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ENERGY OF MOTION

Vasyl Tchaban

Lviv Polytechnic National University, Lviv, Ukraine; Rzeszow University, Rzeszow, Poland

vtchaban@polynet.lviv.ua

Abstract. It is shown that motion is accompanied by energy and co-energy, which are inseparable. In a linear media they are equal and can replace each other. In a nonlinear media they are different, and each of them executes its own functions. Universal expression for co-energy of the physical system is offered. The concept of kinetic energy in variation principles does not fit. Examples refer to the electromagnetic field and relativistic theory of gravitation. The paradox of transversal and longitudinal relativistic masses is refuted.

Key words: universal expression of energy, kinetic co-energy, relativistic kinetic co-energy

1. Introduction

The variation principle of Hamilton-Ostrogradsky underlies the power approach to the physical systems. In the case of linear systems it successfully uses the concept of kinetic and potential energy. In the case of appearance of non-linearity potential energy keeps its status, and kinetic one loses it in favour of kinetic co-operative energy, so-called co-energy [1–3]. Frankly speaking, the concept of energy [4] and, moreover, of co-energy is comparatively new and so unknown in modern physics. We consider energy to be just a mathematical value that can be calculated. Energy has many forms. The law of conservation of energy operates in the nature. To check this law, we must be sure that its increase and losses are taken into account. Exactly under this visual angle we seem to succeed something was to raise the curtain a little on essence of co-energy as a quantitative concept inseparable from energy [5].

2. Formula of energy

From the point of view of a field theory the densities of energies are postulated, because it is possible to obtain many of their expressions, and it is rather difficult to decide which of them are correct, that is the simplest of them are assumed as a basis. All of them satisfy practical experience as yet. In [6] we offered only universal expression of energy that guarantees the same results

$$w_i = \int_0^x c(x) x\,dx., \qquad (1)$$

where w_i $(i = k, p, kc)$ are densities of energies: kinetic, potential and kinetic co-energy, accordingly.

The variable $c(x)$ always contains matrices of static parameters that characterize a medium or concentrated element. If variable x is given the meaning of generalized coordinates, then expression (1) reveals the density of potential energy. If it is given the meaning of generalized impulses or speeds, then expression (1) represents the density of kinetic energy or kinetic co-energy. We will consider two examples.

3. Electromagnetic field

In this case it is accepted to express densities of energy through the field vectors

$$\vec{E} = -\partial \vec{A} / \partial t; \quad \vec{B} = \nabla \times \vec{A}, \qquad (2)$$

where $\vec{A}$ is vector potential, and consequently

$$\vec{D} = \mathbf{E}(\vec{E})\vec{E}; \quad \vec{H} = \mathbf{N}(\vec{B})\vec{B}, \qquad (3)$$

where $\vec{E}, \vec{D}, \vec{B}, \vec{H}$ are vectors of the electromagnetic field. $\mathbf{E}, \mathbf{N}$ are matrices of static electric permeabilities and static reluctivities of a medium.

According to (2) it is possible to interpret the vector $\mathbf{B}$ as the generalized coordinate and the vector $\vec{E}$ – as generalized speed. By substituting (3) into (1), the expressions of densities of potential energy and kinetic co-energy are obtained

$$w_p == \int_0^{\vec{B}} \mathbf{N}(\vec{B})\vec{B}\,d\vec{B}; \quad w_{kc} = \int_0^{\vec{E}} \mathbf{E}(\vec{E})\vec{E}\,d\vec{E}; \qquad (4)$$

$$w_k = \int_0^{\vec{D}} \Xi(\vec{D})\vec{D}\,d\vec{D}, \qquad (5)$$

where $\Xi = \mathbf{E}^{-1}$.

In [2, 3] it is shown that on the basis of of the Hamilton-Ostrogradsky power approach it is impossible to obtain the equation in terms of the vectors of the electromagnetic field. By this medium we can only create the equations of vector potential.

Therefore, we are forced to confess that kinetic co-energy is a physical characteristic. But then another idea arises, that is, to give up kinetic energy in behalf on co-energy. But before awarding a sentence to kinetic energy, it was well-proven in [3] that both above mentioned energies are real and inseparable from one another.

