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ПРЯМИЙ ВАРІАЦІЙНИЙ МЕТОД РОЗВ'ЯЗУВАННЯ ОБЕРНЕНОЇ ЗАДАЧІ ТЕПЛОВОЇ ІДЕНТИФІКАЦІЇ ТУНЕЛЬНОЇ ПОРОЖНИНИ В ДОВГОМУ ЦИЛІНДРІ

Розглянуто задачі визначення геометричних параметрів порожнини у твердому тілі за заданим поверхневим температурним полем, зумовленим стаціонарним нагріванням тіла зосередженими тепловими потоками в умовах конвективного теплообміну із зовнішнім середовищем. З використанням граничних інтегральних рівнянь побудована двовимірна математична модель теплового зондування тіла, в межах якої сформульовано пряму та обернені задачі ідентифікації геометричних параметрів порожнини. Методом граничних елементів здійснено числове дослідження поверхневого температурного поля та виявлені його інформативні параметри. Ці параметри можна використовувати як вхідні дані для оберненої задачі ідентифікації. Розв'язування оберненої задачі зведено до задачі мінімізації функціоналу температурного поля поверхні. Розроблено прямий варіаційний метод розв'язування оберненої задачі, в основу якого покладено гранично-елементний метод, який ґрунтується на квазіньютонівсьому методі. З використанням числового експерименту досліджено ефективність розробленого методу. Запропонований підхід можна використовувати для розроблення неруйнівних безконтактних методів виявлення порожнин у твердих тілах на основі даних ІЧ-термографії.

Ключові слова: виявлення та ідентифікація порожнин, теплове зондування, метод граничних елементів, обернені задачі, варіаційні методи.

The problem for identification of the geometrical parameters of the tunnel cavity in a long cylindrical body is considered in this paper. Temperature field of body's external surface, caused by concentrated stationary heat fluxes under conductive heat exchange with an enviroment is used as input data for the identification problem. With the use of boundary integral equations 2-d mathematical model has been built. Within this model direct and inverse problems have been formulated. Boundary-element method has been used to solve and investigate the direct problem. On the base of direct problem's solution the informative parameters of surface temperature field have been chosen. These parameters can be used as an input data for the inverse problem. The inverse problem has been reduced to minimization of some functional depending on the cavity's geometrical parameters and measured surface temperature field. Direct variational method, based on combination of boundary-element method and Quasi-Newton method has been built for solving the inverse problem. With the use of numerical experiment the efficiency of developed method has been studied. The method can be used for development of nondestructive contactless methods for cavities identification in solids with the use of technique of IR-thermography.

Key words: cavity detection and identification, thermal sounding, boundary-element method, inverse problems, variational methods.

Introduction

Many scientists and engineers study a possibility for application of infra-red (IR) thermography for identification of discontinuities (cavities, inclusions, structural defects etc.) in solids [1]. The idea consists in exciting a thermal process in the object's volume, affecting on it by an external heat flow, and in synchronous measuring temperature field on its surface. Internal structural or/and material heterogeneity of the body impacts on the thermal process. So, the data of such measurements contain some information about the object's structure. A problem is how to use these data to obtain quantitative information about defects' geometries. There are different approaches to this problem solving. One consists in formulation of inverse problems using these data mutually with a mathematical model describing the thermal process in the object. Various approaches were used to solve numerically the inverse problems. Among them are methods based on finite Fourier transform [2], finite differences [1] and finite element [3, 4].

Direct and inverse problems for thermal identification a cylindrical tunnel cavity in a long cylindrical body considered in publications [5–7]. A feature of the approach applied there is that the input data for the inverse problems were formed with the use of several different soundings of the object by different heat flows. On this basis the inverse problems were reduced to systems of implicit equations, to solve which the boundary-element method was used.

In this paper the inverse problem is reduce to a variational problem which is solved with the use of a direct quasi-Newton method.

Formulation of the problem

An infinite heat-conductive body B bounded by a cylindrical surface S_0 is considered. Let the axis of the cylinder S_0 be parallel to the axis x_3 of Cartesian coordinate system $\mathrm{K}=(x_1,x_2,x_3)$. The cross-section of S_0 lying in the plane x_1Ox_2 is a sufficiently smooth contour Γ_0. The body B contains a cylindrical tunnel cavity with a boundary S_1. The cross-section of S_1, laying in the plane x_1Ox_2, is a sufficiently smooth contour Γ_1. We can define the contours Γ_0 and Γ_1 in Cartesian coordinate systems K and $\mathrm{K}'=\{x_1', x_2', x_3\}$ correspondingly as $\Gamma_0=\{x_1=g_1(t,a_1,a_2,\mathbf{K},a_n),\ x_2=g_2(t,a_1,a_2,\mathbf{K},a_n)\}$, $\Gamma_1=\{x_1'=f_1(t,q_1,q_2,\mathbf{K},q_m),\ x_2'=f_2(t,q_1,q_2,\mathbf{K},q_m)\}$. Here K' is local Cartesian coordinate system to which we refer contour Γ_1, g_i, f_i, $i=1,2$ are sufficiently smooth functions of parameter $t\in L$, defined on an interval $L\subset\mathbf{R}$, $a_1,a_2,\mathbf{K},a_n$ and $q_1,q_2,\mathbf{K},q_m$ are parameters, which determine geometry of cylindrical surfaces S_0 and S_1, $n,m\in\mathbf{N}$. The cross-section S of the body B is a plane domain bounded by two contours – external Γ_0 and Γ_1 ones.

