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MATHEMATICAL MODELLING OF EFFICIENCY CRITERIA FOR THE TRAFFIC SERVICE

Розроблені математичні моделі критеріїв ефективності і алгоритми оцінювання ефективності обслуговування трафіку гетерогенних комп'ютерних мереж, що побудовані на порівнянні фактичного режиму обслуговування трафіку з оптимальним режимом і оцінюванні збільшення значення критерію середнього ризику у разі відхилення режиму від оптимального режиму.

Mathematical models of criteria, algorithms for efficiency evaluating of the traffic service of heterogeneous computer networks which are constructed on comparison of an actual mode of the traffic service with an optimum mode and which permit to evaluate increase of values of criteria of average risk in case of a deviation of a mode from an optimum mode are developed.

I. Introduction. The big attention [1-3] is paid to methods and algorithms of modeling of the traffic of computer networks. But there are not enough works where problems of a choice and mathematical modeling of optimization criteria of the traffic service [4, 5] are discussed. In this actual scientific direction there are no almost work which would be devoted evaluating to efficiency of the traffic service in open information networks. Considered work is focused on elimination of this blank.

II. Purpose of work. The purpose of work is the choice of metric space, a substantiation of the principle of efficiency evaluating, development of mathematical models of criteria and algorithms for evaluating efficiency of the traffic service in heterogeneous computer networks. Main principle is comparison of an actual mode of the traffic service and an optimum mode for evaluating relative increase of criterion values of average risk in case of a deviation of a mode from an optimum mode.

III. Statement of a problem. It is supposed, that are known: function of losses because of refusals in service of packages of the data at a transport level of reference model of open information systems, function of expenses for maintenance of the necessary throughput of a transport level of system of service of the traffic, coefficient of using of throughput of this system. On these entrance data the choice of optimum throughput of system of service of the traffic which provides a minimum of criterion of average risk by the method, which was worked out by us in work [5], is carried out. Results of the problem decision of optimization serve as a basis for development of the analysis method of efficiency of the traffic service in heterogeneous computer networks on the criterion of a minimum of average risk.

IV. Decision of the problem. In a role of criterion of inefficient use of system throughput of the traffic service at a transport level it is convenient to choose a relative parameter of a deviation of an actual mode from an optimum mode in kind

$$\delta(C_0, C_{opt}) = \frac{D(C_0) - D_{\min}(C_{opt})}{D_{\min}(C_{opt})}, \tag{1}$$

where $D(C_0)$ and $D_{\min}(C_{opt})$, accordingly, values of criterion of average risk in not optimum and optimum modes of the traffic service in a heterogeneous computer network.

Necessary conditions of use of criterion (1) look like

$$\begin{cases} \delta_0(C_0 = C_{opt}) = 0, \\ \delta_1(C_0 \neq C_{opt}) > 0. \end{cases} \tag{2}$$

Therefore a major principle is definition $D_1(C_0 \neq C_{opt})$ and use of its value in the Eq.1.

The theorem 1. Normalized values *Z* of criterion of average risk depend only on factors ρ_{opt} and ρ_0 uses of system throughput of the traffic service at a transport level in optimum and not optimum modes, as

$$Z(\rho_{opt}, C_{opt}, C_0) = \frac{\sqrt{\rho_{opt}}}{1+\rho_{opt}}\left(\frac{\rho_0}{\rho_{opt}} + \frac{\rho_{opt}}{\rho_0}\right). \tag{3}$$

The proof of the theorem 1. For an optimum mode the equation of balance of expenses and losses is fair

$$P\frac{a}{C_{opt}} = (1-P)bC_{opt} = \frac{D_{\min}(C_{opt})}{2},$$

where *P* is probability of that the line will be occupied in an optimum mode. From this equation it is easy to define values of parameters *a* and *b* through $D_{\min}(C_{opt})/2$

$$\begin{cases} a = \dfrac{D_{\min}(C_{opt})}{2}\dfrac{C_{opt}}{P} \\ b = \dfrac{D_{\min}(C_{opt})}{2}\dfrac{1}{(1-P)C_{opt}} \end{cases} \tag{4}$$

