

чується відповідним вибором порядку числової схеми та кроку дискретизації. Наведена модифікація методу є його подальшим удосконаленням у підвищенні точності і зменшенні обчислювальних витрат.

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MAGNETIC FIELD AND IMPEDANCE OF A CIRCUIT WITH GROUND RETURN FOR A TWO-LAYER OVERHEAD CONDUCTOR

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For a circuit with ground return consisting of a long rectilinear overhead conductor the vector magnetic potential is introduced. Transformation of the differential equations describing the magnetic potential by means of Fourier's transformation yields the vector potential in the form of an analytical formula. Next, the magnetic field strength represented by improper integrals is determined. Appropriate representation of those calculations reduces them to the calculation of Laplace's transformation, thus yielding analytical formulae describing the magnetic field in the circuit with ground return.

The impedance of a circuit with ground return for a two-layer overhead conductor is represented as the total of external and internal impedances. The external impedance is determined using the vector magnetic potential to compute the induced electric field strength. The internal impedance of the conductor is determined using the solution of the Helmholtz's equation for electric field strength.

Finally a computer simulation in Delphi of the discussed question is presented.

Introduction. A circuit with ground return consists of a two-layer overhead conductor placed at the height of y_k . – Fig. 1. In the conductor sinusously alternating current of pulsation ω and complex r.m.s. value I is forced. The conductor has one of its terminals grounded.

The cylindrical conductor is two-layered with a core of radius R_1 and conductivity γ_1 , and with an external layer of radius R_1 and R_2 and conductivity γ_2 – Fig. 2. It is parallel to the ground surface.

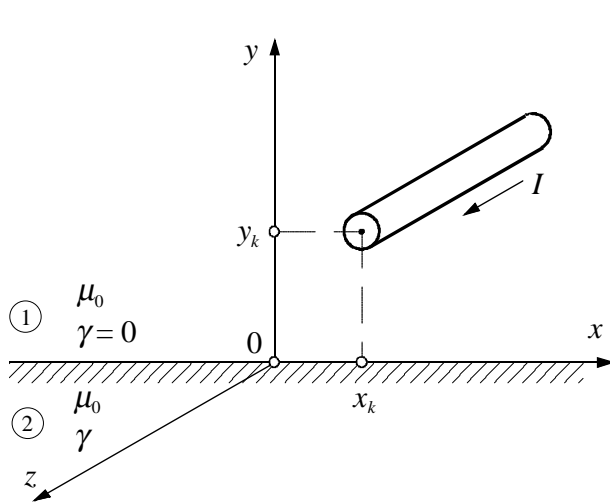
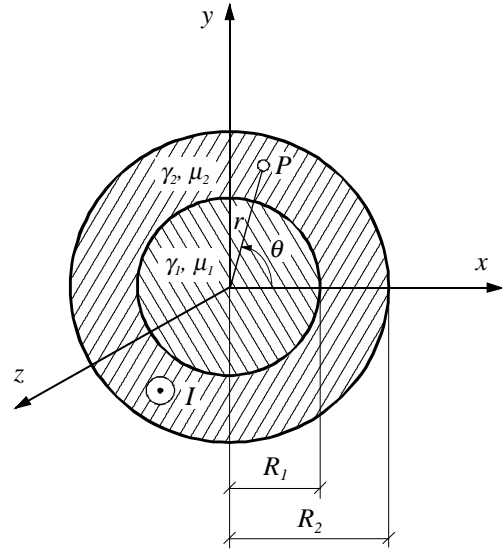


Fig. 1. A circuit with ground return

Fig. 2. Two-layer conductor with current I

Moreover, we assume that the ground constitutes a homogenous environment of magnetic permittivity μ_0 and constant conductivity γ , and we assume that the ground surface is a plane.

If we leave out phenomena taking place at the ends of the system, the electromagnetic field in the system in question is two-dimensional and has the same form in all planes perpendicular to the conductor axis, i.e. it is a function of two variables x and y . Magnetic vector potential is used in the analysis of the magnetic field.