Radius-vector on the plane x_1Ox_2 can be defined as $\mathbf{x}=x_1\mathbf{e}_1+x_2\mathbf{e}_2=x_1'\mathbf{e}_1'+x_2'\mathbf{e}_2'$, where $\mathbf{e}_1,\mathbf{e}_2$ and $\mathbf{e}_1',\mathbf{e}_2'$ are the orts of coordinate systems K and K' correspondingly.

Assume geometry of external surface is known i.e. functions g_i is defined. Instead the geometry of internal surface S_1 is unknown and functions f_i should to be determined. To do that one can approximate f_i by some specific functions of variable t which depend on unknown parameters $q_1,q_2,\mathbf{K},q_m$. Then the problem of determination of contour Γ_1 will be reduced to determination of a vector $\mathbf{q}=(q_1,q_2,\mathbf{K},q_m)^T$, $q_i\in Q_i$, where $Q_i\subset\mathbf{R}^+$ – the ranges of possible values of geometrical parameters $q_1,q_2,\mathbf{K},q_m$.

To formulate inverse problems a posteriori information is needed. One can obtain it with the use the known method of thermal sounding of the body by external concentrated heat flows [5-7],

To do that the body B is heated by the external stationary heat flow $\mathbf{J}=J_0J(\mathbf{x})\mathbf{j}$, $\mathbf{x}\in\Gamma_0$ that incident on the surface S_0. Here $\mathbf{j}$ is a given unit vector laying on the plane x_1Ox_2, $J_0>0$ is a constant, which defines the maximum of the probing flux, $J(\mathbf{x})$ is a function, such that $J(\mathbf{x})\neq 0$ when $\mathbf{x}\in\Gamma_w\subset\Gamma_0$ and $J(\mathbf{x})=0$ when $\mathbf{x}\notin\Gamma_w$ and $\max(J(\mathbf{x}))=1$, where Γ_w is an arc that defines a heating spot on the body's surface S_0. External S_0 and internal S_1 surfaces are in convective heat exchanging with their environments which temperatures are T_{m1} and T_{m2} respectively.

Under these conditions 2-D temperature field $T(\mathbf{x}), \mathbf{x} \in S$ arises in the body B. Since the cavity affects on heat flow in the body, temperature field $T^e(\mathbf{x})$, $\mathbf{x} \in \Gamma_0$, measured on the surface S_0, provides information about cavity's geometry. By changing the direction $\mathbf{j}$ of sounding flow, its intensity J_0 and function of the intensity distribution $J(\mathbf{r})$ on Γ_w, one can get each time different temperature distributions on external surface. By measuring the surface temperature $T_w^e(\mathbf{x}), \mathbf{x} \in \Gamma_0$ for each sounding parameter $\mathbf{J}_w = (\mathbf{j}, J_0, J(\mathbf{x}), \mathrm{S}_w)$, one gets a set $\mathrm{T}^e = \left\{T_w^e(\mathbf{x}), w = w_1, w_2, ..., w_N\right\}$ of empirical functions corresponding to the set $\mathrm{J} = \left\{\mathbf{J}_w(\mathbf{x}), w = 1, 2, \mathbf{K}, w_N\right\}$: $\mathrm{J} \to \mathrm{T}^e$. Here N is number of independent soundings.

The set T^e and relations $\mathbf{J}_w(\mathbf{x})$ **a** $T_w^e(\mathbf{x})$ contain posterior information about cavity's geometry. We can use them for variational formulation of inverse problem of determination of vector $\mathbf{q}$ of cavity's geometrical parameters.

To do that consider the functional:

$$\Omega(\mathbf{q}) = \sum_{k=1}^{N} \int_{\Gamma_0} \left(T_{w_k}(\mathbf{x};\mathbf{q}) - T_{w_k}^e(\mathbf{x})\right)^2 d\mathbf{x}, \tag{1}$$

where $T_{w_k}(\mathbf{x};\mathbf{q})$ – solution of boundary value problem

$$\Delta_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) = 0, \ \ \mathbf{x} \in S \tag{2}$$

$$\begin{aligned} n_0 \cdot \nabla_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) &= \frac{h_0}{k}\left(T_{m0} - T_{w_k}(\mathbf{x};\mathbf{q})\right) - n_0 \cdot \mathbf{J}_{w_k}, \quad \mathbf{x} \in \Gamma_{w_k}, \\ n_0 \cdot \nabla_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) &= \frac{h_0}{k}\left(T_{m0} - T_{w_k}(\mathbf{x};\mathbf{q})\right), \quad \mathbf{x} \in \Gamma_0 \setminus \Gamma_{w_k}, \\ n_1 \cdot \nabla_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) &= \frac{h_1}{k}\left(T_{m1} - T_{w_k}(\mathbf{x};\mathbf{q})\right), \quad \mathbf{x} \in \Gamma_1, \end{aligned} \tag{3}$$

for given $\mathbf{q} \in \mathbf{Q} = Q_1 \times Q_1 \times \mathbf{K} \times Q_m$. Here h_0 and h_1 – convective heat transfer coefficients on surfaces S_0 and S_1, T_{m0} and T_{m1} – temperatures of environment near these surfaces, k – coefficient of thermal conductivity.