The system of parameters (4) allows finding dependence of average expenses on throughput of line C_0 in a not optimum mode as

$$D\left[C_0, C_{opt}, D_{\min}(C_{opt})\right] = \frac{D_{\min}(C_{opt})}{2}\left[\frac{C_{opt}}{C_0} + \frac{C_0}{C_{opt}}\right] \tag{5}$$

Let's take into account that the maximal value of the minimal average expenses

$$D_{\max\min} = \sqrt{ab} = \frac{D_{\min}(C_{opt})}{2}\sqrt{\frac{1}{P(1-P)}}, \tag{6}$$

and optimum value

$$C_{opt} = \sqrt{\frac{a}{b}}\sqrt{\rho_{opt}} = \sqrt{\frac{1-P}{P}}C_{opt}\cdot\sqrt{\rho_{opt}}. \tag{7}$$

Let's execute normalization (5) on (6) and we shall take into account thus optimum value (7), we shall receive

$$\begin{aligned} Z(\rho_{opt}, C_0, C_{opt}) &= \frac{D\left[C_0, C_{opt}, D_{\min}(C_{opt})\right]}{D_{\max\min}} = \\ &= \frac{\sqrt{\rho_{opt}}}{1+\rho_{opt}}\left(\frac{C_{opt}}{C_0} + \frac{C_0}{C_{opt}}\right) \end{aligned} \tag{8}$$

Let's take into account in (8) that

$$\rho_{opt} = R/C_{opt}, \tag{9}$$

$$\rho_o = R/C_0, \tag{10}$$

Finally we shall receive

$$Z(\rho_{opt}, C_{opt}, C_0) = \frac{\sqrt{\rho_{opt}}}{1+\rho_{opt}}\left(\frac{\rho_0}{\rho_{opt}} + \frac{\rho_{opt}}{\rho_0}\right),$$

as was to be shown.

Consequence 1.1. From the Eg.3 follows, that at $\rho_0 = \rho_{opt}$

$$Z_{\min}(\rho_{opt}) = \frac{2\sqrt{\rho_{opt}}}{1+\rho_{opt}}, \tag{11}$$

It means that the minimal normalized value of criterion of average risk depends only on optimum value of system throughput of the traffic service at a transport level.

Consequence 1.2. From a limiting parity

$$\lim_{\rho_0 \to \rho_{opt}} Z(\rho_0, \rho_{opt}) = Z_{\min}(\rho_{opt}) = \frac{2\sqrt{\rho_{opt}}}{1+\rho_{opt}} \tag{12}$$

follows, that at all $\rho_0 \neq \rho_{opt}$ the normalized average expenses

$$Z(\rho_0 \neq \rho_{opt}) > \frac{2\sqrt{\rho_{opt}}}{1+\rho_{opt}}, \tag{13}$$

Therefore this law is offered to be taken as a principle efficiency evaluating of the traffic service at a transport level in heterogeneous computer networks.

The theorem 2. The normalized relative value of criterion of average risk $\delta(\rho_0, \rho_{opt})$, which characterizes a deviation of an actual mode of the traffic service on transport level from an optimum mode, is completely determined by factors ρ_0 And ρ_{opt} systems of service and does not depend on other parameters of a mode because

$$\delta(\rho_0, \rho_{opt}) = \frac{1}{2}\left(\frac{\rho_0}{\rho_{opt}} + \frac{\rho_{opt}}{\rho_0}\right) - 1 \tag{14}$$

The proof the theorem 2. As

$$Z(\rho_0 \neq \rho_{opt}) - Z_{\min}(\rho_{opt}) > 0,$$

and the minimal value

$$Z_{\min}(\rho_{opt}) = \frac{2\sqrt{\rho_{opt}}}{1+\rho_{opt}},$$

that a measure of an inefficiency of a mode of throughput use is expedient to choose increase of the normalized relative expenses in an actual mode in comparison with an optimum mode which plays a role of the standard

$$\delta(\rho_0, \rho_{opt}) = \frac{Z(\rho_0 \neq \rho_{opt}) - Z_{\min}(\rho_{opt})}{Z_{\min}(\rho_{opt})} \tag{15}$$

Substituting in Eq.15 value $Z(\rho_0 \neq \rho_{opt})$ from the Eq.8, and value $Z_{\min}(\rho_{opt})$ from the Eq. 11, we shall receive Eq.14, as was to be shown.