Force is the measurable value of energy. The force density of both kinetic energies will be searched by their gradients

$$\vec{f}_{kc} = -\nabla w_{kc}; \quad \vec{f}_k = -\nabla w_k . \qquad (6)$$

Substituting (4), (5) into (6) we obtain

$$\vec{f}_{kc} = -\nabla \int_0^{\vec{E}} \vec{D} d\vec{E}, \quad \vec{f}_k = -\nabla \int_0^{\vec{D}} \vec{E} d\vec{D} . \qquad (7)$$

Further analysis for simplifying it will be applied to the separate force components

$$f_{kcx} = -\frac{\partial}{\partial x} \int_0^{\vec{E}} \vec{D} d\vec{E}, \; f_{kx} = -\frac{\partial}{\partial x} \int_0^{\vec{D}} \vec{E} d\vec{D}, \; x = x, y, z . \qquad (8)$$

whence

$$f_{kx} = -\int_0^{\vec{D}} \frac{\partial \vec{E}}{\partial x} d\vec{D} - \int_0^{\partial \vec{D}/\partial x} \vec{E} d \frac{\partial \vec{D}}{\partial x}, \; x = x, y, z . \qquad (9)$$

Let us replace differential in the second summand

$$\int_0^{\partial \vec{D}/\partial x} \vec{E} d \frac{\partial \vec{D}}{\partial x} = \vec{E} \frac{\partial \vec{D}}{\partial x} - \int_0^{\vec{D}} \frac{\partial \vec{E}}{\partial x} d\vec{D}, \quad x = x, y, z. \qquad (10)$$

Substituting (10) into (9), we obtain

$$f_{kx} = -\vec{E} \frac{\partial \vec{D}}{\partial x}, \quad x = x, y, z. \qquad (11)$$

Adding summands (8) and taking into account the theorem of integration by parts, we obtain

$$f_{kx} + f_{kcx} = -\frac{\partial}{\partial x}(\vec{E}\vec{D}) = -\frac{\partial \vec{E}}{\partial x}\vec{D} - \vec{E}\frac{\partial \vec{D}}{\partial x}, \quad x = x, y, z . \qquad (12)$$

Comparing (11) and (12), we obtain

$$f_{kcx} = -\frac{\partial \vec{E}}{\partial x} \vec{D}, \quad x = x, y, z . \qquad (13)$$

To compare the power actions of kinetic energy and co-energy, we will somewhat change their expressions

$$f_{kcx} = -\frac{\partial \vec{E}}{\partial x} (\mathbf{E}' \vec{E}); \; f_{kx} = -\vec{E} \left(\mathbf{E}'' \frac{\partial \vec{E}}{\partial x} \right), x = x, y, z , \qquad (14)$$

where $\mathrm{E}'(E)$, $\mathrm{E}''(E)$ are matrices of static and differential electric permeabilities of a nonlinear medium.

As expressions (14) show, the power characteristic of kinetic energy shows itself through differential permeability of a medium $(\mathbf{E}'')$, and the power characteristic of kinetic co-energy – through static one $(\mathbf{E}')$. In other characteristics these expressions coincide. In the linear medium densities of forces coincide too $f_{kx} = f_{kcx}$, because then $\mathrm{E}' = \mathrm{E}''$.

To grasp the essence of kinetic co-energy more thoroughly, let us change the expression of energy (13). For this purpose we will take advantage of Maxwell's postulate

$$\frac{\partial \vec{D}}{\partial t} = \frac{\partial \vec{D}}{\partial x} v_x = d_b, \; x = x, y, z, \qquad (15)$$

where d_b is the density of displacement currents; $v_x = \partial x / dt$ is instantaneous speed of movement, then we obtain the expression

$$f_{kx} = -\frac{\vec{E} d_b}{v_x}, \quad x = x, y, z. \qquad (16)$$

We will consider the important mechanical analogies: $\vec{A} \to \vec{r}, \vec{E} \to \vec{v}, \vec{D} \to \vec{p}, d_b \to \vec{f}, e \to m$, where there are vectors of distance, velocity, impulse, forces and mass are presented accordingly. From this point of view, the vell-known expression $\vec{f} = d\vec{p}/dt = m'' d\vec{v}/dt$, is noticeable in (15), where m'' is differential mass. Therefore, the appearance of the matrix of differential permeabilities in (16) is fully natural. Therefore, it is necessary to consider the fact that a force action of the electromagnetic field is the action of kinetic energy, but not kinetic co-energy.