Impedance of the loop with ground return equals the external impedance Z_z and the internal impedance of the conductor Z_w . Then the impedance of the loop with ground return per unit length

$$Z = Z_z + Z_w. \quad (1)$$

Vector potential. In the system depicted in Fig. 1 the magnetic vector potential is parallel to the conductor axis that is $\mathbf{A}(x, y) = \mathbf{1}_z A(x, y)$. In Ref. [2] M. Krakowski gives the Poisson's equation (Formula (11.83), p.257) in area "1" above the ground and Helmholtz's equation (Formula (11.84), p.258) in area "2", that is in the ground. According to the denotations used in fig. 1, in particular having introduced distance x_k , these equations are of the following form:

$$\frac{\partial^2 A_1(x, y)}{\partial x^2} + \frac{\partial^2 A_1(x, y)}{\partial y^2} = -\mu_0 I \delta(x - x_k) \delta(y - y_k), \quad (2)$$

and

$$\frac{\partial^2 A_2(x, y)}{\partial x^2} + \frac{\partial^2 A_2(x, y)}{\partial y^2} = \beta^2 A_2(x, y), \quad (2a)$$

where $\delta(x - x_k)$ and $\delta(y - y_k)$ are Dirac's discrete functions, while

$$\beta^2 = j\omega\mu_0\gamma. \quad (2b)$$

Transformation of each following term in equations (2) and (2a) by means of Fourier's transform in relation to variable x , then formulation and transformation of boundary_conditions, finally determination of inverse Fourier's transform according to the way applied in Ref. [2,

p. 258–260], yields the vector potential in area „1” (the formula is similar to formula (11.59) in Ref. [2, p.260].

$$A_1(x, y) = \frac{\mu_0 I}{\pi} \left[Q_1 + \frac{1}{4} \ln \frac{(x - x_k)^2 + (y + y_k)^2}{(x - x_k)^2 + (y - y_k)^2} \right], \quad (3)$$

where function Q_1 can be written as

$$Q_1 = \int_0^{\infty} \frac{e^{-\varpi(y+y_k)} \cos[\varpi(x-x_k)]}{\varpi + \sqrt{\varpi^2 + \beta^2}} d\varpi. \quad (3a)$$

After the substitution:

$$\left. \begin{aligned} \varpi &= \alpha u, & d\varpi &= \alpha du, \\ \alpha &= \sqrt{\varpi \mu_0 \gamma}, & \beta &= \sqrt{j \alpha} \end{aligned} \right\} \quad (4)$$

and

$$\left. \begin{aligned} p &= \alpha (y + y_k), \\ q &= \alpha (x - x_k) \end{aligned} \right\}, \quad (4a)$$

the function Q_1 can be written as

$$Q_1(p, q) = \int_0^{\infty} \frac{e^{-pu} \cos qu}{u + \sqrt{u^2 + j}} du. \quad (5)$$

Magnetic field. The magnetic field in area „1” (above the ground) is computed from equation [4]

$$\mathbf{H}_1(x, y) = \frac{1}{\mu_0} \mathbf{rot} A_1(x, y) = \mathbf{1}_x H_{1x}(x, y) + \mathbf{1}_y H_{1y}(x, y), \quad (6)$$

where the components of the magnetic field strength

$$\begin{aligned} H_{1x}(x, y) &= \frac{1}{\mu_0} \frac{\partial A_1(x, y)}{\partial y} = \frac{\alpha}{\pi} I dQ_{1p}[\alpha(y + y_k), \alpha(x - x_k)] + \\ &+ \frac{1}{2\pi} I \left[\frac{y + y_k}{(x - x_k)^2 + (y + y_k)^2} - \frac{y - y_k}{(x - x_k)^2 + (y - y_k)^2} \right] \end{aligned} \quad (6a)$$

and

$$\begin{aligned} H_{1y}(x, y) &= -\frac{1}{\mu_0} \frac{\partial A_1(x, y)}{\partial x} = -\frac{\alpha}{\pi} I dQ_{1q}[\alpha(y + y_k), \alpha(x - x_k)] - \\ &- \frac{1}{2\pi} I \left[\frac{x - x_k}{(x - x_k)^2 + (y + y_k)^2} - \frac{x - x_k}{(x - x_k)^2 + (y - y_k)^2} \right] \end{aligned} \quad (6b)$$

In formula (6a) the derivative of function Q_1 in relation to variable q

$$dQ_{1q}(p, q) = \frac{\partial Q_1(p, q)}{\partial q} = -\int_0^{\infty} \frac{ue^{-pu} \sin qu}{u + \sqrt{u^2 + j}} du, \quad (6c)$$

while in (6b) the derivative of function Q_1 in relation to variable p

$$dQ_{1p}(p, q) = \frac{\partial Q_1(p, q)}{\partial p} = -\int_0^{\infty} \frac{ue^{-pu} \cos qu}{u + \sqrt{u^2 + j}} du. \quad (6d)$$

