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Research of influence of calculation precision on the effectiveness of stochastic optimization methods

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Abstract. This paper describes the influence of the varying computation precision when performing calculations using the optimizing algorithms. A comparative analysis of the computation speed and obtained result accuracy of the Rastrigin's direct cone method with adapting of the step length and the angle of the cone's disclosure for varying precision was performed. It is shown that the speed of the optimization algorithm practically does not depend on used computation precision. The difference is observed only in accuracy of the obtained results.

The investigation of optimizing algorithms behavior under the presence of noise, in particular due to rounding errors was conducted. It is shown that the optimizing algorithm under research becomes unsuitable after some noise level. Characteristics of the optimization algorithm during calculations with a single precision proved to be better then the characteristics of the algorithm when performing calculations with double precision.

The analysis of possibilities of the effective graphics processors (GPU) application in order to conduct optimization was carried out. In particular, the difference in the speed of the GPU when performing calculations with a single and double precision was considered.

To ensure the efficiency of calculations based on optimization algorithms, it is recommended to carry out calculations with the use of single precision, and increase the calculation precision in case of impossibility to achieve the desired accuracy of the result.

There is considering the significantly higher performance of graphics processors when doing calculations with a single precision in comparison with calculations with double precision it is expedient to use a single calculation precision when graphic processors are used to solve considered problem. Double precision can be used if it is difficult to get sufficiently correct solution by single precision calculations.

The results of numerical experiments confirm that the use of lower precision to perform optimization for macromodels creation has a slight influence on the speed of achieving of predetermined optimization accuracy.

Key words: stochastic optimization methods, graphics processors, optimization speed.

INTRODUCTION

Optimization algorithms are now widely used to solve mathematical problems of different range. One of the common problems users have to deal with when using computer calculations based on optimization algorithms are local minima that appear as a result of rounding errors, especially if goal function is not described analytically.

From the other side, optimization algorithms generally require significant amount of computations to be performed. Thus different techniques to perform calculations faster are used, in particular by utilizing high computational power of GPU cards.

In order to make optimization algorithm efficient it would be good to perform the calculations using single precision values, because such calculations are executed much faster than when using double precision values.

A comparative analysis of the optimization algorithms characteristics using different precision of calculations is presented in this article.

The analysis shows that the speed of the optimization process in terms of required algorithm iterations does not noticeably depend on the used precision of calculations. Different precision of calculations affects only the achievable precision of the result. Thus in order to make optimization algorithms work faster it is recommended to start calculations using single precision values and switch to double precision only when sufficient precision of the result is not obtained.

THE ANALYSIS OF RECENT RESULTS AND PUBLICATIONS

Currently the optimization approaches are used more and more widely to solve mathematical problems of different range, in particular for analyzing of large systems of algebraic equations, numerical integration and macromodeling [3]. When above-mentioned approach are implemented using computer technique users have to deal with local minima that appear as a result of rounding errors, as well as in cases when the goal function is not described analytically, that leads to a growth of computational complexity of the tasks mentioned above.

Optimization algorithms generally require significant amount of calculations, especially when the goal function is a multi-dimensional function with long and narrow flat valleys, which is common for many areas, in particular for macromodel construction [1, 7].

Not so many optimization algorithms can be effective in such conditions [14, 15]. The Rastrigìn's direct cone optimization method with special procedures for automatic adaptation of the length of optimization step and the angle of the cone's disclosure [2] was used in this research. As to our mind it is the most effective method for practical implementation to solve the problems appearing during macromodelling of dynamic systems. It was used by authors to create macromodels of objects of different types [1].

The essence of the researched direct cone method is as follows:

1. Orientation at the starting point. The hyper sphere with the center at the point $\vec{x}_0$ and radius h should be constructed. On the surface of the hyper sphere m random points of uniform law of distribution should be selected. Among them the point where the value of the goal function is lowest should be found out. This point is designated as $\vec{x}_1$, and is used to determine the initial memory vector:

$$\vec{W} = \frac{\vec{x}_1 - \vec{x}_0}{h}. \qquad (1)$$

2. At each step the hyper cone is built. It has the apex at the point $\vec{x}_i$, the angle of the disclosure y and axis $\vec{W}$. This hyper cone will trunk some surface from the hyper sphere with center at the point $\vec{x}_i$ and radius h. On this surface m random and uniformly distributed point are selected. Among them the point $\vec{x}_{i+1}$, in which the value of the goal function is the lowest is selected. The memory vector should be recalculated using the following law:

$$\vec{W}_{i+1} = a \cdot \vec{W}_i + b \cdot \frac{\vec{x}_{i+1} - \vec{x}_i}{h}, \qquad (1),$$

where: $0 \le a < 1$, $0 < b \le 1$.