The function $T_{w_k}(\mathbf{x};\mathbf{q})$ is depend on undefined variables $\mathbf{q}$, so functional (1) is a function of m real variables defines in $\mathbf{Q}$. Although explicit analytical representation of function $\Omega(\mathbf{q})$ is unknown, its values can be calculated for any $\mathbf{q}$ and for any given set of probing parameters J, T^e with relation $\mathrm{J} \to \mathrm{T}^e$. That's why $\Omega(\mathbf{q})$ can be considered as implicit function, defines in $\mathbf{Q}$.

Let $\mathbf{q}^* = \left(q_1^*, q_2^*, \mathbf{K}, q_m^*\right)^T$ be real-life values of geometrical parameters of the cavity. Then the value of functional (1), calculated for $\mathbf{q}^*$ will be minimal: $\Omega\left(\mathbf{q}^*\right) < \Omega(\mathbf{q})$, $\forall \mathbf{q} \in \mathbf{Q}$. This feature can be used for inverse problem formulation.

Direct problem formulation. Let the values of parameters $a_1, a_2, \mathbf{K}, a_n$ and $q_1, q_2, \mathbf{K}, q_m$ which define geometry of surfaces S_0 and S_1 are given. It is necessary for each member $\mathbf{J}_w(\mathbf{x}) \in \mathrm{J}$ of given set of sounding parameters to find the temperature field $T_w(\mathbf{x}), \mathbf{x} \in \Gamma_0$ on the external body's surface S_0: $\mathbf{J}_w(\mathbf{x}) \to T_w(\mathbf{x})$.

So, the solution of the direct problem gives the set $\mathrm{T} = \{T_w(\mathbf{x}), w = w_1, w_2, ..., w_N\}$ of surface's temperature fields corresponding to the given set $\mathrm{J} = \{\mathbf{J}_w(\mathbf{x}), w = w_1, w_2, \mathbf{K}, w_N\}$ of sounding heat flows: $\mathrm{J} \to \mathrm{T}$. To solve the direct problem it is sufficiently to solve the boundary-value problem (2), (3) for each $\mathbf{J}_w(\mathbf{x}) \in \mathrm{J}$ and given

Inverse problem formulation. Let $Q_i \subset \mathbf{R}^+$ are given ranges of values of geometrical parameters $q_i \in Q_i$; J and T^e are given set of sounding parameters $\mathbf{J}_w(\mathbf{x})$ and set of empirical functions $T_w(\mathbf{x}), \mathbf{x} \in \Gamma_0$ of surface temperature distributions. It necessary to find a vector $\mathbf{q}^* = \left(q_1^*, q_2^*, \mathbf{K}, q_m^*\right)^T$ of real-life values of geometrical parameters of cavity that minimizes functional (1).

Quasi-Newton algorithm for minimization

The solution of formulated inverse problem can be written in form

$$\mathbf{q}^* = \arg\min_{\mathbf{q} \in \mathbf{Q}} \Omega. \tag{4}$$

Using direct variational methods one can obtain an approximate solution by repeating numerical solving of direct problem (2) (3) for properly chosen sequence of vectors $\mathrm{G} = \left\{\mathbf{q}^{(0)}, \mathbf{q}^{(1)}, \mathbf{K}\right\}$ which converges to solution (4): $\mathrm{G} \to \mathbf{q}^*$.

To construct minimizing sequences G one can use Newton method for unconstrained optimization which based on information about curvature of the objective function $\Omega(\mathbf{q})$. According to this approach values of unknown parameters on the next iteration are calculated by formula [8]

$$\mathbf{q}^{(l+1)} = \mathbf{q}^{(l)} + c^{(l)} \mathbf{b}^{(l)}, \tag{5}$$

where $c^{(l)}$ is a real scalar ($c^{(l)} \in (0,1]$) and $\mathbf{b}^{(l)}$ is a vector which determines a step and direction for next iteration respectively, $l = 0, 1, \mathbf{K}$.