Substitution into formulae (6c) and (6d) of the following

$$\left. \begin{aligned} \sin qu &= \frac{1}{2j}(e^{jq u} - e^{-jq u}) \\ \cos qu &= \frac{1}{2}(e^{jq u} + e^{-jq u}) \\ \frac{-u}{u + \sqrt{u^2 + j}} &= j(u\sqrt{u^2 + j} - u^2) \end{aligned} \right\} \quad (7)$$

yields respectively

$$dQ_{1q}(p, q) = \frac{1}{2} \left\{ \int_0^{\infty} (u\sqrt{u^2 + j} - u^2) e^{-su} du \Big|_{s=p-jq} - \int_0^{\infty} (u\sqrt{u^2 + j} - u^2) e^{-su} du \Big|_{s=p+jq} \right\}, \quad (8)$$

$$dQ_{1p}(p, q) = \frac{1}{2} j \left\{ \int_0^{\infty} (u\sqrt{u^2 + j} - u^2) e^{-su} du \Big|_{s=p-jq} + \int_0^{\infty} (u\sqrt{u^2 + j} - u^2) e^{-su} du \Big|_{s=p+jq} \right\}. \quad (9)$$

For $Re\{s\} = p > 0$ the integral from formulas (8) and (9) is the Laplace's transform of the integrand, i.e

$$\int_0^{\infty} (u\sqrt{u^2 + j} - u^2) e^{-su} du = L\{u\sqrt{u^2 + j} - u^2\} = L\{u\sqrt{u^2 + j}\} - L\{u^2\}. \quad (10)$$

Laplace's transform from function u^2 and function $u\sqrt{u^2 + j}$ is calculated by Mathematica 3.0 program – Ref. [6].

Thus the components of the magnetic field strength are written as analytical formulas which enables us to calculate vector $\mathbf{H}_1(x, y)$ in any point of area „1” above the ground

External impedance. External impedance Z_z per unit length of a loop with ground return is determined with assumption that the conductor in the circuit with ground return from Fig. 1 is of infinite length. Then the vector magnetic potential in area „1” above the ground is given by formula (3).

External impedance of the conductor is given (formula (8.76) in Ref. [4, p.168]) by the following formula:

$$Z_z = -\frac{\int \mathbf{E}_{inl} \cdot d\mathbf{l}}{I}, \quad (11)$$

where \mathbf{E}_{in} is the strength of electric field induced due to time changes of the external magnetic field, while the integration curve is the line C on the conductor surface.

The induced electric field strength (for $\mathbf{A}_1 = \mathbf{1}_z A_1$)

$$\mathbf{E}_{inl} = -j\omega \mathbf{A}_1 = -j\omega A_1 \mathbf{1}_z = E_{inlz} \mathbf{1}_z, \quad (12)$$

from where after the substitution of formula (3) one obtains

$$E_{inlz}(x, y) = -\frac{\omega\mu_0 I}{\pi} \left[Q(p, q) + \frac{j}{4} \ln \frac{(x-x_k)^2 + (y+y_k)^2}{(x-x_k)^2 + (y-y_k)^2} \right], \quad (13)$$

where function

$$Q(p, q) = j Q_1(p, q) = j \int_0^{\infty} \frac{e^{-pu} \cos qu}{u + \sqrt{u^2 + j}} du, \quad (13a)$$

i.e. that the function is determined by the formula given by M.Krakowski in Ref. [4 (Formula (9.137), p. 207)].

Substitution into formula (13a) of the following

$$\left. \begin{aligned} \cos qu &= \frac{1}{2}(e^{jq u} + e^{-jq u}) \\ \frac{1}{u + \sqrt{u^2 + j}} &= -j(\sqrt{u^2 + j} - u) \end{aligned} \right\}, \quad (14)$$

yields function Q given by the formula

$$Q(p, q) = \frac{1}{2} \left\{ \int_0^{\infty} (\sqrt{u^2 + j} - u) e^{-su} du \Big|_{s=p-jq} - \int_0^{\infty} (\sqrt{u^2 + j} - u) e^{-su} du \Big|_{s=p+jq} \right\}. \quad (15)$$

For $\operatorname{Re}\{s\} = p > 0$ the integral in formula (15) is the Laplace's transform of the integrand, i.e.

$$\int_0^{\infty} (\sqrt{u^2 + j} - u) e^{-su} du = L\{\sqrt{u^2 + j} - u\} = L\{\sqrt{u^2 + j}\} - L\{u\}. \quad (16)$$

Laplace's transform from function u and function $\sqrt{u^2 + j}$ is calculated by Mathematica 3.0 program – Ref. [6].

