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SYNTHESIS OF AUTOMATIC CONTROL SYSTEMS BY USING BINOMIAL AND BUTTERWORTH STANDARD FRACTIONAL ORDER FORMS

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Abstract: The article deals with the application of binomial and Butterworth standard fractional order forms in the synthesis of control systems. This work aims at improving the method for the synthesis of fractional controllers for the systems of any structure, on condition of applying desired standard fractional order forms. Due to the usage of standard fractional forms, a range of possible settings for fractional order controllers in the synthesis of control system loops has been expanded and the better desired quality of transition processes in comparison with the integer order controllers has been provided. It has been proved that given the obtained research results for establishing the control system loops,, Butterworth standard fractional forms with q = 0,9 ÷ 1,3, as well as the binomial ones with q = 0,1 ÷ 2 can be recommended to apply since they meet the requirements of control objects. Some of the obtained results that can be recommended for practical use when configuring control systems are presented below. In terms of the research conducted, a maximum deviation between the simulation results and the desired ones does not exceed 1%. Thus, due to the proposed approach, the efficiency of the synthesized systems has been increased.

Key words: synthesis of a fractional order controller, fractional order transfer function, standard fractional forms.

1. Introduction

Synthesis of control systems (CS) using root methods makes a wide use of standard forms of a pole distribution in the complex plane. The dynamic characterristics of any control system are determined by a transfer function (TF) of the system. If a synthesized CS has only poles, the form of the transition function of an original coordinate will be determined by them, i.e. if CS is presented by a transfer function without zeros, its dynamic processes are completely defined by an expression of the characteristic polynomial $H(s)$. For the systems described by integer characteristic polynomials, it is always possible to choose a desired (standard) characteristic polynomial $H_{st}(s)$ of the integer order. The number of $H_{st}(s)$ is large [1], but binominal standard forms ($(H_{bin}(s))$), as well as Butterworth standard forms ($H_{But}(s)$) of the integer order n [1] are most often chosen for CS.

Among a variety of CS, there may be the systems described by the characteristic polynomials of fractional order q [2, 4, 5, 6, 7, 9, 11, 12, 13] (q is a fraction). The synthesis of controllers for such systems is based on defined parameters of logarithmic amplitude-frequency and phase-frequency characteristics, known as Bode diagrams [5]. The research into the fractional PID controllers is considered in [8, 10].

An approach to the synthesis of fractional order CS may have analogy with the root methods, provided that the desired characteristic polynomial $H_{des}(s)$ is described by any expression with known transition functions. This desired characteristic polynomial can be called standard on the analogy with the integer order ones, although it does not reflect the pole placement in the complex plane. This is due to the fact that the value of the poles for fractional order systems is not informative. For example, from the expression below we see

$$H(s) = (s + w_{oc})^q \tag{1}$$

that the pole $s_i = -w_{oc}$, i.e. similar to q = 1; 2; 3... The matter is probably about the systems with different dynamic properties, and the pole values in both cases are identical. At the same time, the transition functions for both systems will be different, as well as their Bode diagrams.

Apparently, the results of the controllers synthesis on the basis of any standard form can be used to obtain other controllers than the PID ones.

The aim of this work is improving the method for the synthesis of fractional controllers for CS of any structure, on condition of applying desired standard fractional order forms [3].

2. Synthesis procedure

The approach to the synthesis of CS controllers, which makes use of the so-called standard binominal and Butterworth forms of fractional order [13] is proposed in this article. This approach can be used when it is necessary to provide desired properties of the control coordinate: d is the overshoot value, $t_{0.95}$ represents the time of the first achieving 95% of the invariable value of the coordinate. The algorithm of this approach is as follows:

1. According to the given block diagram of the closed loop, its TF $W_{cl}(s)$ is calculated;

2. The expression $W_{cl}(s)$ is transformed, dividing its numerator and denominator by the expression numerator. Thus, an expression with the numerator equal to one is obtained, and the denominator corresponds to the characteristic polynomial, which includes unknown parameters;

3. On choosing a binomial or Butterworth standard form as the desired one in terms of the desired parameters of the transition process, i.e. the overshoot d and the rise time $t_{0.95}$, we bring forward the transformation of the expression found in 2 above into the TF expression of a characteristic polynomial of the selected standard form $W_{st}(s)$;

4. From the condition of identity of the denominator of $W_{cl}(s)$ with that of $W_{st}(s)$, a system of equations will be obtained;

5. The system of equations having been solved, the expressions for finding of unknown parameters of the characteristic polynomial are obtained, including a TF $W_c(s)$ fractional controller.