4. Relativistic co-energy

Variation methods are the important part of the relativistic theory of gravitation. But, as it has been shown above, variation methods applied to the nonlinear systems use the concept of kinetic co-energy and, actually, potential energy. Unfortunately, the concept of kinetic co-energy does not exist in the relativistic theory. And the consequences of this fact will be shown below.

As basic description of space non-linearity we will consider the functional dependence of an impulse from velocity $\mathbf{p} = \mathbf{p}(\mathbf{v})$! In the single-component measurement

$$p = \frac{m_0 v}{\sqrt{1 - v^2 / c^2}} = p(v) , \qquad (17)$$

thus m_0, c, v are rest mass, velocity of light in vacuum, real point velocity.

Relativistic co-energy of movable point mass will be still written down in the form of (1)

$$w_{kc} = \int_0^v m'(v) v dv, \qquad (18)$$

where $m'(v)$ is static relativistic mass obtained according to (17)

$$m'(v) = \frac{p(v)}{v} = \frac{m_0}{\sqrt{1 - v^2 / c^2}} , \qquad (19)$$

Substituting (19) into (18), after integration the sought expression of relativistic co-energy [8] is obtained

$$w_{kc} = m_0 c^2 \left(1 - \sqrt{1 - v^2 / c^2}\right). \qquad (20)$$

The expression of relativistic kinetic energy will be received according to (5)

Taking into consideraration (17), (19), we receive

$$w_k = \int_0^p v dp. \qquad (21)$$

The differential of impulse will be found according to (17)

$$dp = \frac{dp}{dv} dv = m''(v) dv, \qquad (22)$$

where $m''(v)$ is differential relativistic mass

$$m''(v) = \frac{dp(v)}{dv} = \frac{m_0}{(1 - v^2 / c^2)^{3/2}}. \qquad (23)$$

Substituting (22), (23) into (21), after integration we approach the known expression of relativistic kinetic energy [4, 7]

$$w_k = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2 / c^2}} - 1 \right). \qquad (24)$$

At $v = 0$ both kinetic co-energy (20) and kinetic energy (24) equal zero, and at $v = c$ they behave differently

$$\begin{aligned} w_k \big|_{v=0} &= w_{kc} \big|_{v=0} = 0; \\ w_k \big|_{v=c} &= \infty; \quad w_{kc} \big|_{v=c} = m_0 c^2. \end{aligned} \qquad (25)$$

In [7] two important simple theorems are proved:

$$\begin{aligned} & w_{kc} \big|_{v<<c} = \frac{m_0 v^2}{2}; \; w_k \big|_{v<<c} = \frac{m_0 v^2}{2}; \\ & w_k + w_{kc} = pv. \end{aligned} \qquad (26)$$

It is possible to transform the expression of kinetic energy (24) into the form of kinetic co-energy (20)

$$\begin{aligned} w_k &= m'(v) c^2 \left(1 - \sqrt{1 - v^2 / c^2}\right); \\ w_{kc} &= m_0 c^2 \left(1 - \sqrt{1 - v^2 / c^2}\right). \end{aligned} \qquad (27)$$

As we can see, static relativistic mass appears in the expression of kinetic energy, and so does rest mass in the expression of kinetic co-energy.

Let us consider the role of kinetic relativistic energy and co-energy in a relativistic dynamics. Therefore, we can obtain the forces through the gradient of corresponding energies

$$F_k = -\frac{\partial w_k}{\partial x}; \quad F_{kc} = -\frac{\partial w_{kc}}{\partial x}. \qquad (28)$$

Substituting (20) and (24), we obtain

$$\begin{aligned} F_k &= \frac{m_0}{\left(1 - v^2 / c^2\right)^{3/2}} a = m''(v) a; \\ F_{kc} &= \frac{m_0}{\sqrt{1 - v^2 / c^2}} a = m'(v) a, \end{aligned} \qquad (29)$$

where a is acceleration $a = dv / dt$.

Using energy and co-energy for relativistic velocities, we came to different results. But if the velocity $v = c$, both forces are equal, because $m'(v) = m''(v) = m_0$

To answer a question which force, F_k or F_{kc}, is real for us and which is unreal, in our opinion, it is necessary to apply the time derivative of impulse that for both energies w_k and w_{kc} is the same

$$F = \frac{dp}{dv} \frac{dv}{dt} = m''(v) a = F_k. \qquad (30)$$

The appearance of kinetic co-energy in relativism removes its theory from some self-contradictions. It is impossible here to omit two immediate tasks being its constituent elements.