According to the formula (11) and assuming that $2y_k \gg R_2$, external impedance of the conductor per unit length is given by the formula

$$Z_z = -\frac{E_{inlz}(x_k, y_k - R_2)}{I} = \frac{\varpi \mu_0}{\pi} \left[Q(2\alpha y_k, 0) + \frac{j}{2} \ln \frac{2y_k}{R_2} \right]. \quad (17)$$

Internal impedance. Electric field strengths in a two-layer conductor are determined by taking into consideration the given current I . In Ref. [2] the authors have proved that these fields depend on the variable r of the cylindrical coordinate system and are given by following formulas:

– for $0 \leq r \leq R_1$

$$E_1(r) = \mathbf{1}_z E_1(r) = \mathbf{1}_z I C_1 \mathcal{J}_0(\beta_1 r), \quad (18)$$

where $\beta_1 = \sqrt{-j\varpi\mu_1\gamma_1}$,

– for $R_1 \leq r \leq R_2$

$$E_2(r) = \mathbf{1}_z E_2(r) = \mathbf{1}_z I [C_2 \mathcal{J}_0(\beta_2 r) + C_3 \mathcal{N}_0(\beta_2 r)], \quad (18a)$$

where $\beta_2 = \sqrt{-j\varpi\mu_2\gamma_2}$,

$\mathcal{J}_0(\beta r)$, $\mathcal{N}_0(\beta r)$ – Bessel functions of first and second kind, 0 order,

Constants C_1 , C_2 and C_3 are obtained from the following system of equations:

$$\begin{bmatrix} J_0(\beta_1 R_1) & -J_0(\beta_2 R_1) & -N_0(\beta_2 R_1) \\ -\frac{\gamma_1}{\beta_1} J_0'(\beta_1 R_1) & \frac{\gamma_2}{\beta_2} J_0'(\beta_2 R_1) & \frac{\gamma_2}{\beta_2} N_0'(\beta_2 R_1) \\ 0 & \frac{\gamma_2}{\beta_2} J_0'(\beta_2 R_2) & \frac{\gamma_2}{\beta_2} N_0'(\beta_2 R_2) \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2\pi R_2} \end{bmatrix}. \quad (18b)$$

The internal impedance of the conductor segment equals the quotient of the complex voltage value along the line on that segment surface divided by the complex value of the current in the conductor (Ref. [4], formula (8.77), p. 167):

$$Z_{wl} = \frac{\int_C E_2(r = R_2) \cdot dl}{I}. \quad (19)$$

Thus the internal impedance per unit length of the conductor

$$Z_w = C_2 J_0(\beta_2 R_2) + C_3 N_0(\beta_2 R_2). \quad (19a)$$

Conclusions. To enable the determination of the magnetic field and impedance of a circuit with ground return for a two-layer overhead conductor a project in Delphi has been elaborated – Fig. 3.

