COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 4, No. 2, 2014

UNCONVENTIONAL METHODS OF ANALYSING DIODE RECTIFIERS WITH ASYMMETRICAL SUPPLY

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Abstract: Output voltage parameters of diode rectifiers such as mean and root mean square (RMS) value are decisive for proper operation of DC systems fed from these rectifiers. It is necessary to know these parameters for the assessment of the operation conditions of these systems. In the paper a formula for the output voltage of three-phase diode rectifiers fed by asymmetrical voltages is derived. The derivation of a universal formula for an output voltage mean value of *n*-phase diode rectifiers with the use of the mathematical induction method is also presented.

Key words: three-phase diode rectifiers, asymmetrical supply voltages, mean value, output voltage, mathematical induction method.

1. Introduction

Three-phase diode rectifiers are supplied by low voltage networks usually with an unearthed neutral (isolated neutral point - IT) (see Fig.1). These devices are used for supply of DC circuits. They can also be used in the systems for the measurement of insulation resistance in AC IT networks [1]. The main parameter of these devices, namely, a mean value of rectifier output voltage (i.e., a DC component), is essential for both applications.

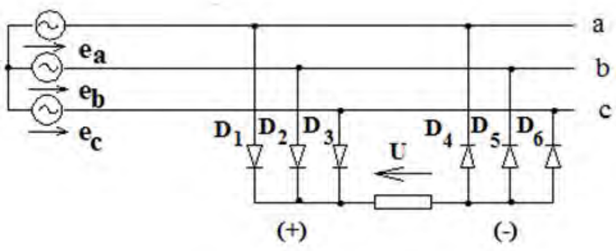

Fig. 1. Circuit diagram of three-phase diode bridge rectifier.

At normal operation conditions, i.e. with symmetric supplying voltages $E_{ab}=E_{bc}=E_{ca}=E_{ph\text{-}ph}$ (RMS values of line voltages) the mean value of rectifier output voltage is given by the well-known formula [2]

$$U_{mean} = \frac{3\sqrt{2}E_{ph-ph}}{p} \qquad (1)$$

In a full-wave rectifier fed from an *n*-phase symmetrical network this parameter is derived as

$$U_{mean} = 2\frac{1}{2p} n \cdot 2 \int_{\frac{p}{2}-\frac{p}{n}}^{\frac{p}{2}} \sqrt{2}E_{ph} \sin a \, da = \frac{n}{p} 2\sqrt{2}E_{ph} \sin\frac{p}{n} = \frac{\sqrt{2}n}{p} E_{ph-ph} \qquad (2)$$

However, in some cases a three-phase voltage source may not be symmetrical. It can be checked that at asymmetrical conditions the following general formula is true

$$U_{mean} = \frac{\sqrt{2}\cdot\left(E_{ab}+E_{bc}+E_{ca}\right)}{p} \qquad (3)$$

The formula (1) is a specific case for symmetrical supply.

2. Unconventional approach to the formula derivation

The analysis of operation of diode bridge rectifiers supplied by an asymmetrical voltage source can be carried out in a few ways. An unconventional approach based on geometrical considerations for three-phase systems is proposed here. Two quite simple methods of formula (3) derivation are presented.

Method 1

Formula (3) can be proved in a quite simple way with the help of the example of waveforms of three line voltages feeding the bridge rectifier. In Fig.2 line voltage e_{ca} waveform is plotted with a reverse sign which is justified from the point of view of rectifier output. Let line voltages be $e_{ab}(t)=\sqrt{2}E_{ab}\sin wt$, $e_{bc}(t)=\sqrt{2}E_{bc}\sin(wt-b)$, $e_{ca}(t)=\sqrt{2}E_{ca}\sin(wt-g)$. The area between the envelope of waveforms e_{ab}, e_{bc}, e_{ca} and a time axis within one time period corresponds to the rectifier output mean voltage. On the other hand, according to formula (3), this voltage is a sum of mean values of half rectified sine waves of line voltages e_{ab}, e_{bc}, e_{ca}. The mean value of the half rectified line voltage, e.g. e_{ab} , corresponds to the area between its half waveform envelope and the time axis. Therefore, it should be proved that referring to Fig.2 the following equation is true

$$\begin{aligned} &2(S_1+S_2+S_3+S_4+S_5+S_6+S_7+S_8)= \\ &=(S_9+S_1+S_2+S_4+S_5)+ \\ &+(S_2+S_3+S_4+S_5+S_6+S_7)+ \\ &+(S_4+S_6+S_7+S_8+S_{10}) \end{aligned} \quad (4)$$