1. The appearance of kinetic energy in the special theory of relativity has wholly understandable natural character. Variation methods are widely involved in the general theory of relativity [7]. And in variation methods in the presence of non-linearity, such as, exactly, relativistic effects, kinetic energy does not work. It was convincingly proved by practice of mathematical modeling [1–3]. Therefore, the authors of general theory of relativity were subconsciously forced to approach to the concept of co-energy and actually to apply expression (2) instead of natural one (5). Therefore, they had to consider co-energy as the energy of old concepts, though expressions of both differ substantially (see (29)).

2. At passing to the dynamics at different energies w_k and w_{kc}, the crisis of relativistic mass occured, in regard to both longitudinal and transverse mass. In our exposition the transvers mass is nothing but static $m'(v)$ (19), and longitudinal mass is differential $m''(v)$ (23).

And, however, as only kinetic co-energy satisfies variation principles, it is expedient to consider it as a veritable physical characteristic.

Variation methods as mediators of law of conservation of energy operate with the concepts of kinetic co-energy only, because it is it that is primary. Getting through a certain medium or vacuum filling some given space, it reaches to place of destination as the kinetic energy. In a certain medium the part of co-energy is spent on internal resistance of its "non-linearity" that is why here the condition $w_k < w_{kc}$ is always satisfied. In relativism, the nature of vacuum non-linearity or space curvature, on the contrary, feeds co-energy, and as a result here we always have $w_k > w_{kc}$. In the absence of non-linearity we have $w_k = w_{kc} = pv/2$. Thus, the phenomenon of energy that accompanies motion appears as kinetic co-energy, and shows up as kinetic energy and corresponding power action (see (30)) which is a measurable value!

An analysis disclosed that the power action of kinetic energy showed up through the differential parameters of medium, and the force action of kinetic co-energy did so through static ones. In all other situations these expressions coincide.

Here a question can arise: why kinetic co-energy appears only in the nonlinear systems? – Proceeding from its role in physical interactions, it is present either in linear or in nonlinear cases. In the first case they equal each other, that is why, they take each other's place, giving us the impression that we operate only with one of them.

At the consideration of spatial curvature a bit too much attention is spared to the relativistic mass instead of an impulse. While the impulse is a basic vector characteristic, the mass is only a derivative statical or differential parameter. It can be seen at subconscious level in the theory of gravitation: the mass actually yielded to the impulse *p*, speed *v*, energy *w*

$$p = wv/c^2; \quad w = c\sqrt{p^2 + m_0^2 c^2}. \qquad (31)$$

The first expression presents the impulse, and the second shows the Hamiltonian function.

5. Conclusions

1. The phenomenon of energy that accompanies motion appears as kinetic co-energy, and becomes apparent as kinetic energy and corresponding force action.

2. Kinetic co-energy is present in any movable physical system, regardless of the degrees of detailed elaboration of the physical phenomena. The newest theory of gravitation cannot do without it too. On the basis of the concept of kinetic co-energy the paradox of longitudinal and transvers relativistic masses is refuted.

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ЕНЕРГІЯ РУХУ

Василь Чабан

Показано, що рух супроводжується енергією і коенергією, які невіддільні одна від одної. У лінійному середовищі вони дорівнюють одна одній, тому заступають одна одну. У нелінійному середовищі вони різні, і кожна з них виконує свої функції. Запропоновано універсальний вираз коенергії фізичної системи. Поняття кінетичної енергії у варіаційні принципи не вписується. Приклади стосуються електромагнетного поля і релятивістської теорії гравітації. Спростовано парадокс поперечної і поздовжньої релятивістських мас.

Vasyl Tchaban – Sc. D., full professor at Lviv Polytechnic National University (Ukraine), Editor-in-Chief of Technical News journal. He obtained his Doctor-eng. habil degree in Electrical Engineering at Moscow Energetic University (Russian) in 1987. His research interests lie in the areas of mathematical modelling of electrome-chanical processes and electromagnetic field theory, surrealistic short story writter. He is the autor of about 500 scentific publications and about 500 surrealistic short stories, including 36 books (9 monographs, 11 didactic, 7 humanistic and 9 of the arts), over 1200 aphorisms.