Vector $\mathbf{b}^{(l)}$ is calculated by formula

$$\mathbf{b}^{(l)} = -\left(\mathbf{H}^{(l)}\right)^{-1} \cdot \left(\mathbf{G}^{(l)}\right)^T, \tag{6}$$

where $\mathbf{H}^{(l)} = \left\{H_{kl}^{(l)}\right\}$ is Hessian matrix, which elements $H_{kl}^{(l)}$ are calculated as

$$H_{kl}^{(l)} = \left.\frac{\partial^2 \Omega}{\partial q_k \partial q_l}\right|_{q_1 = q_1^{(l)}, q_2 = q_2^{(l)}, \mathbf{K}, q_m = q_m^{(l)}}, \tag{7}$$

and $\mathbf{G}^{(l)} = \left\{G_k^{(l)}\right\}$ is a gradient of objective function $\Omega(\mathbf{q})$, which elements are calculated as

$$G_k^{(l)} = \left.\frac{\partial \Omega}{\partial q_k}\right|_{q_1 = q_1^{(l)}, q_2 = q_2^{(l)}, \mathbf{K}, q_m = q_m^{(l)}}. \tag{8}$$

In classic Newton method scalar $c^{(l)} = 1$, but in Quasi-Newton methods scalar $c^{(l)}$ can be determined with the use of known 1-d optimization methods,

$$c^{(l)} = \arg\min_{c > 0} \Omega\left(\mathbf{q}^{(l)} + c\mathbf{b}^{(l)}\right), \tag{9}$$

for instance, golden section search [9].

As an analytical representation of the objective function $\Omega(\mathbf{q})$ is unknown, to calculate partial derivatives of it on each iteration one can use the finite difference analogues of these derivatives. So for the calculation of components of the gradient of function $\Omega(\mathbf{q})$ we use the formula

$$G_k = \frac{\Omega(q_1, q_2, \dots, q_k + \Delta_k, \dots, q_m) - \Omega(q_1, q_2, \dots, q_k - \Delta_k, \dots, q_m)}{2\Delta_k}, \tag{10}$$

where Δ_k stands for a small as compared to q_k positive constant.

To calculate a component G_k it is necessary to determine the functions $T_{w_i}(\mathbf{x}; q_1, q_2, \mathbf{K}, q_k + \Delta_k, \mathbf{K}, q_m)$ and $T_{w_i}(\mathbf{x}; q_1, q_2, \mathbf{K}, q_k - \Delta_k, \mathbf{K}, q_m)$, $\mathbf{x} \in \Gamma_0$, $\forall i \in \overline{1,N}$ by solving the direct problem. In general, calculations of $\mathbf{G}^{(l)}$ require solving of direct problem at least $2Nm$ times on each iteration.

To calculate components of Hessian matrix one can use next formula

$$\begin{gathered} H_{kl} = \frac{\Omega(q_1, q_2, \mathbf{K}, q_k + \Delta_k, \mathbf{K}, q_l + \Delta_l, \mathbf{K}, q_m)}{\Delta_l \Delta_k} - \\ - \frac{\Omega(q_1, q_2, \mathbf{K}, q_k + \Delta_k, \mathbf{K}, q_l, \mathbf{K}, q_m)}{\Delta_l \Delta_k} - \frac{\Omega(q_1, q_2, \mathbf{K}, q_k, \mathbf{K}, q_l + \Delta_l, \mathbf{K}, q_m)}{\Delta_l \Delta_k} + \\ + \frac{\Omega(q_1, q_2, \mathbf{K}, q_k, \mathbf{K}, q_l, \mathbf{K}, q_m)}{\Delta_l \Delta_k}. \end{gathered} \tag{11}$$

These calculations require solving direct problems at least $4Nm^2$ times at each iterations.

Thus, using of Newton method for solving inverse problem by variational method requires at least $2Nm(1+2m)$ solutions of direct problem and calculations of functional (1) at each iteration.

Due to Quasi-Newton methods in formula (6) instead of Hessian matrix $\mathbf{H}$ one uses close to $\mathbf{H}$ some matrix $\tilde{\mathbf{H}}$, which is calculated iteratively on basis of components G_k of gradient $\mathbf{G}$ at this iteration. Herewith, inverse matrix $\tilde{\mathbf{H}}^{-1}$ can be calculated simultaneously. This can significantly reduce the amount of computation. We used the algorithm BFGS [8] which realized the iterative process

$$\left(\tilde{\mathbf{H}}^{(l)}\right)^{-1} = \left(\mathbf{I} - \frac{\boldsymbol{\gamma}^{(l)}\left(\boldsymbol{\alpha}^{(l)}\right)^T}{\left(\mathbf{a}^{(l)}\right)^T \cdot \boldsymbol{\gamma}^{(l)}}\right) \cdot \left(\tilde{\mathbf{H}}^{(l-1)}\right)^{-1} \cdot \left(\mathbf{I} - \frac{\boldsymbol{\alpha}^{(l)}\left(\boldsymbol{\gamma}^{(l)}\right)^T}{\left(\boldsymbol{\alpha}^{(l)}\right)^T \cdot \boldsymbol{\gamma}^{(l)}}\right) + \frac{\boldsymbol{\gamma}^{(l)}\left(\boldsymbol{\gamma}^{(l)}\right)^T}{\left(\boldsymbol{\alpha}^{(l)}\right)^T \cdot \boldsymbol{\gamma}^{(l)}} \tag{12}$$

where $\gamma^{(l)} = \mathbf{q}^{(l)} - \mathbf{q}^{(l-1)}$, $\boldsymbol{\alpha}^{(l)} = \mathbf{G}^{(l)} - \mathbf{G}^{(l-1)}$, $\mathbf{I}$ – identity matrix, $l = 1, 2\mathbf{K}$.

