Для розглянутої системи квазіперіодичні коливання також є неможливими. Це випливає з того, що рівняння кола, якщо записати його у вигляді системи третього порядку

$$\begin{cases} \dfrac{dx}{dt} = y \\[2mm] \dfrac{dy}{dt} = -f(x) - 2\delta y + F(z)\,, \\[2mm] \dfrac{dz}{dt} = 1 \end{cases} \tag{9}$$

характерне тим, що $\mathrm{div}\,f(x, y, z) = -2\delta$ не змінює знака. Тут $f(x, y, z)$ – це вектор-функція правої частини (8). У цьому випадку, як відомо, субгармонічні коливання є неможливими.

Залишається тільки питання про можливість існування хаотичних рухів. На жаль, тут простих критеріїв не існує, тому ця проблема залишається не вирішеною.

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SYMMETRIZATION OF ASYMMETRICAL THREE-PHASE SYSTEMS WITH PERIODIC NONSINUSOIDAL VOLTAGE SOURCE TAKING THEIR INNER IMPEDANCES INTO CONSIDERATION
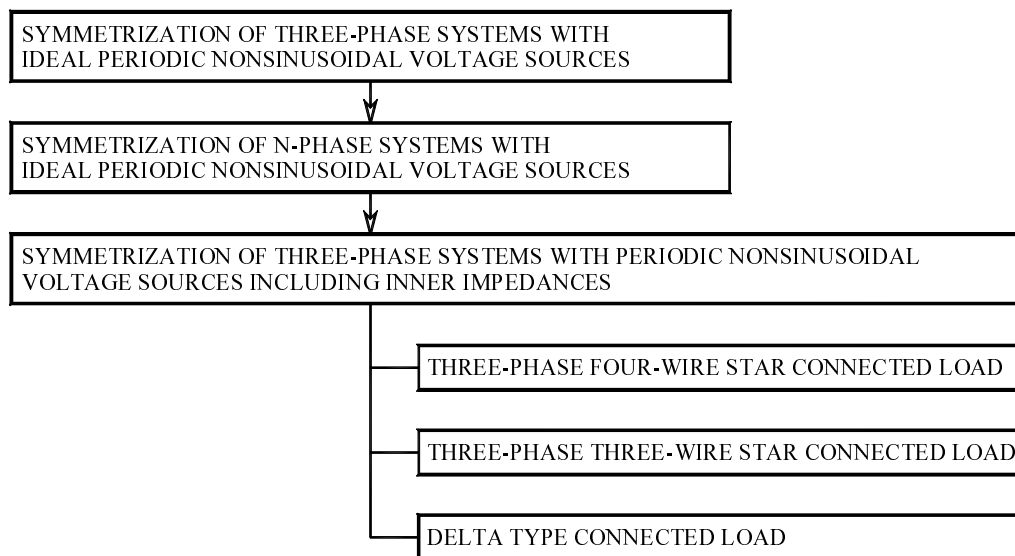
The purpose of this paper is to present the principles of symmetrization of a system supplied from three-phase periodic nonsinusoidal voltage source with asymmetrical phase impedance. Symmetrization of the system is achieved by means of connection of the symmetrization system composed of LC two-terminal networks. Their structure will be determined as a result of proper synthesis process [1, 2, 4]. In the paper, the system including three-phase asymmetrical star connected load (Fig. 2) has been taken into account. It is assumed that voltage source has the inner asymmetry impedance, i.e.: $\bigwedge_{h} Z_{aah} \neq Z_{bbh} \neq Z_{cch}$ and asymmetrical load is described by non-singular full matrix of admittance Y_h for the considered harmonics. If the system works without symmetrization its line currents i_a, i_b, i_c are asymmetrical. With the proper symmetrization network currents become symmetrical and their RMS values and active power on source impedances become lower, but the source generates the active

power $P_2 > P_1$. Deformation of currents of separate phases of the system is reduced, where as the generally understood source power factor is increased. The solution of presented problem may be obtained by means of solving the optimization problem determining the proper purpose function or by means of symmetrical components theory. In this paper the symmetrization problem has been solved by using the symmetrical components theory and compensation of reactive power for each of voltage harmonics under consideration.