This algorithm has the following advantages:

– the possibility of obtaining the desired transition characteristics matching the binomial, Butterworth and other standard distribution forms of the characteristic equation roots for CS with zeros in TF;

– the possibility of synthesizing an astatic control system, based on the dependent, modal and combined control principles.

We have considered the possibility [13] of using the fractional variations of a standard distribution form of the characteristic equation roots: both binomial and Butterworth as the standard ones for CS loops optimization. Below, we give some of the results obtained that can be recommended for practical use when configuring CS.

3. Standard fractional forms

Let us consider a Butterworth standard fractional order form represented by CS with a TF component:

$$W_{st.But}(s) = \frac{w_{oc}}{s^q + w_{oc}}, \qquad (2)$$

and a binomial standard fractional order form represented by TF

$$W_{st.bin}(s) = \frac{\omega_{oc}}{(s + \omega_{oc})^q}, \qquad (3)$$

where w_{oc} is the desired value of the average geometric root of an ACS component, which determines its performance.

By using MATLAB environment, we have obtained the transition functions and Bode diagrams [13] that meet the Butterworth standard fractional order form $H_{But}(s) = s^q + w_{oc}$ for q =0.1÷1.9, and the binomial standard fractional order form $H_{bin}(s) = (s + w_{oc})^q$ for q =0,1 ÷ 2 if w_{oc} = 1; 10; 100 s^{-1}. Fig.1 a, b presents transition functions, and Fig.2 a, b shows Bode diagrams of the research for w_{oc} = 10 s^{-1}, and parameters of the obtained transition functions are given in Table 1 and Table 2 respectively.

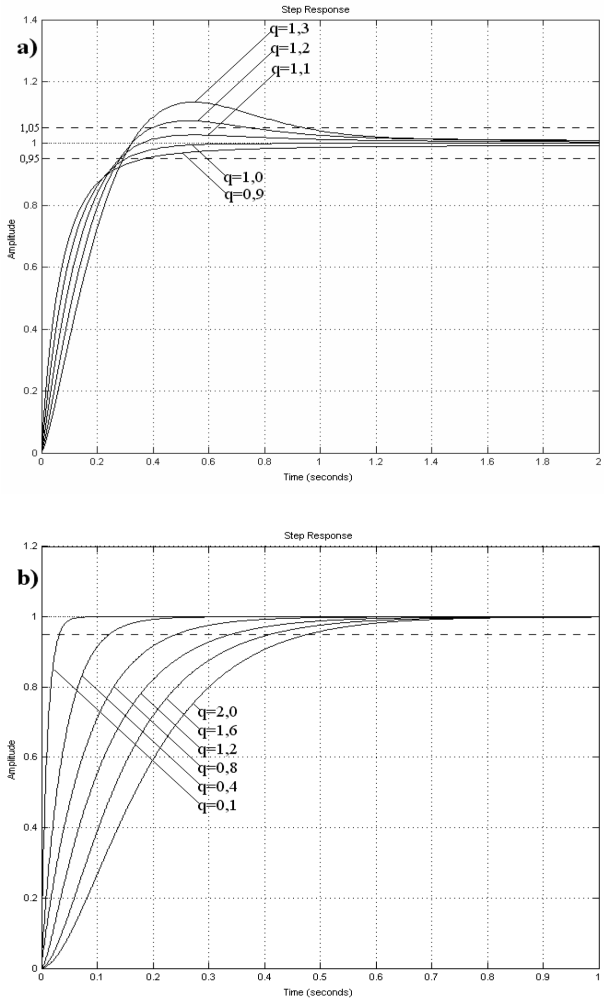

Fig. 1. Transition functions:
$H_{But}(s)$ for q =0,9; 1,0; 1,1; 1,2; 1,3 (a) and
$H_{bin}(s)$ for q =0,1; 0,4; 0,8; 1,2; 1,6; 2,0 (b), if w_{oc} =10s^{-1}.