As an initial approximation $\mathbf{q}^{(0)}$ for vector $\mathbf{q}$ we choose any vector $\mathbf{q}^{(0)} \in \mathbf{Q}$. As an initial approximation $\tilde{\mathbf{H}}^{(0)}$ for matrix $\tilde{\mathbf{H}}$, used in (6), we used Hessian matrix calculated for vector $\mathbf{q}^{(0)}$ due to formula (11) at $q_i = q_i^{(0)}$, $\forall i = \overline{1,m}$. As the stopping criteria for iterative process the conditions $\left\|\mathbf{q}^{(l+1)} - \mathbf{q}^{(l)}\right\| \le e_{\mathbf{q}}$ and $\left|\Omega\left(\mathbf{q}^{(l+1)}\right) - \Omega\left(\mathbf{q}^{(l)}\right)\right| \le e_{\Omega}$, were used, where $e_{\mathbf{q}}, e_{\Omega} > 0$ are small enough real number.

Due to this we need to use an effective algorithm for solving the direct problem. In publications [5–7] boundary element method has been used for this. Here we also apply this method.

Boundary-element method for direct and inverse problems solving

Solving of the inverse problem due to proposed algorithm reduces to multiple solving of corresponding direct problem (2), (3). Since the analytical solution of this problem is unknown, it is necessary to apply numerical methods. In publications [5-7] with the use of numerical experiments computational efficiency of direct boundary-element method for such class of problems has been shown. Also, it has been shown that the direct and inverse problems of identification advisable to formulate regarding to the perturbation of surface temperature field caused by presence of cavities

$$q_w(\mathbf{x},\mathbf{q}) = \frac{T_w(\mathbf{x},\mathbf{q}) - \bar{T}_w(\mathbf{x})}{T_0}, \tag{13}$$

where $\bar{T}_w(\mathbf{x})$ – temperature field in the body $\bar{\mathrm{B}}$, which differs from body B in that cavity is absent there; $T_0 \equiv R_0 J_0/k$ – typical temperature; R_0 – typical body size of B (e.g., it's diameter).

Function $\bar{T}_w(\mathbf{x})$ is the solution of boundary value problem

$$\Delta_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) = 0, \ \ \mathbf{x} \in \mathrm{V} \tag{14}$$

$$\begin{aligned} n_0 \cdot \nabla_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) &= \frac{h_0}{k}\left(T_{m0} - T_{w_k}(\mathbf{x};\mathbf{q})\right) - n_0 \cdot \mathbf{J}_{w_k}, \quad \mathbf{x} \in \mathrm{S}_{w_k}, \\ n_0 \cdot \nabla_{\mathbf{x}} T_{w_k}(\mathbf{x};\mathbf{q}) &= \frac{h_0}{k}\left(T_{m0} - T_{w_k}(\mathbf{x};\mathbf{q})\right), \quad \mathbf{x} \in \mathrm{S}_0 \setminus \mathrm{S}_{w_k}. \end{aligned} \tag{15}$$

A perturbation $q_{w_k}(\mathbf{x},\mathbf{q})$ satisfies boundary integral equation

$$\begin{aligned} &\frac{1}{2} q_{w_k}(\boldsymbol{\eta}) + \int_{\Gamma_0} \left(\Phi(\boldsymbol{\eta},\xi) + m_0 \Theta(\boldsymbol{\eta},\xi)\right) q_{w_k}(\xi)\, dl(\xi) + \int_{\Gamma_1} \left(\Phi(\boldsymbol{\eta},\xi') + m_1 \Theta(\boldsymbol{\eta},\xi')\right) q_{w_k}(\xi')\, dl(\xi') = \\ &= -\int_{\Gamma_1} \left(\Phi(\boldsymbol{\eta},\xi') + m_1 \Theta(\boldsymbol{\eta},\xi')\right) \bar{q}_{w_k}(\xi')\, dl(\xi') - \int_{\Gamma_1} \Theta(\boldsymbol{\eta},\xi')\, m_1 q_{m1}(\xi')\, dl(\xi'), \end{aligned} \tag{16}$$