Introduction. In works presented earlier, the problem of symmetrization for periodic nonsinusoidal waveforms has been considered mainly for three-phase and multi-phase systems supplied from ideal periodic nonsinusoidal voltage sources [3, 5, 6] (Fig. 1). The paper presents a system which contains a periodic nonsinusoidal voltage source with inner impedances and three-phase asymmetrical star connected load. It is generally assumed that a three-phase periodic voltage source is real, and source voltages are symmetrical in accordance with a basic harmonic, i.e.:

$$e_a(t) = \sqrt{2}\,\text{Re}\sum_{h=1}^{N} E_{ah}\exp\left(jh\omega t\right), \quad e_b(t) = e_a\left(t - \frac{T}{3}\right), \quad e_c(t) = e_a\left(t - \frac{T}{3}\right) \tag{1}$$

where E_{ah} – RMS complex value of the voltage for the successive harmonics: $h = 1,2,...,N$, and $\bigwedge_{h} Z_{aah} \neq Z_{bbh} \neq Z_{cch}$.

```
┌─────────────────────────────────────────────────────────┐
│ SYMMETRIZATION OF THREE-PHASE SYSTEMS WITH               │
│ IDEAL PERIODIC NONSINUSOIDAL VOLTAGE SOURCES             │
└─────────────────────────────────────────────────────────┘
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│ SYMMETRIZATION OF N-PHASE SYSTEMS WITH                   │
│ IDEAL PERIODIC NONSINUSOIDAL VOLTAGE SOURCES             │
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│ SYMMETRIZATION OF THREE-PHASE SYSTEMS WITH PERIODIC NONSINUSOIDAL │
│ VOLTAGE SOURCES INCLUDING INNER IMPEDANCES                    │
└──────────────────────────────────────────────────────────────┘
             │
             ├──────┤ THREE-PHASE FOUR-WIRE STAR CONNECTED LOAD │
             │
             ├──────┤ THREE-PHASE THREE-WIRE STAR CONNECTED LOAD │
             │
             └──────┤ DELTA TYPE CONNECTED LOAD │
```

*Fig.1. The stages of solution of the symmetrization problem for systems
with periodic nonsinusoidal voltage sources.*

Problem of the symmetrization. Generally, when two terminal elements (the symmetrizing system in Fig. 2) which symmetrize line currents are not turned on in the system, i_a, i_b, i_c are asymmetrical. Problem of symmetrization of this system may be presented in the following way: asymmetrical static linear star connected load described by a diagonal matrix Y_h for the considered harmonics supplied from the real source of asymmetrical voltage should be, by means of LC two-terminal elements with interfacial voltage (Fig. 2), turned into the symmetrical system from the point of view of the ideal source terminals (terminals "1"). It is also required that after joining the symmetrizing system, passive power cannot be consumed from the source for each considered

harmonic. For the system from Fig. 2 for each considered harmonic the following dependences are true:

$$
\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}_h = \left(\begin{bmatrix} j_k\,B_{ab} & 0 & 0 \\ 0 & j_k\,B_{bc} & 0 \\ 0 & 0 & j_k\,B_{ca} \end{bmatrix}_h \right) \begin{bmatrix} U_{ab} \\ U_{bc} \\ U_{ca} \end{bmatrix}_h \tag{2}
$$

then:

$$
\begin{bmatrix} I_a^{''} \\ I_b^{''} \\ I_c^{''} \end{bmatrix}_h = \left(\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_h \cdot \begin{bmatrix} j_k\,B_{ab} & 0 & 0 \\ 0 & j_k\,B_{bc} & 0 \\ 0 & 0 & j_k\,B_{ca} \end{bmatrix}_h \right) \begin{bmatrix} U_{ab} \\ U_{bc} \\ U_{ca} \end{bmatrix}_h ; \tag{3}
$$

where:

$$
\begin{bmatrix} U_{ab} \\ U_{bc} \\ U_{ca} \end{bmatrix}_h = \begin{bmatrix} E_a - E_b \\ E_b - E_c \\ E_c - E_a \end{bmatrix}_h - \begin{bmatrix} Z_{za} & -Z_{zb} & 0 \\ 0 & Z_{zb} & -Z_{zc} \\ -Z_{za} & 0 & Z_{zc} \end{bmatrix}_h \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h ; \tag{4}
$$

because:

$$
U_{abh} = U_{ah} - U_{bh} = E_{ah} - Z_{zah}I_{ah} - E_{bh} + Z_{zbh}I_{bh} ,
$$
$$
U_{bch} = U_{bh} - U_{ch} = E_{bh} - Z_{zbh}I_{bh} - E_{ch} + Z_{zch}I_{ch} ,
$$
$$
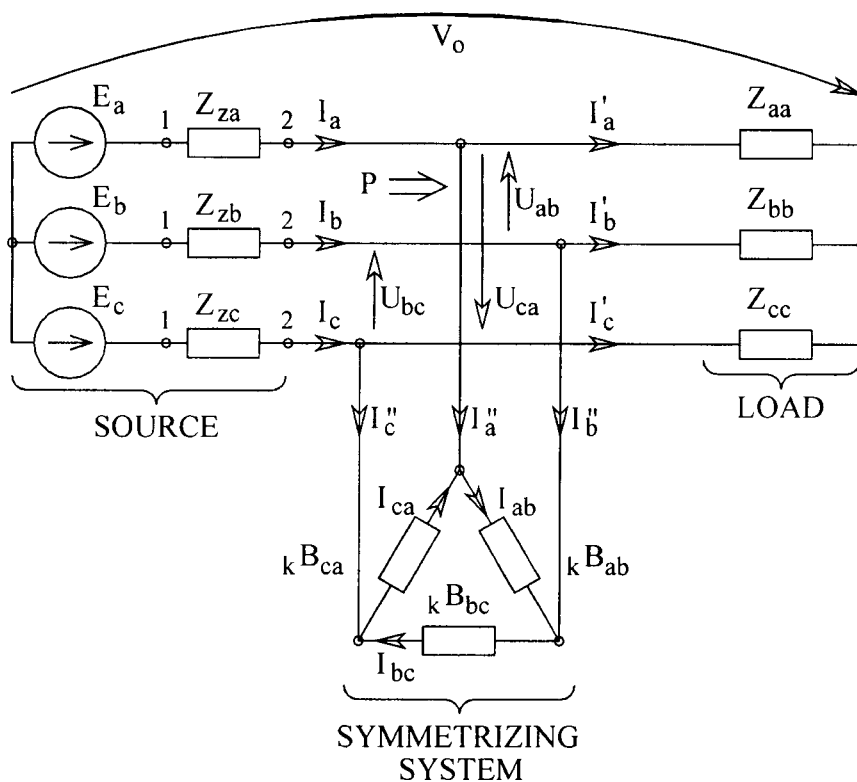U_{cah} = U_{ch} - U_{ch} = E_{ch} - Z_{zch}I_{ah} - E_{ah} + Z_{zah}I_{ah} .
$$

Fig. 2. System under consideration.

After placing the dependence (4) into (3) we obtain the following formula (5):

$$
\begin{bmatrix} I_a^{''} \\ I_b^{''} \\ I_c^{''} \end{bmatrix}_h = \left(\underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{M} \cdot \begin{bmatrix} j_k\,B_{ab} & 0 & 0 \\ 0 & j_k\,B_{bc} & 0 \\ 0 & 0 & j_k\,B_{ca} \end{bmatrix}_h \right) \times
$$

$$
\times \left(\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{N} \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}_h - \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{N} \underbrace{\begin{bmatrix} Z_{za} & 0 & 0 \\ 0 & Z_{zb} & 0 \\ 0 & 0 & Z_{zc} \end{bmatrix}_h}_{Z_Z} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h \right)
$$

and in the matrix form:

$$
\mathbf{I}_h^{''} = \mathbf{M}j_k\mathbf{B}_h\mathbf{N}\mathbf{E}_h - \mathbf{M}j_k\mathbf{B}_h\mathbf{N}\mathbf{Z}_{zh}\mathbf{I}_h . \tag{6}
$$

Moreover, the following dependences are also true:

$$
I_{ah}^{'} + I_{bh}^{'} + I_{ch}^{'} = 0 , \tag{7}
$$

$$
\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}_h - \begin{bmatrix} Z_{za} & 0 & 0 \\ 0 & Z_{zb} & 0 \\ 0 & 0 & Z_{zc} \end{bmatrix}_h \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h - \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}_h \begin{bmatrix} I_a^{'} \\ I_b^{'} \\ I_c^{'} \end{bmatrix}_h = \begin{bmatrix} V_0 \\ V_0 \\ V_0 \end{bmatrix}_h , \tag{8}
$$

and after transformations:

$$
\begin{bmatrix} I_a^{'} \\ I_b^{'} \\ I_c^{'} \end{bmatrix}_h = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_h \left(\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}_h - \begin{bmatrix} Z_{za} & 0 & 0 \\ 0 & Z_{zb} & 0 \\ 0 & 0 & Z_{zc} \end{bmatrix}_h \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h - \begin{bmatrix} V_0 \\ V_0 \\ V_0 \end{bmatrix}_h \right) \tag{9}
$$