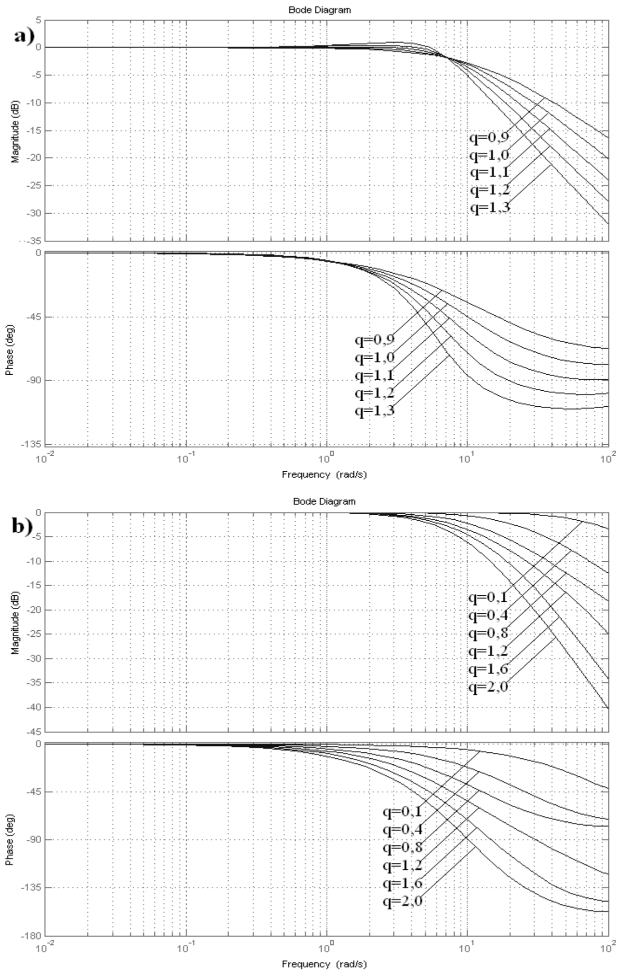

Fig. 2. Bode diagrams: $H_{But}(s)$ for q =0,9; 1,0; 1,1; 1,2; 1,3 (a) and $H_{bin}(s)$ for q =0,1; 0,4; 0,8; 1,2; 1,6; 2,0 (b), if w_{oc} =10s^{-1}.

Table 1

Butterworth standard fractional order form

$H_{But}(s)=s^q+\omega_0$					
№	q	w_{oc} [s^{-1}]	δ [%]	$t_{0.95}$ [s]	$t_{settling}$ [s]
1	0,9	1	-	4,75	4,75
2	1,0	1	0	3,014	3,014
3	1,1	1	2,7	2,272	2,272
4	1,2	1	7,3	1,9136	5,0925
5	1,3	1	13,4	1,72	5,53
6	0,9	10	-	0,365	0,365
7	1,0	10	0	0,3	0,3
8	1,1	10	2,7	0,28	0,28
9	1,2	10	7,3	0,28	0,75
10	1,3	10	13,3	0,29	0,94
11	0,9	100	-	0,02985	0,02985
12	1,0	100	0	0,0319	0,0319
13	1,1	100	2,7	0,0361	0,0361
14	1,2	100	7,3	0,0424	0,1106
15	1,3	100	11,36	0,0506	0,1628

Table 2

Binomial standard fractional order form

$H_{bin}(s)=(s+\omega_0)^q$				
№	q	w_{oc} [s^{-1}]	$t_{0.95}$ [s]	$t_{settling}$ [s]
1	0,1	1	0,3018	0,3018
2	0,4	1	1,2033	1,2033
3	0,8	1	2,4095	2,4095
4	1,2	1	3,405	3,405
5	1,6	1	4,117	4,117
6	2,0	1	4,789	4,789
7	0,1	10	0,0319	0,0319
8	0,4	10	0,1219	0,1219
9	0,8	10	0,2418	0,2418
10	1,1	10	0,322	0,322
11	1,2	10	0,341	0,341
12	1,6	10	0,4108	0,4108
13	2,0	10	0,478	0,478
14	0,1	100	0,00463	0,00463
15	0,4	100	0,01387	0,01387
16	0,8	100	0,02592	0,02592
17	1,2	100	0,0358	0,0358
18	1,6	100	0,0432	0,0432
19	2,0	100	0,0507	0,0507

Let us consider different synthesis options of a fractional controller for CS in relation to the proposed method reasoned by:

– the peculiarities of the object under control,

– the wish to obtain a desired transition process (monotonous or with overshoot).