Here $\boldsymbol{\eta} = \mathbf{x}/R_0$, $\xi = \mathbf{x}/R_0$, $\xi' = \mathbf{x}'/R_0$, $\Theta(\boldsymbol{\eta},\xi) \equiv \frac{1}{2p}\left(\ln\left(1/|\boldsymbol{\eta} - \xi|\right) - \ln R_0\right)$ and $\Phi(\boldsymbol{\eta},\xi) \equiv \Theta(\boldsymbol{\eta},\xi)/\partial \mathbf{n}(\xi)$ – potentials of simple and double layers, $\mathbf{n}$ – exterior unit normal to the surface $\mathrm{S}_0 \cup \mathrm{S}_1$, $m_0 = R_0 h_0/k$ and $m_1 = R_0 h_1/k$ – reduced coefficients of heat transfer on surfaces S_0 and S_1, $q_{m1} \equiv T_{m1}/T_0$ – reduced temperature of environment near surface S_1, $\bar{q}_{w_k}(\boldsymbol{\eta}) = \bar{T}_{w_k}(\boldsymbol{\eta})/T_0$ – dimensionless temperature on the surface of body $\bar{\mathrm{B}}$, which satisfies boundary integral equation

$$\frac{1}{2} \bar{q}_{w_k}(\boldsymbol{\eta}) + \int_{\Gamma_0} \left(\Phi(\boldsymbol{\eta},\xi) + m_0 \Theta(\boldsymbol{\eta},\xi)\right) \bar{q}_{w_k}(\xi)\, dl(\xi) = \int_{\Gamma_0} \Theta(\boldsymbol{\eta},\xi)\left(J(\xi)\,\mathbf{j} \cdot \mathbf{n} + m_0 q_{m1}\right) dl(\xi), \tag{17}$$

Boundary integral equations (14) – (15) determine a mathematical model of probing body B by concentrated heat fluxes.

Using the boundary element method, we reduce the boundary integral equation (17), (16) to the matrix equations [7]

$$\mathbf{M}_{(11)} \bar{\boldsymbol{\theta}}_{(1)} = \bar{\mathbf{B}}_{(1)} \tag{18}$$

$$\begin{aligned} \mathbf{M}_{(11)} \boldsymbol{\theta}_{(1)} + \mathbf{M}_{(12)} \boldsymbol{\theta}_{(2)} &= \mathbf{0}, \\ \mathbf{M}_{(21)} \boldsymbol{\theta}_{(1)} + \mathbf{M}_{(22)} \boldsymbol{\theta}_{(2)} &= \mathbf{B}_{(2)}. \end{aligned} \tag{19}$$

Here $\boldsymbol{\theta}_{(1)}$ and $\boldsymbol{\theta}_{(2)}$ are vectors of nodal values of perturbation of temperature $q_{w_k}(\mathbf{x},\mathbf{q})$ on surfaces S_0 and S_1 of body B, $\bar{\boldsymbol{\theta}}_{(1)}$ – vectors of nodal values dimensionless temperature $\bar{q}_{w_k}(\mathbf{x})$ on the surface of body $\bar{\mathrm{B}}$. Elements of matrices $\mathbf{M}_{(11)}$, $\mathbf{M}_{(12)}$, $\mathbf{M}_{(21)}$ and $\mathbf{M}_{(22)}$ are determined by values of integrals of potentials of simple and double layers over boundary elements []. Elements of vector $\bar{\mathbf{B}}_{(1)}$ are determined by values of integrals over boundary elements of intensity function of probing flux. Elements of vector $\mathbf{B}_{(2)}$ are determined by elements of vector $\bar{\mathbf{B}}_{(1)}$ [6-7].

By successively solving of linear systems (18) and (19) we find the nodal temperature on the external and internal surfaces of body B

$$\boldsymbol{\theta}_{(1)} = \left(\mathbf{B}_{(1)}\mathbf{M}_{22} - \mathbf{B}_{(2)}\mathbf{M}_{21}\right) \cdot \left(\mathbf{M}_{11}\mathbf{M}_{22} - \mathbf{M}_{21}\mathbf{M}_{12}\right)^{-1},$$
$$\boldsymbol{\theta}_{(2)} = \left(\mathbf{B}_{(1)}\mathbf{M}_{12} + \mathbf{B}_{(2)}\mathbf{M}_{11}\right) \cdot \left(\mathbf{M}_{11}\mathbf{M}_{22} - \mathbf{M}_{21}\mathbf{M}_{12}\right)^{-1}. \qquad (20)$$

Solving of the direct problems

We conducted numerical investigation of direct problem for body B that has a form of round cylinder of radius R_0. We examined the case of radial directed heat flux with d - shaped intensity distribution

$$J(h, w) = \exp\left(-\sin^2(h - w) / l^2\right), \qquad (18)$$

where $w \in [0, 2p)$ – polar angle that corresponds to maximum value of probing heat flux, $h \in [0, 2p)$ – polar angle, that corresponds to arbitrary point on the contour Γ_0, l – parameter that determines the width of probing flux. Herewith, the probing flux is defined by two parameters – w та l.

In this paper we considered two cases of cavity's geometry: problem 1, when Γ_1 is a circle and problem 2 when Γ_1 is a radially oriented ellipse. In the first case the cavity's geometry is determined by three parameters – polar angle j_0, radius r_0 of the centre of cavity and radius r_0. In the second case the cavity's geometry is determined by polar angle j_0, radius r_0 of the centre of cavity and two semi-axes of the ellipse a_0 and b_0.

Fig. 1 and Fig. 2 shows the dependencies of the perturbation of temperature field $q_w(h)$ on the surface of body B for cases of circular and elliptical cavities. These dependencies have been defined for each w and fixed values of parameter l and parameters j_0, r_0, r_0, a_0, b_0 respectively.

