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SIMULATION OF INTERFERENCE PROCESSES OF VIBRATION CHARGE IN THE ELECTROMAGNETIC CONTOUR

Здійснено інтегрування рівняння вимушеного коливання заряду в гармонійному електромагнітному осцилографі. Було отримано точні вирази функцій обміну синусоїдного джерела і омічного опору для осцилографа енергії і визначено функцію кореляції з її спектром коливань.

Integrating the equation of forced vibration of the charge in the harmonic electromagnetic oscillator was carried out. The exact expressions for functions of oscillator energy exchange with sinusoidal source and Ohm's resistors were obtained and function correlation with its oscillation spectrum was determined.

Introduction. Vibrations of the harmonic electromagnetic oscillator (*HEMO*) have been quite sufficiently studied, and the corresponding results have been generalised in a great number of monographs, including [1–4]. Nevertheless, the kinetics of the development of oscillator energy exchange with Ohm's resistors and an external sinusoidal electromotion force (*EMF)* has not been thoroughly investigated, and this fact determined the objective of this work. Its topicality confirmed that relaxation processes of oscillator energy playing an important part not only in the physical [5, 6], but also at the study of more general issues [7, 8].

General relations. In this work, one-dimenssional vibrations of *HEMO* in the external sinusoidal force potential $U(t) = U_0 \begin{pmatrix} \sin \Omega t \\ \cos \Omega t \end{pmatrix}$ are considered. It is know, that vibration of charge are described by the differential equation

$$\frac{d^2 q(t)}{dt^2} + 2\gamma \frac{dq(t)}{dt} + \omega_0^2 q(t) = \frac{U(t)}{L} \qquad (1)$$

having solution

$$q(\Omega,\gamma,t) = \left(A\cos\omega t + B\sin\omega t\right)\exp(-\gamma t) + y_m\begin{pmatrix}\cos(\Omega t-\phi)\\ \sin(\Omega t-\phi)\end{pmatrix}. \quad (2)$$

Where ω_0 is resonance frequency of the oscillator, $\left(2\gamma = \frac{R}{L}\right)$ $q_m = \frac{U_0/L}{\omega_0^2-\Omega^2}$ is factor of the linear damping amplitude of the charge vibrations, which is displaced by phase by value $\phi = \phi(\Omega,\gamma)$ relative to the impact of the external source, and $A = q_0 - q_m\cos\phi$, $B = \frac{I_0 - q_m\Omega\sin\phi + q_0\gamma - \gamma q_m\cos\phi}{\omega}$ is constants of integration, if at the initial moment the oscillator had current I_0 at the point having charge q_0.

The conservation of energy is known as:

$$\frac{L}{2}\left(\frac{dq(\gamma,t)}{dt}\right)^2 + \frac{L\omega_0^2 q^2(\gamma,t)}{2} = W_0 + Q(\Omega,\gamma,t) + A(\Omega,\gamma,t). \quad (3)$$

Where $Q(\Omega,\gamma,t) = -2\gamma L\int\left(\frac{dq(\gamma,t)}{dt}\right)^2 dt$ is the dissipative Rayleigh function, which describes the process of energy exchange between the electromagnetic oscillator and the Ohm's resistors, and $A(\Omega,\gamma,t) = \int\left(\frac{dq}{dt}\right)U(\Omega,t)dt$ is energy exchange between the electromagnetic oscillator, whose frequency of natural vibrations equals $\omega^2 = \omega_0^2 - \gamma^2$, and the external force $U(\Omega,t)$ with the impact frequency Ω, respectively.

Theoretical results. After integrating (3), taking into account (2), the active function is conveniently presented as a sum of two addends:

$$A(\Omega,\gamma,t) = A_1(\Omega,\gamma,t) + A_2(\Omega,\gamma,t), \quad (4)$$

where

$$A_1(\Omega,\gamma,t) = \frac{U_0 q_m\Omega}{2}\begin{pmatrix} t\sin\phi + \frac{1}{2\Omega}\cos(2\Omega t-\phi)\\ t\sin\phi - \frac{1}{2\Omega}\cos(2\Omega t-\phi)\end{pmatrix},$$