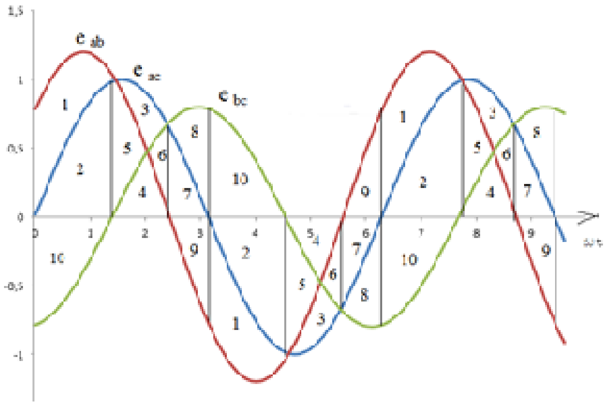

Fig. 2. Waveforms of line voltages feeding three-phase diode rectifier.

Note that the following areas have equal surfaces: $S_9=S_8$, $S_{10}=S_1$, $S_3=S_4$ which follows from three-phase three-wire network properties, e.g., $S_1+S_2=S_2+S_{10}$ as $e_{ab}(t)+e_{bc}(t)+e_{ca}(t)=0$. After substituting these equalities into equation (4), it takes shape

$$\begin{aligned} &2(S_1+S_2+S_3+S_3+S_5+S_6+S_7+S_8)= \\ &=(S_8+S_1+S_2+S_5+S_3)+ \\ &+(S_2+S_3+S_3+S_5+S_6+S_7)+ \\ &+(S_3+S_6+S_7+S_8+S_1) \end{aligned}$$

which is an identity for any possible set of line voltages in a 3-phase network.

Method 2

Formula (3) can be also derived in a different way. It should be kept in mind that at any moment the pair of diodes conducts when fed by highest line voltage. According to Fig.3 presenting waveforms of the network source phase (-to-neutral) voltages, U_{mean} value is given as

$$U_{mean}=\frac{1}{2p}\left(\int_{a1}^{a2} e_{ab}da+\int_{a2}^{a3} e_{ac}da+\int_{a3}^{a4} e_{bc}da+\int_{a4}^{a5} e_{ba}da+\int_{a5}^{a6} e_{ca}da+\int_{a6}^{a1+2p} e_{bc}da\right) \quad (5)$$

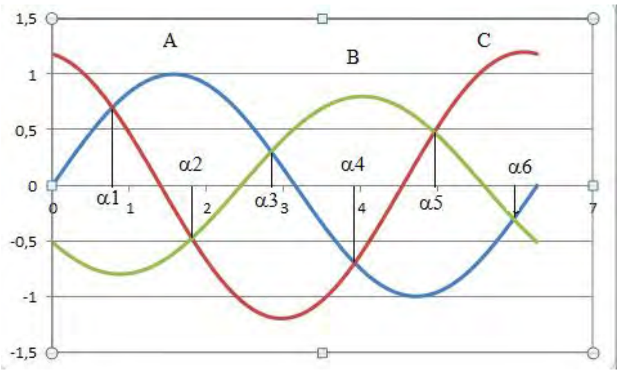

Fig. 3. Waveforms of phase voltages feeding a bridge rectifier. Time functions of phase voltages:

$$e_a=\sqrt{2}E_a\sin wt\,,\ e_b=\sqrt{2}E_b\sin(wt-b)\,,$$

$$e_c=\sqrt{2}E_c\sin(wt-g)\,.$$

This integral is a surface of an area between respective envelopes (upper and lower) as it is shown in Fig.3. As surfaces between each of these two envelopes and the time axis are equal within a time period, it is sufficient to calculate only one of them, for example, the upper one. In this case formula (5) takes shape