4. Synthesis example 1

As one of the options of using the proposed approach to optimization, we may consider CS with a TF control object

$$W_H(s) = \frac{1}{0,8s^{2,2} + 0,5s^{0,9} + 1}. \qquad (4)$$

Control object (4) is borrowed from [9] in order to compare the effectiveness of the proposed synthesis method with the swarm particle optimization method considered there. Fig.3 demonstrates a block diagram of CS with the control object $W_H(s)$, fractional controller $W_c(s)$ and coefficient K_{fc} feedback, and a transition function of the control object which corresponds to the TF is shown in Fig. 4 (curve “1”).

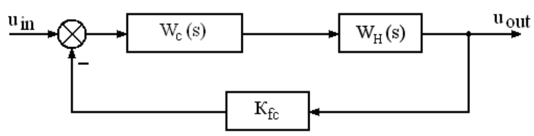

Fig. 3. Block diagram of CS.

In [9], the author set the following parameters of the transition process in CS with TF (4): a maximum 10 % overshoot and 0.3 s rise time. As a result of the synthesis, he obtained a TF of the controller and a transition process with the following parameters: the 0.03 s rise time and 0,5 % overshoot, which differ significantly from the set ones.. It should be noted that the parameters of the synthesized controller are difficult to be implemented in practice.

$$W_c(s) = 442,38 + 324,03s^{-1,5} + 115,27s^{1,41} . \quad (5)$$

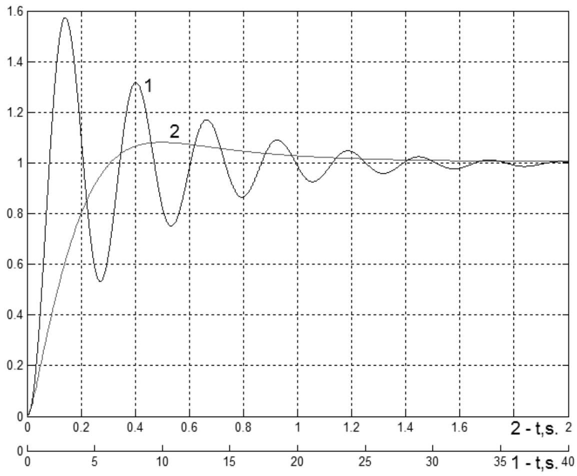

Fig. 4. Transition functions of: a control object – curve "1", an"optimized loop using Batterworth form – curve "2".

Let us synthetize the controller for CS by means of the proposed method, for example, with the following given in Table 1 parameters of the transition process: d =7,3 %, w_{oc} =10 s^{-1} and $t_{0.95}$ =0,28 s.

According to the block diagram (see Fig 3), the TF of the closed loop $W_{cl}(s)$ is as follows:

$$W_{cl}(s) = \frac{W_c(s)\dfrac{1}{0,8s^{2,2}+0,5s^{0,9}+1}}{1+W_c(s)\dfrac{1}{0,8s^{2,2}+0,5s^{0,9}+1}K_{fc}} . \quad (6)$$

Dividing the numerator and denominator of the obtained TF, by the numerator, we will obtain

$$W_{cl}(s) = \frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_c(s)}+K_{fc}} . \quad (7)$$

It is obvious that to provide the given parameters of the transition process, we choose a Butterworth fractional order form $W_{st.But}(s)$, and set the requirement for the transformation of expression (7) into expression (2), with an introduced parameter – the feedback coefficient K_{fc}, which in expression (2) is equal to 1. If $W_{cl}(s)$ (7) and $W_{st}(s)$ (2) are identical, we shall obtain the following:

$$\frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_c(s)}+K_{fc}} = \frac{w_{oc}/K_{\text{fc}}}{s+w_{oc}} . \quad (8)$$

