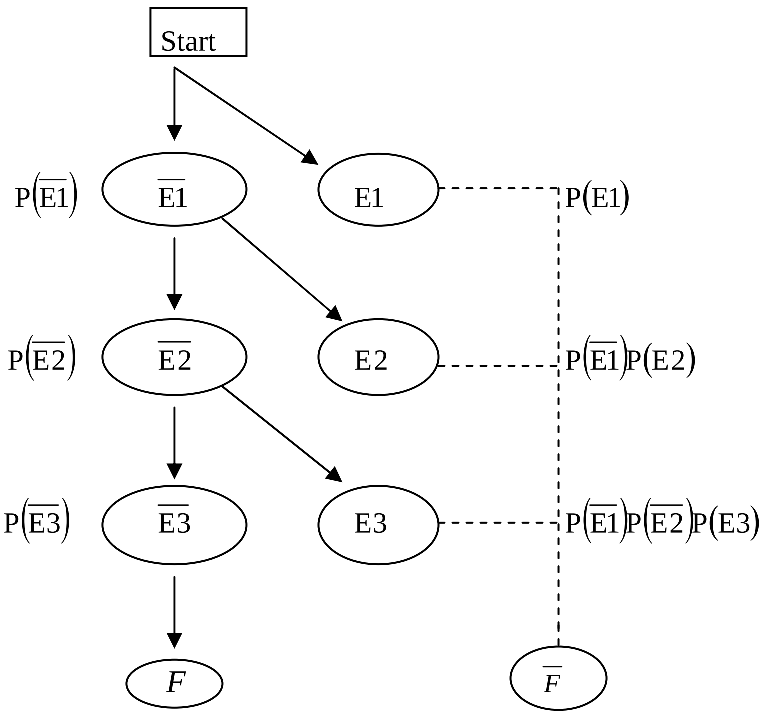

Fig.2 Network diagram to obtain value of the probability of failure $p_f = P(F)$

At first sight it seems that now the event F is not an equivalent of the analogous event – "fire has not been extinguished and, as a consequence, has led to the member failure", which has been analysed in the previous part of this paper. However, if the situation – "fire had not been extinguished but failure of the structure did not occur" (it is important that the designer defines the event ended before her/his study) – is interpreted within the framework of the event $E1$ the equipoise of both compared events is validated. Diagram presented above is also the fault diagram type because the probability of failure is looked for. Adequate probabilities are attributed to particular junctions of the diagram. Movement through this scheme is possible from its start point in the direction of the events F or $\overline{F}$, only by means of the "routes" marked by the lines connecting selected junctions (see Fig. 2). These lines reflect the logical structure of dependences between particular events. Let us notice that such a structure does not depend on detailed meanings accompanying individual events E_i. Moreover, the order of their analysis is also not important. Connections marked by continuous line can be interpreted as a logical gate AND type. It means the conjunction of randomly independent events in which the resultant probability is obtained as the product of partial probabilities taken into account in the analysis. On the other hand, connections marked by dashed line mean logical gate OR type which is the formal description of the alternative of randomly independent events. The resultant probability is now calculated as the sum of all considered partial probabilities. Consequently, probabilities obtained on the base of the scheme presented in Fig.2 are determined as follows:

$$P(F) = P(\overline{E1})P(\overline{E2})P(\overline{E3}) = [1 - P(E1)] \cdot [1 - P(E2)] \cdot [1 - P(E3)] \tag{20}$$

$$\begin{aligned} P(\overline{F}) &= P(E1) + P(\overline{E1})P(E2) + P(\overline{E1})P(\overline{E2})P(E3) = \\ &= P(E1) + [1 - P(E1)]P(E2) + [1 - P(E1)] \cdot [1 - P(E2)]P(E3) \end{aligned} \tag{21}$$

Correctness of the solution can be verified by checking the equation $P(F) = 1 - P(\overline{F})$. Methodology of the construction and the analysis of an analogous, but more complicated, network diagram for the case of fire which has an opportunity to expand to neighbouring fire compartments is presented by one of the authors in [6].