$$A_2(\Omega,\gamma,t) = \frac{U_0}{2}e^{-\gamma t}\left\{\Theta_1\begin{pmatrix}\delta_1\cos\Delta_+ t + \delta_2\sin\Delta_+ t\\ \delta_1\cos\Delta_+ t + \delta_2\sin\Delta_+ t\end{pmatrix} + \Theta_2\begin{pmatrix}\delta_3\cos\Delta_- t + \delta_4\sin\Delta_- t\\ \delta_1\cos\Delta_+ t + \delta_2\sin\Delta_+ t\end{pmatrix}\right\},$$

$$\Delta_\pm = \omega \pm \Omega,\ \Theta_{1,2} = \left[\gamma^2 + \Delta_\pm^2\right]^{-1},\ \begin{matrix}\delta_1 = -\gamma\alpha_1 - \Delta_+\alpha_2, & \delta_2 = -\gamma\alpha_2 + \Delta_+\alpha_1,\\ \delta_3 = -\gamma\alpha_1 - \Delta_-\alpha_2, & \delta_4 = -\gamma\alpha_2 + \Delta_-\alpha_1,\end{matrix}$$

$$\alpha_1 = -\gamma A + \omega B,\ \alpha_2 = -\gamma B - \omega A.$$

Rayleigh function can also be conveniently presented as a sum, but of three addends:

$$Q(\Omega,\gamma,t) = Q_1(\Omega,\gamma,t) + Q_2(\Omega,\gamma,t) + Q_3(\Omega,\gamma,t) \quad (5)$$

where

$$Q_1(\Omega,\gamma,t) = -2\gamma L e^{-2\gamma t}\left\{\frac{\alpha_1^2+\alpha_2^2}{-4\gamma}+\frac{1}{4\omega_0^2}\Big[-\gamma(\alpha_1^2-\alpha_2^2)+2\omega\alpha_1\alpha_2\cos 2\omega t\right.$$

$$\left.+\Big(\omega(\alpha_1^2-\alpha_2^2)-2\gamma\alpha_1\alpha_2\Big)\sin 2\omega t\Big]\right\}, \qquad Q_2(\Omega,\gamma,t) = -\gamma L q_m^2\Omega^2\begin{pmatrix} t-\frac{1}{2\Omega}\sin[2(\Omega t-\phi)] \\ t+\frac{1}{2\Omega}\sin[2(\Omega t-\phi)]\end{pmatrix},$$

$$Q_3(\Omega,\gamma,t) = 2\gamma L q_m\Omega e^{-\gamma t}\left\{\Theta_1\begin{pmatrix}[\theta_1\sin\Delta_+t+\theta_2\cos\Delta_+t] \\ [\theta_5\cos\Delta_-t-\theta_6\sin\Delta_-t]\end{pmatrix}+\Theta_2\begin{pmatrix}[\theta_3\sin\Delta_-t+\theta_4\cos\Delta_-t] \\ [\theta_7\cos\Delta_-t-\theta_8\sin\Delta_-t]\end{pmatrix}\right\},$$

$$\begin{array}{l}\theta_1=-\gamma\beta_1+\Delta_+\beta_2, \quad \theta_2=-\gamma\beta_2-\Delta_+\beta_1, \\ \theta_3=-\gamma\beta_3+\Delta_-\beta_4, \quad \theta_4=-\gamma\beta_4-\Delta_-\beta_3,\end{array},$$

$$\begin{array}{l}\beta_1=\alpha_1\cos\phi-\alpha_2\sin\phi, \quad \beta_2=\alpha_1\sin\phi-\alpha_2\cos\phi, \\ \beta_3=-\alpha_1\cos\phi-\alpha_2\sin\phi, \quad \beta_4=-\alpha_1\sin\phi+\alpha_2\cos\phi,\end{array}.$$

Discussion. ***Kinetics of vibrations charge in the undamped HEMO.*** In this case $\gamma = 0$, and the following conclusions can be drawn from expressions (4)–(5):

a) At low frequencies $\Omega\langle\langle\omega_0$, in the course of time, energy exchange between the external source and the undamped *HEMO* takes on the character of pulsations lasting $\tau_A = \frac{\pi}{\Omega}$, each of them consisting of pulse strings $A(\Omega,t)$ lasting $\tau_i = \frac{2\pi}{\omega_0}$ each.