The searching process continues until the value of the goal function goes down. In order to improve features of the Rastrigin's direct cone method several additional procedures were developed. The most important and frequently used improvements are adaptation of the length of the search step h and the angle of the cone disclosure y [2].

OBJECTIVES

Our task is to research the efficiency of optimization process with different precision of calculation.

The research uses a variant of the Rosenbrock's function of the following form [5]:

$$\begin{aligned} f(\vec{x}) &= (1-x_1)^2 + 100\left(x_2 - x_1^2\right)^2 + 100\left(x_3 - x_2^2\right)^2 + \\ &+ 100\left(x_4 - x_3^2\right)^2 + (1-x_5)^2 + 100\left(x_6 - x_5^2\right)^2 + \\ &+ 100\left(x_7 - x_6^2\right)^2 + 100\left(x_8 - x_7^2\right)^2 + \\ &+ (1-x_9)^2 + 100\left(x_{10} - x_9^2\right)^2. \end{aligned} \qquad (2)$$

The specified goal function has a long and narrow flat valley that is typical for optimization problems.

To assess the effectiveness of the proposed optimìzation method it is expedient to use the following three dependencies:

1. The dependence of the number of iterations and obtained accuracy of results on the precision of calculations. In this case optimization algorithm will be allowed to stop according to standard criteria. In our case, the following criterion has been used: a failure to improve the value of the goal function within 30 iterations of the algorithm;
2. The dependence of the optimization speed on the precision of calculations;
3. The dependence of the optimization speed of calculations on the precision under presence of random noise imitating the rounding errors.

MAIN RESULTS OF THE RESEARCH

To study the dependence of the number of iterations and reached accuracy of the results on selected calculation precision a search of the global minimum of the function (3) in 2d-space was carried out. For a specified precision of calculations 10 experiments were conducted and data averaging was used.

Obtained dependencies of number of iterations carried out by the algorithm and resulting deviation of the searched point from the optimal solution on calculation precision are depicted in the Fig. 1. According to the chart (Fig. 1), under higher calculation precision the deviation of position from the exact solution is smaller. However, in such a case number of iterations of the algorithm is greater. This result is fully consistent with expectations.

To compare the optimization speed when calculations with different precision are used let us carry out three series of experiments: for double precision (~ 16 significant digits), single precision (~ 8 significant digits), and for 4 significant digits.

In each experiment type five attempts of the optimization procedure of the goal function was carried out and the average value for the specified number of iterations of optimization algorithm was done.

In Fig. 2 the dependence of the goal function values on the number of executed iterations of optimization algorithm for varying precision is presented.

According to presented dependencies (Fig. 2), the nature of optimization process does not depend practically on settled calculation precision. In all cases the value of the goal function decreases gradually, while the computation speed becomes slower in the process of the minimum approaching.

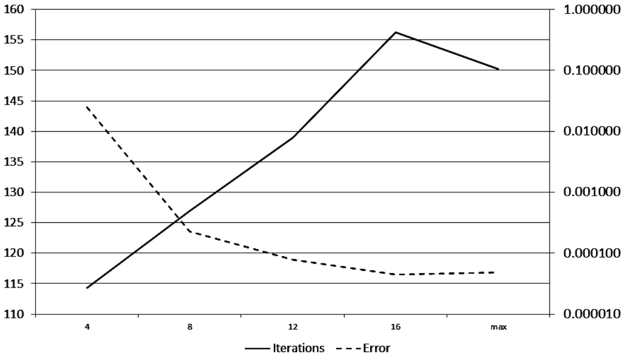

Fig. 1. Number of iterations carried out by the algorithm and resulting deviation of the searched point as a function of calculation precision. The number of decimal places that are taken into account in the calculations are depicted along the horizontal axis. The left vertical axis displays the number of iterations, the right vertical axis – the deviation of the obtained position of the minimum point from the optimal solution

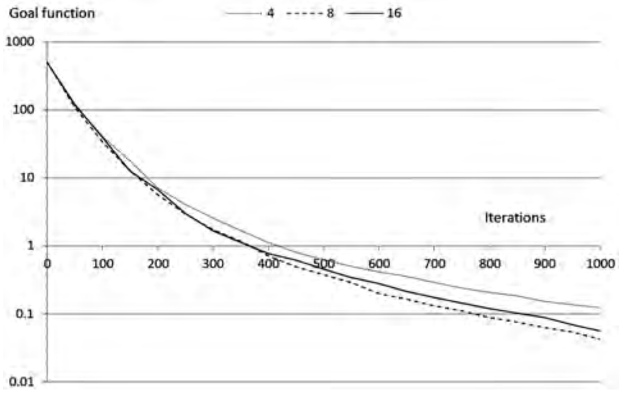

Fig. 2. The dependence of the goal function on the number of conducted iterations of optimization algorithm under different precision of calculations

In order to research a dependence of optimization speed on selected precision of calculation under presence of random noise the function (1) should be modified in the following form:

$$\tilde{f}(\vec{x}) = f(\vec{x})(1+k\xi), \qquad (4)$$

where: ξ a random value uniformly distributed along with the interval [-1, 1], k is a weighting coefficient.