Probability-based approach to evaluation of member fire resistance. The value of probability of member failure (provided that this failure is caused by fully developed fire), called p_f in our article, is in classical safety analysis estimated in a completely different way. Such a failure is now recognized as an up-crossing of the level of random member resistance $R_{fi,\Theta}$ (reduced in given steel temperature Θ_a) by random value of action effect E_{fi}, which is the result of summing, in accordance with accidental combination rule, of all unfavourable partial effects generated by particular loads applied to the structure. In consequence we have two fully separate random variables, E_{fi} and $R_{fi,\Theta}$. For this reason, considering the density function of two-dimensional normal probability distribution $f(E_{fi}, R_{fi,\Theta})$ is necessary to precisely evaluate the real safety level in given steel temperature. Let us underline that in code formats in general only simplified approach, in which one-dimensional marginal density distributions $f(E_{fi})$ and $f(R_{fi,\Theta})$ are taken into account, is applied. It is commonly known, that such a methodology always leads to the evaluations which are safe indeed, but uneconomical. However, if the new random variable:

$$\gamma_\Theta = \frac{R_{fi,\Theta}}{E_{fi}} \tag{22}$$

is defined then taking into consideration only the density function of one-dimensional probability distribution $f(\gamma_\Theta)$ is sufficient. The event that $\gamma_\Theta \geq 1$ is in this case interpreted as a member survival; whereas, the value $\gamma_\Theta < 1$ means its failure. Let us consider as an example the situation when some steel structural element is designed. In simple load cases member resistance $R_{fi,\Theta}$ is directly proportional to steel yield point $f_{y,\Theta} = f_y(\Theta_a)$, where $f_{y,\Theta} = k_{y,\Theta} f_{y,20}$. Values of the reduction factor $k_{y,\Theta}$ for particular steel temperatures Θ_a are given in EN 1993-1-2 [10]; whereas, the quantity $f_{y,20}$ is the measure of steel yield point determined in room temperature (20^oC). Thus, in such cases:

$$R_{fi,\Theta} = k_{y,\Theta} R_{fi,20} \tag{23}$$

Value of probability p_f can be calculated by means of the global safety factor β_Θ concept:

$$\beta_\Theta = \frac{\ln \breve{\gamma}}{\upsilon_\Theta} \tag{24}$$

where

$$\breve{\gamma} = \frac{k_{y\Theta} \breve{R}_{fi,20}}{\breve{E}_{fi}} \quad \text{and} \quad \upsilon_\Theta = \sqrt{\upsilon_R^2 + \upsilon_E^2} \tag{25}$$

The modal value of member resistance $\breve{R}_{fi,20}$, calculated with reference to room temperature, is in considered case also proportional to adequate modal value of random steel strength $\breve{f}_{y,20}$. Parameters of distribution of random action effects (modal value and coefficient of variation) are estimated as follows:

$$\breve{E}_{fi} \cong \overline{G} + \sum_i \overline{Q_i} \qquad \text{and} \qquad \upsilon_E \cong \sqrt{\upsilon_G^2 + \sum_i \upsilon_{Qi}^2} \tag{26}$$

where G means permanent load; whereas, Q_i i-th variable load. Finally:

$$p_f = \Phi(-\beta_\Theta) \tag{27}$$

Symbol $\Phi(\ )$ in this formula denotes a cumulative distribution function of standardized normal distribution, in other words the *Laplace* function, accessible in statistical tables. It should be noticed that both action effect E_{fi} and member resistance $R_{fi,\Theta}$ depend on the steel temperature Θ_a. That is the reason that these variables are correlated in statistical sense and significantly complicated analysis is necessary to precisely describe the shape of function $p_f = p_f(\Theta_a)$. However, a simplified approach, in which evaluations of probability p_f in relation to fixed values of steel temperature are obtained, is proposed by authors to be applied. The fact that in such design methodology the steel temperature Θ_a cannot be taken into account as a random variable, because it is now only the design parameter (which is not random in formal sense), must be underlined. Dependence $p_f = p_f(\Theta_a)$ can be determined indirectly by means of multiple recurrent calculations, made for succeeding Θ_a values. On the other hand, values of ultimate failure probability $p_{f,ult}$, acceptable by the user of the structure, can be determined at her/his discretion. It is necessary to pay attention that these values are explicitly connected with adequate required values of $\beta_{\Theta,req}$ index. However, according to EN 1990 [11], when ordinary safety requirements are taken into consideration (so-called reliability class RC2), value $\beta_{\Theta,req} = 3{,}8$ should be accepted. It is an equivalent of the ultimate failure probability value equal to $p_{f,ult} = 7{,}235 \cdot 10^{-5}$. For another reliability classes different ultimate parameters are defined, particularly:

- for class RC1 – reduced safety requirements:
  $\beta_{\Theta,req} = 3{,}3$, then $p_{f,ult} = 48{,}342 \cdot 10^{-5}$
- for class RC3 – special safety requirements:
  $\beta_{\Theta,req} = 4{,}3$ then $p_{f,ult} = 0{,}854 \cdot 10^{-5}$.

Consequently, the safety condition can be described as follows:

$$p_f \le p_{f,ult} \tag{28}$$

or in an equivalent way:

$$\beta_\Theta \ge \beta_{\Theta,req} \tag{29}$$

Temperature Θ_a for which $p_f = p_{f,ult}$ occurs (then also $\beta_\Theta = \beta_{\Theta,req}$) is named the critical temperature of the member $\Theta_{a,cr}$. Let us notice that the checking the global safety condition by means of formula (28) or (29) resolves itself into the verification of the following inequality:

$$\Theta_a < \Theta_{a,cr} \tag{30}$$

The steel temperature Θ_a is not the only parameter allows the designer to verify the safety condition of steel member in fire. In many cases determination of time period t_{fi}, which can be used by the user of a structure under fully developed fire conditions to safely evacuate from the fire compartment (with the acceptable by herself/himself and fixed value of ultimate probability of failure $p_{f,ult}$), seems to be more useful. Particular steel temperature value $\Theta_{a,cr}$ may be in this way obviously linked with the fire moment in which member failure occurs $t_{fi,d} = t_{fi}\left(\Theta_{a,cr}\right)$. The time period, calculated from fire flashover t_0 to $t_{fi,d}$ moment, is commonly called the member fire resistance. In classical engineering approach the intensity of member fire exposure is described as a function of the fire time by means of the assumption of a model temperature-time curve $\Theta_a - t_{fi}$. Every fire moment, which has been chosen by the designer, may be explicitly connected with adequate steel temperature; therefore, such a relation can be interpreted as a mapping in mathematical sense. Consequently, formulae (28), (29) and also (30), can be described otherwise, in time units:

$$t_{fi,d} \geq t_{fi,req} \tag{31}$$

where the required value of member fire resistance $t_{fi,req}$ for buildings with given kind of utility is taken from the regulations of national law.

Conclusions. In authors' opinion the approach presented in this paper allows the designer to assess the real safety level under fully developed fire conditions in the way which seems to be more objective and complete in comparison with the classical solutions, applied in national codes. Moreover, design methodology proposed in the article is still user friendly and not too much time consuming for structural designers. Partial safety factors, which are commonly used in current standard recommendations has been replaced by the maximum, possible to accept, values of ultimate probability of failure $p_{f,ult}$. Determination of such values gives us the opportunity to make the adequate safety analysis also if different levels of reliability requirements has to be taken into account. It is consistent with formal suggestions given in EN 1990 [11]. For these reasons the solutions described above can be considered as a base helpful in the process of calibration and verification of typical parameters applied in the reliable fire safety analysis.

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