$$\begin{aligned} U_{mean}&=2\cdot\frac{1}{2p}\cdot\left(\int_{a1}^{a3} e_a da+\int_{a3}^{a5} e_b da+\int_{a5}^{a1+2p} e_c da\right)= \\ &=\frac{\sqrt{2}}{p}\cdot\left[E_a\cdot(\cos a_1-\cos a_3)+\right. \\ &+E_b\cdot(\cos(a_3-b)-\cos(a_5-b))+ \\ &\left.+E_c\cdot(\cos(a_5-g)-\cos(a_1+2p-g))\right] \end{aligned} \quad (6)$$

All components of the last expression can be determined with the use of phasor diagrams illustrating the operation of the rectifier (Fig.4).

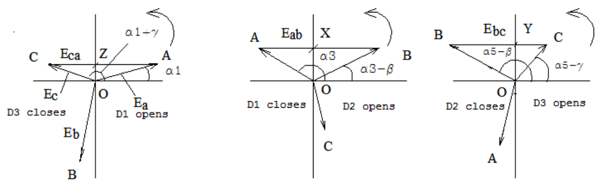

Fig. 4. Phasor diagrams for rectifier diodes commutation.

Within the interval (α_1, α_3) the diode D1 conducts. At a phase angle α_1 the momentary values of e_c and e_a voltages become equal. Thus at this moment line voltage e_{ca} assumes zero value, so its phasor is parallel to the time axis. The expression $E_a\cdot\cos a_1$ is equal to the length of a line section AZ, whereas $E_a\cdot\cos a_3$ (its absolute value) corresponds to the length of AX. Similarly, $E_b\cos(a_3-b)$ corresponds to BX,

$E_b \cos(a_5 - b)$ to BY, $E_c \cos(a_5 - g)$ to CY and $E_c \cos(a_1 - g)$ to CZ. As the sum of lengths of all these six line sections is equal to the sum of maximum values (amplitudes) of all network line voltages, then finally

$$\begin{aligned} U_{mean} &= \frac{\sqrt{2}}{p}\Big[E_a\left(\cos a_1 - \cos a_3\right) + \\ &\quad + E_b\left(\cos(a_3 - b) - \cos(a_5 - b)\right)\Big] + \\ &+ \frac{\sqrt{2}}{p}E_c\left[\cos\left(a_5 - g\right) - \cos\left(a_1 + 2p - g\right)\right] = \\ &= \sqrt{2}\,\frac{E_{ab} + E_{bc} + E_{ca}}{p} \end{aligned} \tag{7}$$

3. Universal formula for *n*-phase rectifier

Formula (3) suggests that in the general case the mean value of the output voltage of the diode bridge rectifier fed from an asymmetrical *n*-phase network is

$$U_{mean-n} = \frac{\sqrt{2}\left(E_{1,2} + E_{2,3} + \ldots + E_{n-1,n}\right)}{p} \tag{8}$$

where $E_{i,j}$ is the line voltage between phases *i,j* (RMS value).

This general formula can be derived with the help of, for example, a mathematical induction method. As proved above, the formula (8) is true for any three-phase network. Let us assume that it is true for an *n*-phase voltage source. Basing on this assumption, this formula for *n+1* phases is also considered to be true.

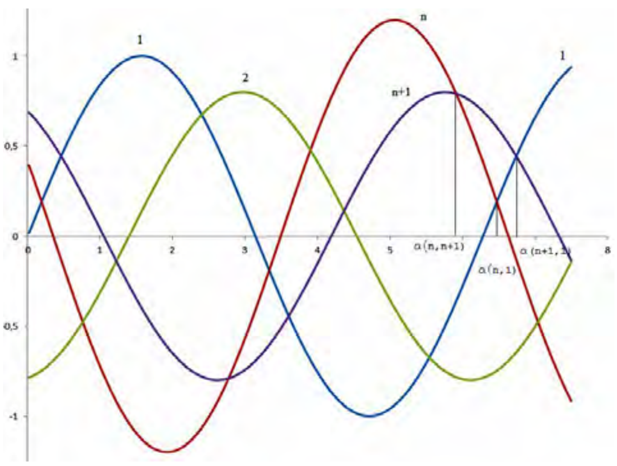

Fig. 5. Waveforms of n+1 phase voltages feeding a bridge rectifier.