from relationships (7) i (9) the value V_{0h} can be determined as:

$$
V_{0h} = \frac{1}{\displaystyle\sum_{\alpha,\beta=a}^{c} Y_{\alpha\beta h}} \left[Y_A E_a + Y_B E_b + Y_C E_c - Y_A Z_{za} I_a - Y_B Z_{zb} I_b - Y_C Z_{zc} I_c \right]_h ; \tag{10}
$$

where:

$$
Y_{Ah} = Y_{aah} + Y_{bah} + Y_{cah} \; ; \; Y_{Bh} = Y_{abh} + Y_{bbh} + Y_{cbh} \; ; \; Y_{Ch} = Y_{ach} + Y_{bch} + Y_{cch} .
$$

Then:

$$
\begin{bmatrix} I_a^{'} \\ I_b^{'} \\ I_c^{'} \end{bmatrix}_h = \underbrace{\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_h}_{Y} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \underbrace{\frac{1}{\displaystyle\sum_{\alpha,\beta=a}^{c} Y_{\alpha\beta h}} \begin{bmatrix} Y_A & Y_B & Y_C \\ Y_A & Y_B & Y_C \\ Y_A & Y_B & Y_C \end{bmatrix}_h}_{L_l} \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}_h \right) -
$$

$$-\underbrace{\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_h}_{Y} \left(\begin{bmatrix} Z_{za} & 0 & 0 \\ 0 & Z_{zb} & 0 \\ 0 & 0 & Z_{zc} \end{bmatrix} - \underbrace{\frac{1}{\sum\limits_{\alpha,\beta=a}^{c} Y_{\alpha\beta h}} \begin{bmatrix} Y_A Z_{za} & Y_B Z_{zb} & Y_C Z_{zc} \\ Y_A Z_{za} & Y_B Z_{zb} & Y_C Z_{zc} \\ Y_A Z_{za} & Y_B Z_{zb} & Y_C Z_{zc} \end{bmatrix}_h}_{Z_1} \right)_h \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h , \quad (11)$$

and in the matrix form:

$$\mathbf{I}'_h = \mathbf{Y}_h(\mathbf{1} - \mathbf{L}_{1h})\mathbf{E}_h - \mathbf{Y}_h(\mathbf{Z}_{zh} - \mathbf{Z}_{1h})\mathbf{I}_h. \quad (12)$$

Because:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_h = \begin{bmatrix} I'_a \\ I'_b \\ I'_c \end{bmatrix}_h + \begin{bmatrix} I''_a \\ I''_b \\ I''_c \end{bmatrix}_h \quad (13)$$

after placing the dependences (6) and (12) into (13) and after the suitable transformations the following formula can be obtain:

$$\mathbf{I}_h = \left[\mathbf{1} + \mathbf{Y}_h(\mathbf{Z}_{zh} - \mathbf{Z}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{NZ}_{zh}\right]^{-1}\left[\mathbf{Y}_h(\mathbf{1} - \mathbf{L}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{N}\right]\mathbf{E}_h = \mathbf{Y}_{sh}\mathbf{E}_h. \quad (14)$$

By applying symmetrical components of voltages and currents we receive the equation:

$$\mathbf{SI}_S = \left[\mathbf{1} + \mathbf{Y}_h(\mathbf{Z}_{zh} - \mathbf{Z}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{NZ}_{zh}\right]^{-1}\left[\mathbf{Y}_h(\mathbf{1} - \mathbf{L}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{N}\right]\mathbf{SE}_S; \quad (15)$$

where:

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

– symmetrical component matrix; $\alpha = e^{j120^\circ}$; $\alpha^2 = e^{-j120^\circ}$;

$$\mathbf{I_S} = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}_h$$

– components of symmetrical conduct currents of the source for h-harmonic;

$$\mathbf{E_S} = \begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix}_h$$

– components of symmetrical conduct voltages of the source for h-harmonic.