Vibrates with frequency Δ_-, the vibrations being modulated with long-wave oscillations having period $\frac{2\pi}{\Omega}$. Their maxima form during those intervals of time, when the impulse of energy transfer from the source to the oscillator is in phase with the amplitude of its vibrations. If the corresponding oscillations of functions $A(\Omega,t)$ and $q(\Omega,\gamma,t)$ come to be in antiphase with time, minima of long-wave modulation of the vibration spectrum form during the given intervals of time (Fig. 1). Therefore, at $\Omega\langle\langle\omega_0$ pulsation of the amplitude of forced vibrations of the oscillator does not occur.

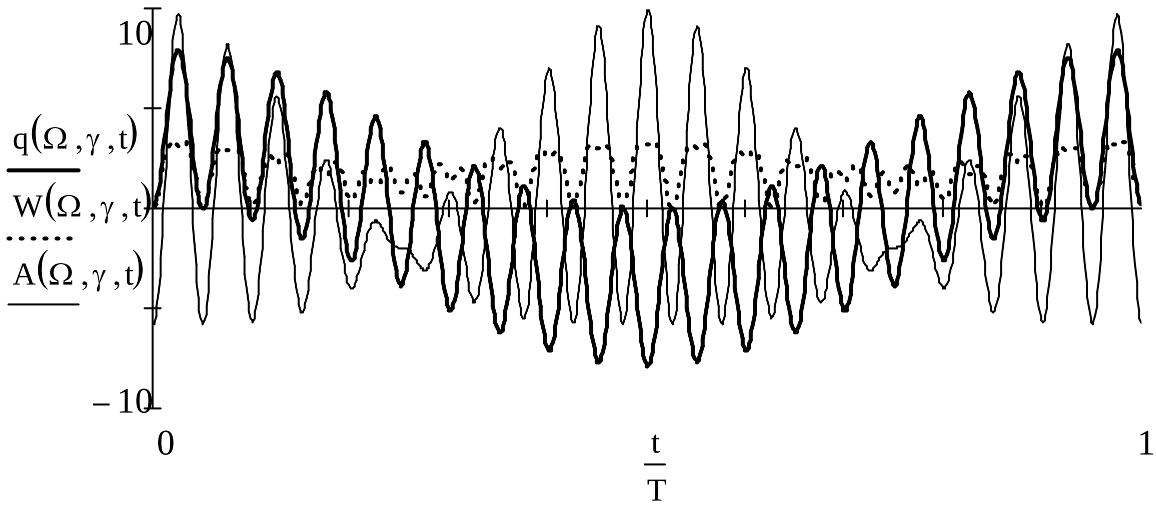


Fig. 1. $\Omega = 0.075\omega_0$, $T = \dfrac{2\pi}{\Omega}$. $W(\Omega,\gamma,t) = W_k(\Omega,\gamma,t) + W_p(\Omega,\gamma,t)$

b) Spectrum of the pulsations of the amplitude of the forced vibrations of undamped *HEMO* forms at the higher frequencies of the source impact on the oscillator and is the most clearly observed on approaching the condition of resonance, $\Omega \to \omega_0$. In this case (Fig. 2), the oscillator performs forced vibrations with period $T_q = \dfrac{2\pi}{\Delta_+}$, and period of pulsations of the amplitude of these vibrations is defined by the envelope of function $A(\Omega,t)$, which already has period $T_A = \dfrac{2\pi}{\Delta_-}$. Minima of the pulsations' amplitudes of the oscillator vibrations form due to the energy transfer backwards from the oscillator to the source. During these intervals of time the oscillator stops vibrating, because during these intervals $A(\Omega,t)\langle 0$ and total dynamic energy of the oscillator $U(\Omega,t) \to 0$. At the resonance coincidence of the frequencies, $\Omega = \omega_0$, is observed and the period of the amplitude pulsation increases without limit.