According to the assumption (designations are explained in Fig.5, in which the e_{n+1} voltage waveform should not be taken into account at this stage) for the *n*-phase rectifier the output voltage mean value U_{mean-n} is equal to

$$\begin{aligned} U_{mean-n} &= 2\cdot\frac{1}{2p}\cdot\left(\int_{a_{n,1}-2p}^{a_{1,2}} e_1 da + \ldots + \int_{a_{n-1,n}}^{a_{n,1}} e_n da\right) = \\ &= \frac{\sqrt{2}}{p}\cdot\Big[E_1\cdot\left(\cos a_{n,1} - \cos a_{1,2}\right) + \ldots \\ &+ E_n\cdot\left(\cos\left(a_{n-1,n} - g_n\right) - \cos\left(a_{n,1} - y_n\right)\right)\Big] = \\ &= \frac{\sqrt{2}}{p}\cdot\Big[\left(E_1\cdot\cos a_{n,1} - E_n\cdot\cos\left(a_{n,1} - g_n\right)\right) + \ldots + \Big] + \\ &+ \frac{\sqrt{2}}{p}\cdot\left[E_n\cdot\cos\left(a_{n-1,n} - g_n\right) - E_{n-1}\cdot\cos\left(a_{n-1,n} - g_{n-1}\right)\right] = \\ &= \frac{\sqrt{2}}{p}\cdot\left(E_{n,1} + E_{1,2} + \ldots + E_{n-1,n}\right) \end{aligned} \tag{9}$$

According to Fig.5, for the *n+1* phase rectifier this parameter is

$$\begin{aligned} U_{mean-n+1} &= 2\cdot\frac{1}{2p}\cdot\Bigg[\int_{a_{n+1,1-2p}}^{a_{1,2}} e_1 da + \ldots + \int_{a_{n-1,n}}^{a_{n,n+1}} e_n da \\ &+ \int_{a_{n,n+1}}^{a_{n+1,1}} e_{n+1} da\Bigg] = \\ &= \frac{\sqrt{2}}{p}\cdot\Big[E_1\cdot\left(\cos\left(a_{n+1,1}\right) - \cos\left(a_{1,2}\right)\right) + \ldots + \\ &+ E_n\cdot\left(\cos(a_{n-1,n} - g_n) - \cos\left(a_{n,n+1} - g_n\right)\right)\Big] + \\ &+ \frac{\sqrt{2}}{p}\cdot E_{n+1}\cdot\left[\cos\left(a_{n,n+1} - g_{n+1}\right) - \cos\left(a_{n+1,1} - g_{n+1}\right)\right] = \\ &= U_{mean-n} - \frac{\sqrt{2}}{p}\cdot\Big[E_1\cdot\cos a_{n,1} - E_n\cdot\left(\cos\left(a_{n,1} - g_n\right)\right) + \\ &+ E_n\cdot\cos\left(a_{n,n+1} - g_n\right)\Big] + \\ &+ \frac{\sqrt{2}}{p}\cdot\Big[E_{n+1}\cdot\cos\left(a_{n,n+1} - g_{n+1}\right) + E_1\cdot\cos a_{n+1,1} - \\ &- E_{n+1}\cdot\cos\left(a_{n+1,1} - g_{n+1}\right)\Big] = \\ &= \frac{\sqrt{2}}{p}\left[\left(E_{1,2} + \ldots + E_{n-1,n}\right) + E_{n,n+1} + E_{n+1,1}\right] \end{aligned} \tag{10}$$

This means that formula (8) for *n+1* phases is also true. This formula has a clear geometrical illustration.