The desired parameters of the transition process with a 7,3 % overshoot and $t_{0,95}$ =0,28 s are selected from Table 1, which are possible due to the standard form (line №9) with the parameters q =1,2; w_{oc} =10 s^{-1} and K_{fc} = 1. Thus

$$W_{st.But}(s) = \frac{10}{s^{1,2}+10} . \quad (9)$$

Substituting (9) into (8) we shall obtain

$$\frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_C(s)}+1} = \frac{10}{s^{1,2}+10} .$$

Equating the identical left and right components of the characteristic polynomials, a TF of the fractional order controller will be obtained:

$$W_c(s) = 8s^{1,0} + 5s^{-0,3} + 10s^{-1,2} .$$

With the use of this controller, we obtain a transition process with the following parameters: d =8,1 %, $t_{0.95}$ =0,271 s. (Fig. 4, curve "2"), i.e. the deviation from the given parameters is less than 1 %.

5. Synthesis example 2

Let us consider another fractional controller synthesis with the given parameters of the transition process, as in the previous example: d =7,3 % i $t_{0.95}$ =0,28 s, but for CS with a TF control object:

$$W_H(s) = \frac{1}{0,5s^{0,9}+1} . \quad (10)$$

Fig. 5 (curve "1") shows the transition function of the control object that meets this TF.

In this case, the TF of the closed loop ($W_{cl}(s)$) takes the following form:

$$W_{cl}(s) = \frac{W_c(s)\dfrac{1}{0,5s^{0,9}+1}}{1+W_c(s)\dfrac{1}{0,5s^{0,9}+1}K_{fc}} .$$

Having transformed as in the previous example, and set standard form (9) for K_{fc} = 1, we shall obtain

$$\frac{1}{\dfrac{0,5s^{0,9}+1}{W_c(s)}+1} = \frac{10}{s^{1,2}+10} .$$

Hence, the TF of the controller is:

$$W_c(s) = 5s^{-0,3} + 10 \cdot s^{-1,2} . \quad (11)$$

So, as a result of the synthesis, a fractional order controller has been obtained without a proportional component of the I^2 type.

With the use of this controller, we obtain a transition process with the following parameters: d =8,1 %,

$t_{0.95}$ =0,271 s (Fig. 5, curve "2"), i.e. the deviation from the given parameters is less than 1 %.

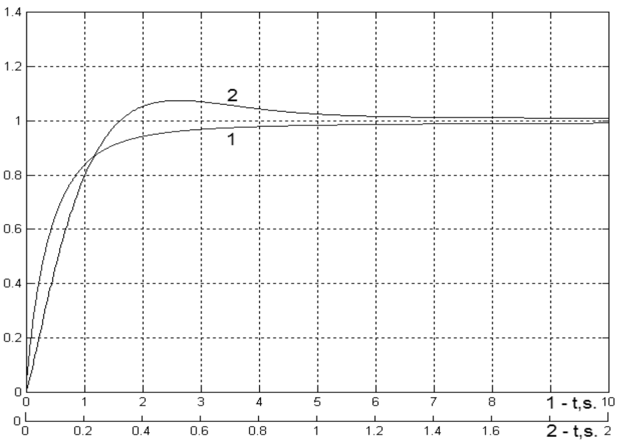

Fig. 5. Transition functions of a control object– curve "1", an optimized loop –curve "2".

6. Synthesis example 3

Let us consider another option of fractional controller synthesis with the given transition function parameters d =0% and $t_{0.95}$ =0,32s for CS with TF control object (4) (Fig. 6, curve "1"). It is obvious that in this case, on condition that there is no overshoot present, a binomial standard fractional form should be used. According to Table 2, the given parameters of the transition process are provided by a standard form with the parameters in line 10 of the Table, if w_{oc} =10 s^{-1} and K_{fc} =1. Then

$$W_{st}(s) = \frac{w_{oc}{}^{1.1}/K_{fc}}{(s+w_{oc})^{1.1}}. \qquad (12)$$

From the condition of identity, $W_{cl}(s)$ and $W_{st}(s)$ the following equation will be obtained:

$$\frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_c(s)}+K_{fc}} = \frac{w_{oc}{}^{1,1}/\mathrm{K_{fc}}}{(s+w_{oc})^{1,1}}. \qquad (13)$$