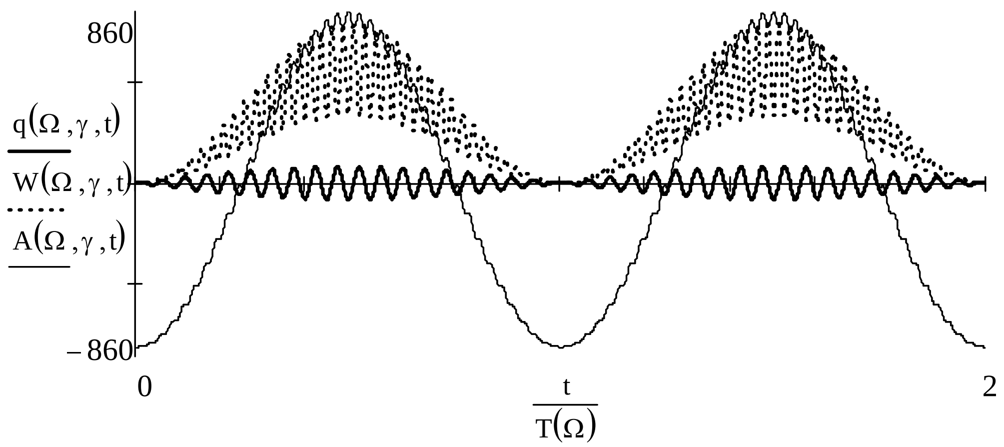
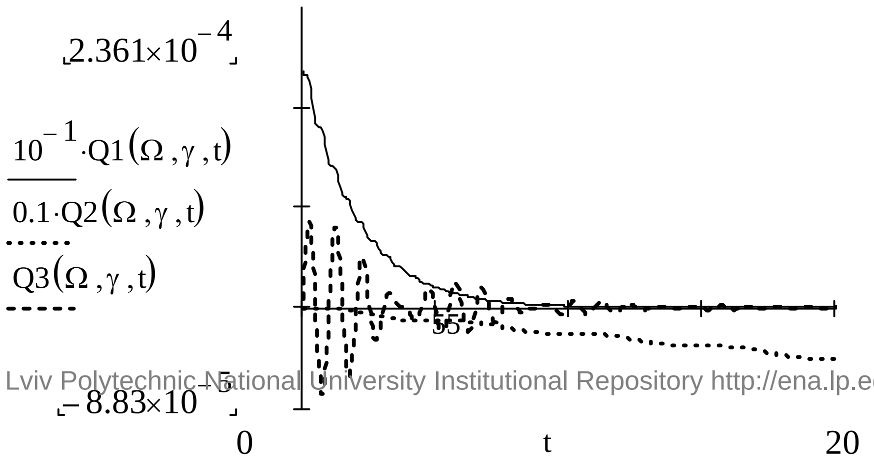


Fig. 2. $\Omega = 0.95\omega_0$, $T = \dfrac{2\pi}{\Delta_-}$

Kinetics of energy relaxation of damped HEMO. In this case $\gamma \neq 0$, therefore, the oscillator exchanges energy both with the Ohm's resistors and the source of external excitation. Thus, the distinguished three components of function $Q(\Omega,\gamma,t)$ have completely different characters of time dependence.

The first component, $Q_1(\Omega,\gamma,t)$, describes the dynamics of the Ohm's resistors to the natural vibrations of the linear *HEMO*. For free *HEMO*, function $Q_1(\Omega,\gamma,t)\big|_{\Omega=0} = Q(\gamma,t)$ is the envelope of its total energy [5]. Therefore, phase-plane portrait of free damped vibrations at co-ordinates



$U(\gamma,t), \frac{dU(\gamma,t)}{dt}$ has oscillating character. Time of the energy relaxation during vibrations $\tau_U = 2\tau_q$, where τ_q is the corresponding time for a co-ordinate of the oscillator. Starting of the sinusoidal source does not change general character of dynamic spectrum $Q_1(\gamma,t)$. The second part $Q_2(\gamma,t)$ is negative and gradually increases with time. It describes energy loses of the oscillators in the Ohm's resistors due to their forced vibrations. The third component, $Q_3(\gamma,t)$ starts vibrating with time and expresses interference of vibrational states of the *HEMO* and of the sinusoidal source (Fig. 3). In order to sustain forced vibrations in the system, function $A(\Omega,\gamma,t)$ needs to be actuated. It also consists of two parts, the first one, $A_1(\Omega,\gamma,t)$, describing forced impact of the source on the oscillator, the second one, $A_2(\Omega,\gamma,t)$, – interference of the vibrational states of the oscillator and the source.