Consequence 2.1. For efficiency evaluating of the traffic service at a transport level it is necessary to know productivity *R* of the traffic source, throughput optimum by criterion of a minimum of average risk of service system C_{opt} and actual average throughput C_0 of this system.

Consequence 2.2. Having executed necessary transformations of an analytical parity (7), we shall receive the following evident form of inefficiency criterion an of the traffic service at a transport level

$$\delta(\rho_{0\delta}, \rho_{opt})_0 = \frac{(\rho_0 - \rho_{opt})^2}{2\rho_o \rho_{opt}} \tag{16}$$

From the Eq. 16 follows, that the inefficiencies criterion (15) of the traffic service is constructed on the metrics of normalized Euclid's space. For increase of method sensitivity at small deviations it is possible to use root-mean-square value of criterion (16)

$$\sigma(\rho_0, \rho_{opt}) = \sqrt{\delta(\rho_0, \rho_{opt})} = \frac{\rho_0 - \rho_{opt}}{\sqrt{2\rho_0 \rho_{opt}}}, \tag{17}$$

where a sign of a difference $\rho_0 - \rho_{opt}$ shows, in what side the value deviation of system throughput of the traffic service C0 from optimum value takes place C_{opt}.

Consequence 2.3. Function (17) is asymmetrical concerning value ρ_{opt}, therefore a deviation of a mode aside values with smaller than ρ_{opt} results in the greater damage, i.e.

$$\sigma_1(\rho_0 - \rho_{opt} < 0) > \sigma_2(\rho_0 - \rho_{opt} > 0)$$

Therefore, if on any circumstances it is impossible to use an optimum mode of the traffic service then it is better to use the traffic service on modes at $\rho_0 > \rho_{opt}$.

V. A technique for efficiency evaluating of the traffic service. Let's consider a technique and algorithm of efficiency evaluating the traffic service of on a concrete example. We shall assume, that intensity of a stream of messages $\lambda_0 = 1$ messages/sec, average length of one message $\ell_0 = 60$ bit. /messages, the allowed normalized average expenses Z^* in an optimum mode should not be more than 1/2. It is necessary to construct algorithm of efficiency evaluating of the traffic service to calculate optimum throughput of the communication line and to estimate efficiency of its use at two relative values of a line throughput:

$$\frac{C_{01}}{C_{opt}} = 0{,}6; \ \frac{C_{02}}{C_{opt}} = 1{,}4 \cdot$$

The algorithm of the problem decision contains such steps.

1. We shall calculate optimum operating ratio of throughput of the communication line from the equation which we shall receive from the Eq.11 at *Zmin* = 1/2:

$$Z_{\min}(\rho_{opt}) = \frac{2\sqrt{\rho_{opt}}}{1+\rho_{opt}} = 1/2 \,.$$

From this equation we shall find, that $\rho_{opt} \approx 0{,}072$.

2. Optimum throughput of the communication line we shall find on speed of the information transfer and optimum operating ratio

$$C_{opt} = \frac{R}{\rho_{opt}} = \frac{\lambda_0 \cdot \ell_0}{\rho_{opt}} \approx \frac{60}{0{,}072} \approx 833{,}33 \text{ bit/sec.}$$

3. Value of the normalized expenses for not optimum modes we shall find under the Eq.8

$$Z_1(\rho_{opt}, \frac{C_{01}}{C_{opt}} = 0{,}6) = \frac{\sqrt{0{,}072}}{1+0{,}072}\left(\frac{1}{0{,}6} + 0{,}6\right) \approx 0{,}5667,$$

$$Z_2(\rho_{opt}, \frac{C_{02}}{C_{opt}} = 1{,}4) = \frac{\sqrt{0{,}072}}{1+0{,}072}\left(1{,}4 + \frac{1}{1{,}4}\right) \approx 0{,}5300.$$