After transformation of the equation (15) the finally dependence can be determined:

$$\mathbf{I}_S = \mathbf{S}^{-1}\left[\mathbf{1} + \mathbf{Y}_h(\mathbf{Z}_{zh} - \mathbf{Z}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{NZ}_{zh}\right]^{-1}\left[\mathbf{Y}_h(\mathbf{1} - \mathbf{L}_{1h}) + \mathbf{Mj}_k\mathbf{B}_h\mathbf{N}\right]\mathbf{SE}_S. \quad (16)$$

Because the circuit being under consideration is three-phase and three-wire one, so $I_o = 0$ and $E_o = 0$. When we solve the equation (16), we receive dependences of the type:

$$I_{1h} = Y_{11h}E_{1h} + Y_{12h}E_{2h}, \quad (17)$$

$$I_{2h} = Y_{21h}E_{1h} + Y_{22h}E_{2h}. \tag{18}$$

As it is assumed that the source voltages are symmetrical in accordance with the basic harmonic, so for a given harmonic of 3n+1 order there is a positive-sequence voltage, and for 3n-1 there is a negative-sequence voltage. Thus, for a given harmonic being considered in symmetrical components of the source voltage there is only one component. The system is symmetrical, when the symmetrical component resulting from the harmonic voltage calls a corresponding symmetrical component of the current, i.e.:

$$\bigwedge_{h \in 3n+1} E_{1h} \neq 0 \ , \ E_{2h} = 0 \ \text{thus it should be:} \ \bigwedge_{h \in 3n+1} I_{1h} \neq 0 \ , \ I_{2h} = 0,$$

so the following condition should be fulfilled:

$$\bigwedge_{h \in 3n+1} Y_{21h} = 0. \tag{19}$$

$$\bigwedge_{h \in 3n-1} E_{1h} = 0 \ , \ E_{2h} \neq 0 \ \text{thus it should be:} \ \bigwedge_{h \in 3n-1} I_{1h} = 0 \ , \ I_{2h} \neq 0,$$

so the following condition should be fulfilled:

$$\bigwedge_{h \in 3n-1} Y_{12h} = 0. \tag{20}$$

Moreover, for each harmonic being under consideration a condition that the system cannot consume reactive power from the ideal source (terminals «1») is imposed. It can be noticed that for each harmonic being under consideration we receive a system of three equations with three unknown values of the symmetrizing system susceptances $_k B_{ah}$, $_k B_{bh}$, $_k B_{ch}$ from dividing the equations (19) and (20) into an imaginary part and a real part and from a zero condition of reactive power consumption by the system. The symmetrization problem for ideal sources considered in the article [3] resulted in determination of the symmetrizing system susceptances in an analytical way (results in a closed form). It is impossible to obtain such results for the system supplied from the source with inner impedances. Due to inversion of the matrix with unknown values of the susceptances $_k B_{ah}$, $_k B_{bh}$, $_k B_{ch}$ (formula (16)) – division of the equation (19) and (20) into a real and imaginary part to determine susceptances of the symmetrizing system results in the system of nonlinear equations.

That is why a determination of the symmetrizing susceptances value from the equation (19) and (20) and from a zero condition of passive power consumption by the system has been carried out in a numerical way, for each harmonic being under consideration. The suitable numerical algorithm will be presented at the conference.

Conclusions. On the basis of the considerations presented above the following conclusions can be formulated: – symmetrization carried out according to the described algorithm allows to decrease RMS values of the source line currents considerably;

– symmetrization of the system causes a symmetrical decomposition of the source phase currents, i.e.: $i_b(t) = i_a\left(t - \dfrac{T}{3}\right), \quad i_c(t) = i_a\left(t + \dfrac{T}{3}\right);$

– after symmetrization the source voltage coefficient λ increases, the source supplies a bigger active power, the active power losses decrease considerably on the source inner impedances.

It should be noticed that the source inner impedances can constitute, in a general case, impedances of the transmission system between an ideal source and a load. This fact makes an analysis of a simple power network possible. A suitable calculation example will be presented at the conference.

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OPTIMIZATION OF ELECTRICAL ENERGY QUALITY IN SYSTEMS WITH NONSINUSOIDAL WAVEFORMS

1. INTRODUCTION

Methods and ways of describing power and qualitative properties of systems with nonsinusoidal waveforms have not been unified up to the present. However, the definition of reactive power recommended by International Electrotechnics Committee (IEC) has been changed many times during the last tens of years. Therefore analysis of possibilities of limiting the active power losses and waveform distortion in the systems is complex while the obtained results are often controversial. In the presented project KBN Nr 8T 10a 06811 the following quantities (generally accepted in electrotechnics) have been suggested for describing the analysis and optimization of such systems:

– instantaneous power, active power and apparent power as well as rms values of currents and voltages,

– multicriterial goal functions of the circuit operating conditions which are based on the above-mentioned quantities and they describe the active power (energy) losses and distortion of current voltage waveforms in the systems.