Fig. 3. $\frac{\gamma}{\omega_0} \approx 0.04, \Omega = 0.1\omega_0$

Analysis of spectra $A_1(\Omega,\gamma,t)$ and $A_2(\Omega,\gamma,t)$ (Fig. 4) shows that component $A_2(\Omega,\gamma,t)$ has oscillating character and damps rapidly with time, whereas forced power of the source $A_1(\Omega,\gamma,t)$ rises with time and compensates loses in the oscillatory circuit, causing steady-state oscillations to arise in it.

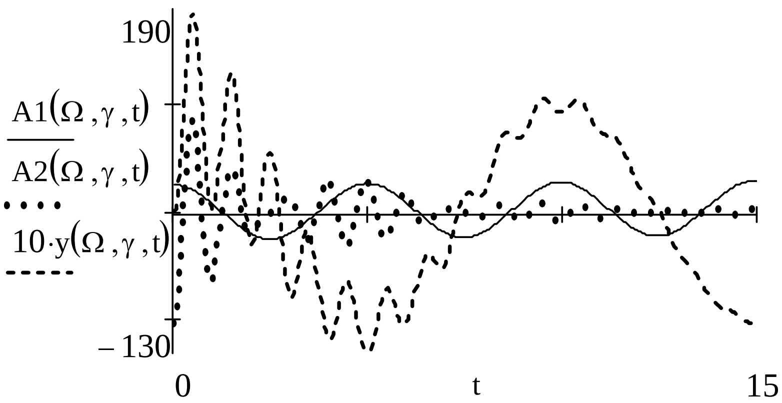
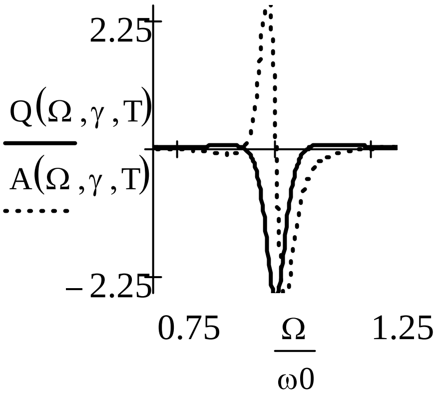


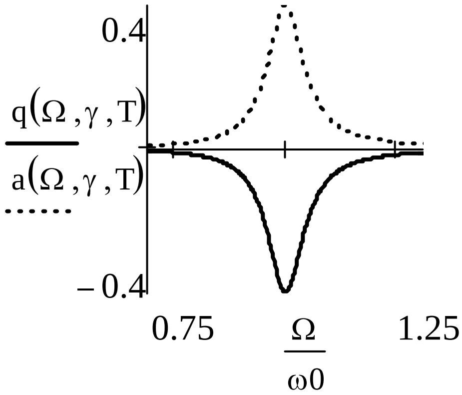
Fig. 4. $\frac{\gamma}{\omega_0} \approx 0.04, \Omega = 0.1\omega_0$

Frequency spectrum of forced vibrations HEMO. Fig. 5, *a*, computed spectra of power functions $Q(\Omega,\gamma,t)$ and $A(\Omega,\gamma,t)$ are adduced as an approbation of the results obtained in this work. It is seen that $Q(\Omega,\gamma,t=T)$ determines the known spectrum of energy absorption by the oscillator, whereas $A(\Omega,\gamma,t=T)$ defines dispersion of the reaction of the oscillating circuit to the external source impact.

Suppression mechanism of the unlimited increase of the forced vibrations' amplitude at the resonance coming has become clear as well. As it can be seen from Fig. 5, *b*, under the specified conditions, components of the active function, $a(\Omega,\gamma,t) = \frac{U_0 q_m \Omega}{2} t \sin\phi$, and the dissipative one, $q(\Omega,\gamma,t) = -\gamma L q_m^2 \Omega^2 t$, are precisely correlated and mutually compensate each other.



а



б

Fig. 5. $\frac{\gamma}{\omega_0} \approx 0.04$

Conclusions. 1. At low-frequency impact of the external source on the oscillator, energy exchange between them has the character of pulsations. Maximum of the modulation of the forced vibrations' spectrum forms on the condition that impulse of energy transfer from the source to the oscillator is either in phase or in anti-phase with the vibrations' amplitude of the latter.

2. In the mode of steady-state oscillations, interference effects in the processes of energy exchange of the oscillator with the interacting systems are suppressed by the damping processes and are not to be taken into consideration.

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