4. We shall define throughputs of not optimum modes

$$C_{01} = 0{,}6\ C_{opt} \approx 0{,}6 \cdot 833{,}33 \approx 500 \text{, bit/sec.}$$

$$C_{02} = 1{,}4\ C_{opt} \approx 1{,}4 \cdot 833{,}33 \approx 1166{,}7 \text{, bit/sec.}$$

5. We shall define increase of the normalized expenses for not optimum modes under the Eq.15

$$\delta_1(0{,}072; 0{,}6) = \frac{Z_1(0{,}072; 0{,}6) - Z_{\min}(0{,}072; 1{,}0)}{Z_{\min}(0{,}072; 1{,}0)} =$$

$$= \frac{0{,}5667 - 0{,}5000}{0{,}5} \approx 0{,}1354,$$

$$\delta_2(0{,}072; 1{,}4) \approx \frac{Z_2(0{,}072; 1{,}4) - Z_{\min}(0{,}072; 1{,}0)}{Z_{\min}(0{,}072; 1{,}0)} \approx$$

$$= \frac{0{,}5300 - 0{,}5000}{0{,}5000} \approx 0{,}06.$$

Thus, a deviation of a line throughput on 40 % from optimum value $C_{opt} = 833{,}33$ bit/sec. aside the greater value results to approximately twice to smaller relative increase of the normalized expenses in comparison with a deviation of throughput on the same size in the smaller side.

6. We shall define operating ratios of the communication line throughput for not optimum modes

$$\rho_{01} = \frac{R}{C_{01}} \approx \frac{60}{500} \approx 0{,}1200,$$

$$\rho_{02} \approx \frac{R}{C_{02}} \approx \frac{60}{1166{,}7} \approx 0{,}0514$$

Thus, increase of operating ratio of throughput in $\frac{\rho_{01}}{\rho_{opt}} \approx \frac{0{,}1200}{0{,}072} \approx 1{,}66$ time results in increase of relative expenses (3) approximately on 13,54 %. Reduction of operating ratio of throughput in $\frac{\rho_{opt}}{\rho_{02}} \approx \frac{0{,}072}{0{,}0514} \approx 1{,}41$ time results in increase of relative expenses (3) approximately on 6 %.

7. We shall define efficiency of use of the communication line throughput by the normalized root-mean-square criterion (17)

$$\sigma_1(0{,}072; 0{,}12) = \frac{0{,}12 - 0{,}072}{\sqrt{2 \cdot 0{,}12 \cdot 0{,}072}} \approx +36{,}8\%,$$

$$\sigma_2(0{,}072; 0{,}0514) \approx \frac{0{,}051 - 0{,}072}{\sqrt{2 \cdot 0{,}051 \cdot 0{,}072}} \approx -24{,}5\%$$

Comparing these values with the appropriate values of item 5, it is possible to draw a conclusion, that the criterion (17) is more sensitive to deviations of a mode, than criterion (16). Besides it allows defining a sign of an absolute error of a deviation that in some cases is convenient circumstance.

8. On the data and results of this example on fig. 1 diagrams of dependences are constructed

$$Z_2(0{,}072; \frac{C_0}{C_{opt}}) = 0{,}25 C_0 / C_{opt} \text{ (a curve1)},$$

$$Z_2(0{,}072; \frac{C_0}{C_{opt}}) = 0{,}25 C_0 / C_{opt} \text{ (a curve 2)},$$