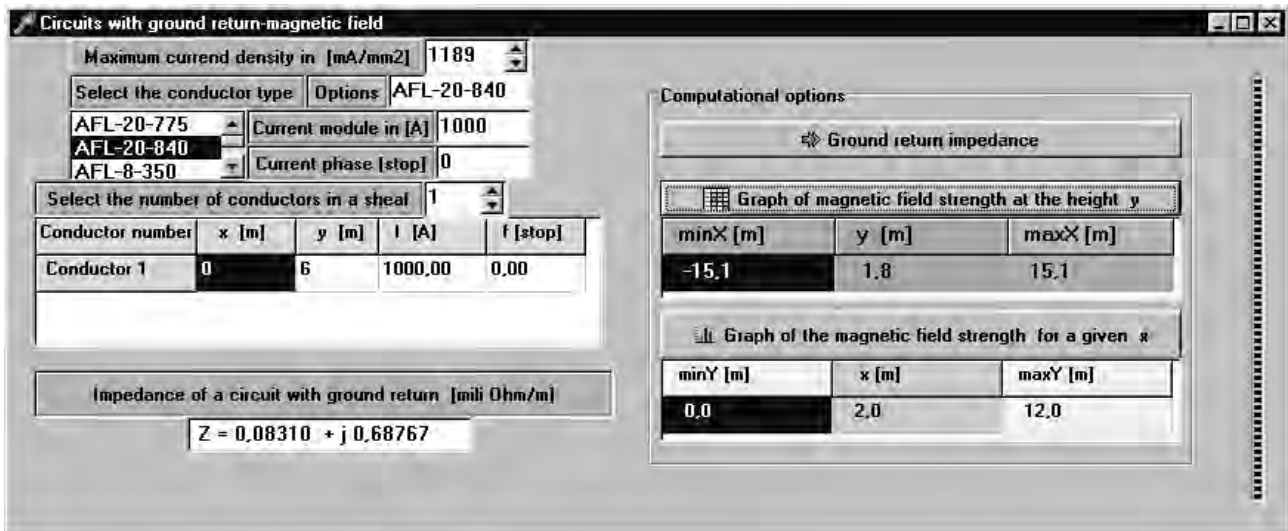


Fig. 3. Form of a numerical simulator for determination of the impedance of a circuit with ground return

For actual computations of the magnetic field we have chosen a two-layer conductor AFL-20-840 placed at the height of $y_k = 6$ m. Fig. 4 depict the distribution of magnetic field strengths along some chosen straight line in the system of the two-layer conductor-ground.

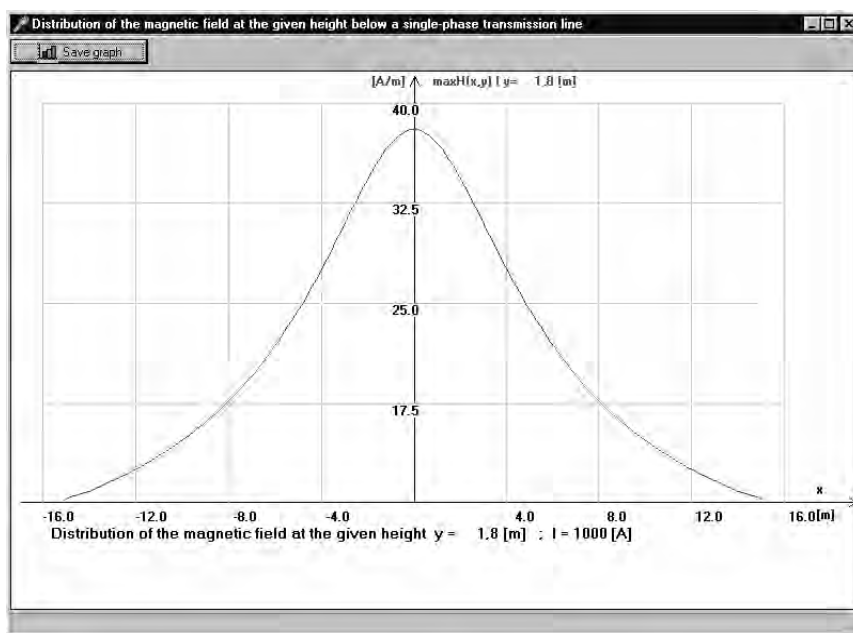


Fig. 4. Distribution of the magnetic field along the straight line
 $y = 1,8 \text{ m}$; $y_k = 6 \text{ m}$, $I = 1 \text{ kA}$.

Table

Impedance of a circuit with ground return

	Type of conductor	Impedance in $\text{m}\Omega \cdot \text{m}^{-1}$
1	AFL-20-670	0,09171 + j 0,69503
2	AFL-20-840	0,08310 + j 0,68767
3	AFL-8-350	0,12823 + j 0,71249
4	AFL-8-525 (line 400 kV)	0,10357 + j 0,70020
5	AFL-6-25	1,18235 + j 0,82786
6	AFL-6-240	0,16817 + j 0,72463
7	AFL-1,7-50 (ground cond.)	0,61650 + j 0,78787

For numerical computation of the impedance of a circuit with ground return we have chosen a few types of two-layer conductors, and the computation results are presented in Table.

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