For K_{fc} = 1 and w_{oc} =10s^{-1} , by dividing the numerator and denominator of the right expression part by $10^{1,1}$, we shall obtain:

$$\frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_p(s)}+1} = \frac{1}{(0,1s+1)^{1,1}}. \qquad (14)$$

In [13], the possibility of approximation of the binomial fractional order form by an expression with the integer order derivative is proved:

$$(0,1s+1)^{1,1} = 1+0,11s+0,00055s^2.$$

Then expression (14) can be written as

$$\frac{1}{\dfrac{0,8s^{2,2}+0,5s^{0,9}+1}{W_c(s)}+1} = \frac{1}{0,00055s^2+0,11s+1}.$$

Thus, a TF of the controller will be as follows:

$$W_c(s) = \frac{0,8s^{2,2}+0,5s^{0,9}+1}{0,00055s^2+0,11s}. \qquad (15)$$

Substituting TF (15) into the initial CS, we shall obtain a system with the transition process shown in Fig.6, curve "3", possessing the following parameters: d = 0 % and $t_{0.95}$ = 0,3195s. Thus, the deviation from the given parameters does not exceed 0,2 %.

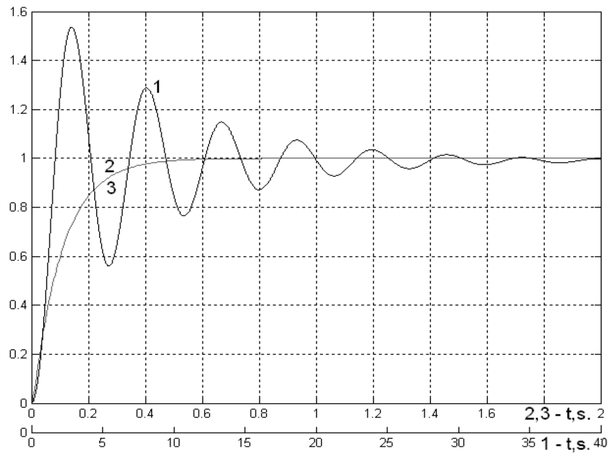

Fig. 6. Transition functions of a control object – curve "1", a binomial form – curve "2", an optimized loop – curve "3".

7. Conclusions

1. The use of standard fractional forms extends the range of possible fractional controllers settings in the synthesis of CS loops, provides better quality of transition processes in comparison with the integer order controllers and thereby encreases the effectiveness of the synthesized systems.

2. On the basic of the results obtained, for the CS loops to be established, the Batterworth standard fractional forms with q =0,9 ÷ 1,3 and the binomial ones with q =0,1 ÷ 2 can be recommended as those able to meet the requirements of the control objects.

3. The proposed approach to the synthesis of CS loops with fractional controllers can provide the desired quality of the transition process. The maximum deviation between the simulation results and the desired ones does not exceed 1 %.

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СИНТЕЗ СИСТЕМ АВТОМАТИЧНОГО КЕРУВАННЯ ЗА ВИКОРИСТАННЯ БІНОМІАЛЬНОЇ ТА БАТТЕРВОРТА СТАНДАРТНИХ ФОРМ ДРОБОВОГО ПОРЯДКУ

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Розглянуто застосування стандартних дробових бінональних форм та форм Баттерворта під час синтезу систем керування. Метою цієї роботи є удосконалення методу синтезу дробових регуляторів довільної структури, за умови забезпечення бажаних стандартних форм дробового порядку. У зв'язку з використанням дробових стандартних форм розширено гамму можливих налаштувань дробових регуляторів під час синтезу контурів ЕМС та забезпечено кращу якість перехідних процесів порівняно з регуляторами цілого порядку. Доведено, що на основі отриманих результатів досліджень для налагодження контурів ЕМС можна рекомендувати дробові стандартні форми: Батерворта за q = 0,9–1,3 і біноміальні за q = 0,1–2, як такі, що задовольняють вимоги об'єктів керування. Наведено деякі з отриманих результатів, які можна рекомендувати для практичного використання під час налаштування систем керування. З огляду на проведене дослідження, максимальне відхилення між результатами, отриманими через моделювання і бажаними не перевищує 1 %. Отже, завдяки запропонованому підходу підвищено ефективність синтезованих систем.

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