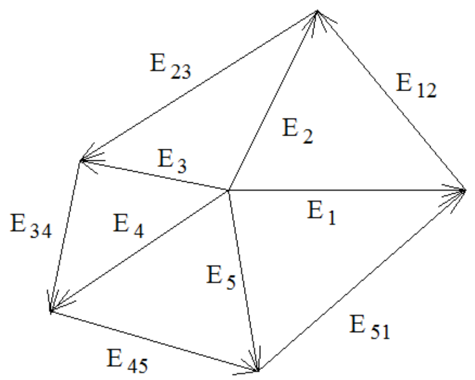

Fig. 6. Vector diagram of phase ($E_1 \ldots E_5$) and line ($E_{12} \ldots E_{51}$) voltages of IT AC multiple-phase asymmetrical network feeding a diode rectifier (an example for n=5 phases).

As it is shown at the diagram, vectors of *n*-phase network line (phase-to-phase) voltages form the perimeter of a polygon. According to formula (8) total length of this perimeter is proportional to the mean value of the output voltage of the rectifier.

4. Testing of three-phase rectifiers

Testing *n*-phase rectifiers has confirmed the formula for the output voltage mean value derived above. In Fig.7 the dependence of DC and AC components of the full-wave rectifier output voltage on the variation of three-phase supply line voltages is shown. In this case the AC component level has also been measured.

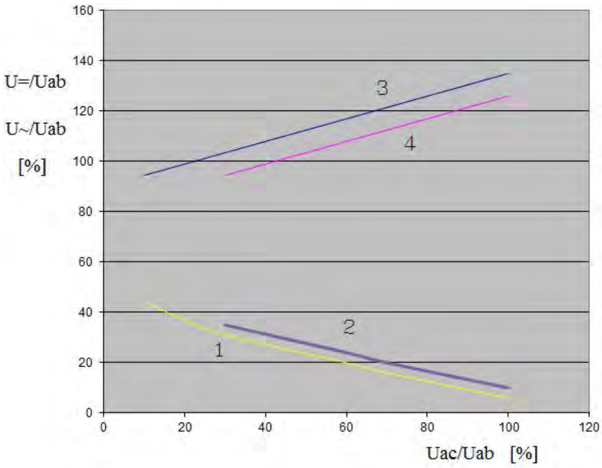

Fig. 7. Dependence of DC and AC components of the output voltage of the full-wave rectifier on supply line voltages variation for the fixed U_{ab} voltage and two selected U_{bc} values : 1- AC component for $U_{bc}/U_{ab}=1$, 2- AC component for $U_{bc}/U_{ab}=0.8$, 3- DC component for $U_{bc}/U_{ab}=1$, 4- DC component for $U_{bc}/U_{ab}=0.8$.

The measurements have shown that with the symmetrical supply the highest DC component and the lowest AC component are obtained.

5. Conclusion

1. It is necessary to know the DC component of the output voltage of the diode rectifiers for the assessment of their operating conditions. It is also a useful parameter for insulation monitoring of IT AC networks.

2. In the case of any *n*-phase both symmetrical and asymmetrical supply the mean value of the output voltage is given by universal formula (8).

3. With the help of geometrical considerations, as well as the mathematical induction method it is possible to prove the formula without the use of more sophisticated tools of mathematical analysis.

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НЕСТАНДАРТНІ МЕТОДИ АНАЛІЗУ ДІОДНИХ ВИПРЯМЛЯЧІВ З АСИМЕТРИЧНИМ ДЖЕРЕЛОМ ЖИВЛЕННЯ

Піотр Ольшовєц

Такі параметри вихідної напруги діодних випрямлячів, як середнє та середньоквадратичне значення є вирішальними для правильного функціонування систем постійного струму, які живляться від цих випрямлячів. Необхідно знати ці параметри для оцінювання умов роботи таких систем. У статті виведено формулу вихідної напруги для трифазного діодного випрямляча, який живиться асиметричною напругою. Також тут представлено виведення універсальної формули для середнього значення вихідної напруги для *n*-фазного діодного випрямляча з використанням методу математичної індукції.

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