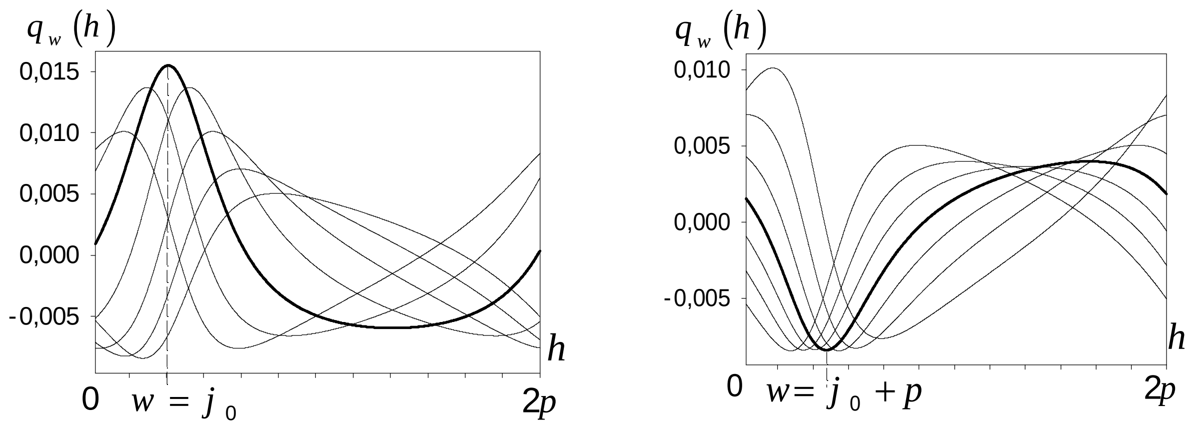

Fig. 1. Perturbations of body's surface temperature field with circular cavity obtained for different probing directions w_k *: a)* $w_k = \frac{p}{6}k,\ k = 0,1,\mathbf{K},6$ *, b)* $w_k = p + \frac{p}{6}k,\ k = 0,1,\mathbf{K},6$

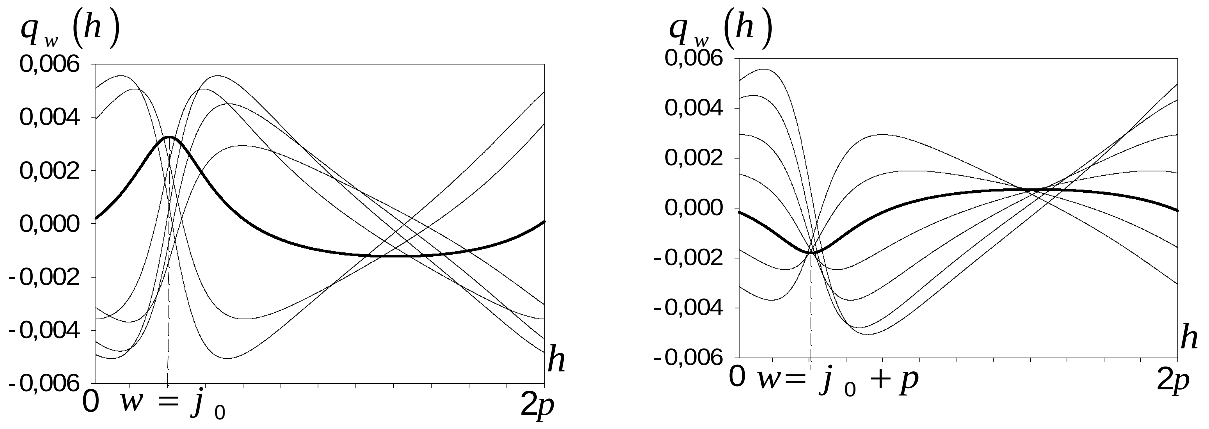

Fig. 2. Perturbations of body's surface temperature field with elliptical cavity obtained for different probing directions w_k *: a)* $w_k = \frac{p}{6}k,\ k = 0,1,\mathbf{K},6$ *, b)* $w_k = p + \frac{p}{6}k,\ k = 0,1,\mathbf{K},6$

In the case of circular cavity caused by reasons of symmetry next properties of temperature field $q_w(h)$ are followed

$$\begin{aligned}&\arg\left(\max_w\left(\max_h\left(q_w(h,j_0)\right)\right)\right) = j_0 = w,\ \arg\left(\max_w\left(\min_h\left(q_w(h,j_0)\right)\right)\right) = j_0 + p = w + p,\\ &\arg\left(\min_w\left(\min_h\left(q_w(h,j_0+p)\right)\right)\right) = j_0 + p = w + p,\ \arg\left(\min_w\left(\max_h\left(q_w(h,j_0+p)\right)\right)\right) = w.\end{aligned} \tag{19}$$

These properties have been proved by numerical experiment.