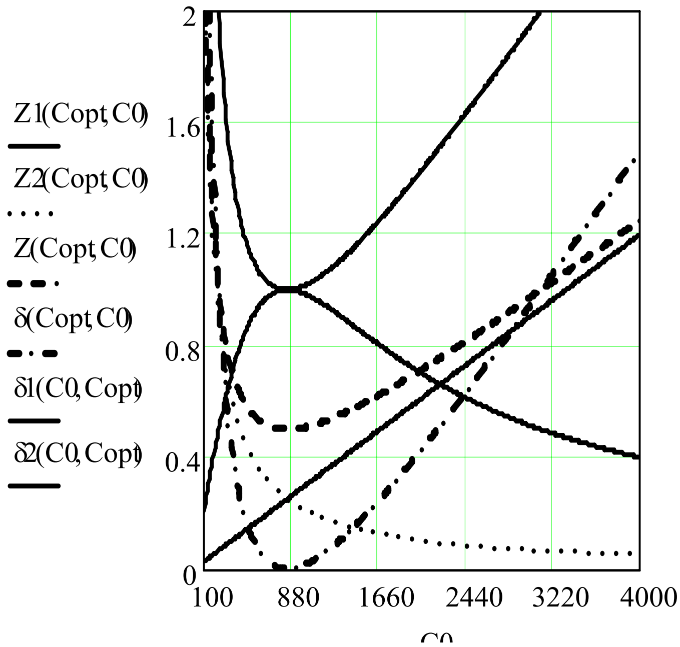

Fig 1. Diagrams of criteria of efficiency

$$Z(0.072;C_0)\approx 0.25\left(\frac{C_{opt}}{C_0}+\frac{C_0}{C_{opt}}\right) \text{ (a curve 3), } \delta(0,072;C_0)\approx 0,5\left(\frac{C_{opt}}{C_0}+\frac{C_0}{C_{opt}}\right)-1 \text{ (a curve 4)}$$

$$\delta 1(0,072;C_0)\approx 0,5\left(\frac{C_{opt}}{C_0}+\frac{C_0}{C_{opt}}\right) \text{(a curve 5)}$$

$$\delta 2(0,072;C_0)\approx 0,5\left(\frac{C_{opt}}{C_0}+\frac{C_0}{C_{opt}}\right)^{-1} \text{(a curve 6).}$$

Criteria *δ1* and *δ2* are used when positive values of criteria are more prefer.

These diagrams well illustrate all basic features of efficiency evaluating of use of the communication line by criterion of a minimum of average expenses (3) with the help of parameters (3), (5), (16) and (17). On the basis of the offered technique the algorithm of efficiency evaluating of the traffic service in heterogeneous computer networks for cases when the necessary initial data are known is developed: a , b , λ_0 , μ_0 , C_0 .

The circuit of algorithm includes 11 mainframes, including the block of 1 initial data and the block of 11 results of calculations:

$$\rho_0, \rho_{opt}, Z(\rho_0, \rho_{opt}), \delta(\rho_0, \rho_{opt}).$$

In system Mathcad experimental researches of influence of change of parameters *a, b* on efficiency of the traffic service and a methodical error of definition of the optimum decision were carried out.

Results of research have shown, that the developed criteria and models allow an adequate estimation of efficiency of the traffic service.

At the same time they show, that by results of direct supervision of the aggregated traffic it is possible to estimate both values of criterion of average risk, and value of parameters *a, b.*

Conclusions. 1. In this work the method and mathematical models of evaluating and the control of efficiency of the traffic service by criterion of a minimum of average risk of losses from idle time of service system and from expectation by packages of service in turn are offered.

2. Theorems 1 and 2 about dependence of the normalized value of criterion of average risk (3) from factors ρ_0 and ρ_{opt} of the traffic service at a transport level in not optimum and optimum modes, and also about laws of a dependences of the normalized relative value of criterion of average risk (8) from factors ρ_0 and ρ_{opt}. are proved.

3. Statements of theorems allow entering criterion (15) inefficiencies of the traffic service, constructed on the metrics of normalized Euclid's space. For increase of sensitivity of a method at small deviations it is possible to use root-mean-square value of criterion as (16), in which sign of a difference $\rho_0 - \rho_{opt}$ shows, in what side the deviation of throughput C_0 value of the traffic service system from optimum value C_{opt} takes place.

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