Six series of experiments were carried out: two experiments in the absence of noise ($k=0$), as well as for the following values of the noise intensity: 0.1 % ($k=0.001$) and 0.2 % ($k=0.002$). In each pair 10 repeating were performed of the function (4) optimization for double precision (~ 16 significant digits) and for single precision (~ 8 significant digits). The experimental results were averaged for each series.

The results of the experiments are in the Fig. 3.

We should note that the third presented research has an interesting results. As it can be seen from the figures a nature of the optimization process does not depend on the used precision of calculations (double or single). Also the optimization speed does not change under the noise level of 0.1 %. It indicates that the selected algorithm of optimization is not sensitive to small noises. But when the noise level is increased up to 0.2 % the nature of optimization process changes dramatically. Actually the algorithm is not able to solve this optimization problem both when double-precision and single precision calculations are used. Moreover, if you use a better precision of calculations the optimization algorithm showes worse results.

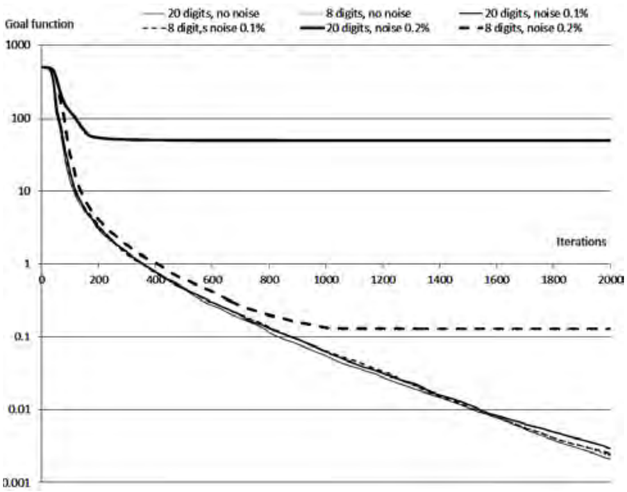

Fig. 3. The dependence of the goal function on number of executed iterations of optimization algorithm for varying precision of calculations and the level of noise

Additional studies have shown that limits of the noise level at which this change can occur depends on the minimum allowed length of the searching step, which is appointed for the algorithm. In particular, under increasing of the length of searching step the permissible noise level increases too.

As to our mind the following explanation of the optimization algorithm approach is possible: if the noise level is essential the calculation of the goal function at some iteration can give us a randomly lowered value (due to the presence of noise). This lowered value is selected as a further initial assumption because it is better than other known values. However, the further calculations of the goal function give only worse results, and the algorithm stops.

We should underline that if the rounding error is considered to be the reason of the noise appearing then usage of higher calculation precision will lead to the lower noise amplitude and it will reduce the risk of preterm stop of the optimization algorithm. It is the opinion of the authors, and it is the reason of the relationship between precision and reached accuracy of the solution presented in the Fig. 1.

CONCLUSIONS

The results of numerical experiments confirm that the use of lower precision to perform optimization for macromodels creation has a slight influence on the speed of achieving of predetermined optimization accuracy (if the algorithm is able to achieve it).

Obviously, if the lower calculation precision is used a preterm stop of optimization algorithm can happen. It will lead to impossibility to obtain a desired level of the result. In this case one should continue the optimization using double-precision calculations, taking the result obtained from single-precision calculations as an initial approximation.

Based on presented facts, it can be recommended to pay attention to the possibility of usage of graphic processors for solving tasks discussed in this paper. For these calculation means a speed of calculations essentially depends on the precision of the calculation.

For example, on architecture "Maxwell" of NVIDIA Corporation the speed of calculations with double precision was equal to 1/32 of the speed of calculations with single precision. For the next generation of the graphic processors with "Kepler" architecture this factor was improved essentially and it was equal to 1/3 [6]. As it can be seen, in spite of all efforts of graphic processor producers to improve characteristics of their devices and to ensure high speed, if calculations with double precision should be carried out, conduction of operations with single precision is still essentially faster. This situation is caused by the significant difference in the complexity of the respective computer modules, and therefore it is expected that the difference in performance for single and double precision will be preserved.

Therefore, considering the significantly higher performance of graphics processors when doing calculations with a single precision in comparison with calculations with double precision it is expedient to use a single calculation precision when graphic processors are used to solve considered problem. Double precision can be used if it is difficult to get sufficiently correct solution by single precision calculations.

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