In the case of elliptical cavity temperature filed $q_w(h)$ depends on: 1) the angle j_0 of ellipse's orientation; 2) the ratio b_0/a_0, which defines the distance from cavity to surface S_0; 3) the width of probing flux. Thus, temperature filed $q_w(h)$ can satisfy properties (19) or next properties

$$\begin{aligned}&\arg\left(\min_w\left(\max_h\left(q_w(h,j_0)\right)\right)\right) = j_0 = w,\ \arg\left(\max_w\left(\min_h\left(q_w(h,j_0)\right)\right)\right) = j_0 + p = w + p,\\ &\arg\left(\max_w\left(\min_h\left(q_w(h,j_0+p)\right)\right)\right) = j_0 + p = w + p,\ \arg\left(\min_w\left(\max_h\left(q_w(h,j_0+p)\right)\right)\right) = w.\end{aligned} \tag{20}$$

As we can see the polar angle j_0 can be determined directly from measurements of surface temperature fields according to properties (19), (20). It enables to reduce a number the unknown parameters to two parameters (r_0, r_0) in the first case and to three (r_0, a_0, b_0) in the second case.

Solving of the inverse problem

Quantitative study the developed variational algorithm for solving the inverse problems in cases of circular and elliptical cavities has been conducted by using numerical experiment [6]. For that the results of direct problems solving (Fig. 1, Fig. 2) have been used as an input data for the inverse problems.

We studied the convergence of the iterative process depending on initial approximation and the number of unknown parameters. To do that we have solved problems 1 and 2 for full set of unknown parameters (three unknown parameters for problem 1 and four unknown parameters for problem 2). Then we solved these problems for reduced sets of unknown parameters (two unknown parameters for problem 1 and three unknown parameters for problem 2). The results of numerical experiments are presented in Tables 1, 3 (problem 1) and 2, 4 (problem 2). In the tables: t_{calc} stands for problems' solving time, N_{calc}

is the needed number of calculations of functional (1) and N_{iter} is the needed number of iteration. The "empirical" data for inverse problems were taken from the solution of the direct problem which was solved the next values of geometrical parameters: $j_0 = p/4$, $r_0 = 0.02$ and $r_0 = 0.03$ (for problem 1) and $j_0 = p/4$, $r_0 = 0.02$ and $a_0 = 0.03$, $b_0 = 0.01$ (for problem 2)

The problems were solved on a computer with processor Intel (R) Core (TM) i5 – 3470 CPU 3.2 GHz, 8 GB DDR III RAM and Windows 8.1 operating system.

Table 1

Initial approximation			Numerical results		
$r_0^{(0)}$	$r_0^{(0)}$	$j_0^{(0)}$	t_{calc}, s	N_{calc}	N_{iter}
0.015	0.02	$p/4$	2437.16	92	17
0.025	0.03	$p/2$	2851.3	104	22
0.03	0.04	$5p/12$	2669.86	104	25
0.035	0.035	$5p/21$	2414.89	100	23
0.05	0.045	0	3534.96	148	33

Table 2

Initial approximation				Numerical results		
$a_0^{(0)}$	$b_0^{(0)}$	$r_0^{(0)}$	$j_0^{(0)}$	t_s	N_{calc}	N_{iter}
0.015	0.008	0.02	$p/4$	2503.35	90	15
0.025	0.01	0.03	$p/2$	3603.65	150	27
0.03	0.015	0.04	$5p/12$	3497.9	145	28
0.035	0.02	0.035	$5p/21$	3937.03	160	31
0.05	0.025	0.045	0	4550.2	195	33

Table 3

Initial approximation		Numerical results		
$r_0^{(0)}$	$r_0^{(0)}$	t_s	N_{calc}	N_{iter}
0.015	0.02	216.99	42	10
0.025	0.03	203.04	39	11
0.03	0.04	173.81	36	11
0.035	0.035	189.73	36	11
0.05	0.045	302.77	60	10

Table 4

Initial approximation			Numerical results		
$a_0^{(0)}$	$b_0^{(0)}$	$r_0^{(0)}$	t_s	N_{calc}	N_{iter}
0.015	0.008	0.02	553.84	104	23
0.025	0.01	0.03	542.05	96	21
0.03	0.015	0.04	523.87	100	21
0.035	0.02	0.035	694.65	124	29
0.05	0.025	0.045	677.75	136	33

Conducted numerical experiments enabled to estimate a computational capability needed for solving the inverse problems with the use of the quasi-Newton direct variational method depending of number of unknown geometrical parameters.

Conclusions

In this paper the problem for identification of geometry of tunnel cavities in a long cylindrical body is considered. The approach is based on exciting of a thermal process in the object's volume, affecting on it by concentrated heat flow, and synchronous measuring temperature field on its surface. A variational formulation of inverse identification problem was done with the use of objective function depending of unknown geometrical parameters and data obtained by thermal sounding of the object. To solve the variational problem the algorithm based on a quasi-Newton method and boundary element method is developed. Numerical experiments performed with the use of the developed algorithm shown its high convergence and accuracy.

Obtained results can be used to create contactless methods for identification of internal structure of solids with the use of IR-thermography.

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