UDC 004.8

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APPLICATION OF ENVELOPE FUNCTION METHOD FOR MULTIBEAM INTERFERENCE EXTREMUMS TO THE ELLIPSOMETRY ELECTROMAGNETIC WAVES BY SINGLE-LAYERED COATINGS

The physical principles of application of the envelope function for multibeam interference spectra extremums to the ellipsometry of light reflection for plane wave incidence on transparent and absorptive monolayered structures were for the first time established.

Key words: method for Fabry-Perot, ellipsometry, electromagnetic waves.

Вперше визначено фізичні принципи застосування обвідної функції до екстремумів спектрів багатопроменевої інтерференції в еліпсометрії відбиття світла на площину падіння хвилі на прозорі й абсорбуючі одношарові структури.

Ключові слова: метод Фабрі-Перо, еліпсометрія, електромагнітні хвилі.

Introduction

Ellipsometry is well known to be a sensitive noncontact method of surface layer structure investigation [1]. The basic principle of the method consists in analysis of the equation for relative complex coefficient

$$r = \frac{\tilde{r}_p}{\tilde{r}_p} = \tan\Psi \cdot \exp\left[i\left(f_p - f_s\right)\right] = \tan\Psi \cdot \exp\left(i\Delta\right), \qquad (1)$$

which describes the changes in wave polarization state after light reflection, here Δ and Ψ are polarization angles. The angles and the solutions of two real equations arisen from (1) allow determining any two unknown parameters of the reflective system. The doubtless successes of the approach which consists in experimental measuring the ellipsometric angles Δ and Ψ, calculating the angles from known film and substrate parameters and comparison of the theoretical and experimental results were achieved mainly due to its automation with computer application. But the problem of determination of the relation between extremums (or envelope functions) of ellipsometric parameters $\Delta_{m,M}$ and $\Psi_{m,M}$ with parameters of reflective medium was still unsolved. First of all, the parameters are extremum values of energetic coefficients $R_{m,M}$ and phase $f_{m,M}$. The problem statement for ellipsimetry application was for the first time formulated and partially solved for $\tan\Psi$ in paper [2]. In the present work the general physical principle of $\Delta_{m,M}$ and $\Psi_{m,M}$ envelope function formation was determined[1].

Basic relations, results and conclusions

Let us consider two plane single interfaces of medium-layer (index 01) and layer-substrate (index 12) type, that is plane parallel structure with thickness d at the surface of semiinfinite substrate with refraction index $\tilde{n}_2$. The essence of the envelope function method for Fabry-Perot interference maxima and minima consists in expression of energetic coefficients of reflection (reflectance) in the form [3]:

$$R = \frac{R_m + b^2\cos^2 F^-}{1 + b^2\cos^2 F^+} = \frac{R_M - a^2\sin^2 F^-}{1 - a^2\sin^2 F^+} \quad \text{where} \quad a^2 = \frac{4s_{01}\Theta}{\left(1 + s_{01}\Theta\right)^2}, \quad b^2 = \frac{4s_{01}\Theta}{\left(1 - s_{01}\Theta\right)^2}, \quad \Theta = \Omega s_{12},$$

$\Omega = \exp(-\mathrm{Im}\tilde{d})$, $F^{\pm} = \dfrac{f_{01} \pm (f_{12} - \mathrm{Re}\tilde{d})}{2}$, $\tilde{d} = \dfrac{4p\,\tilde{n}_1 d}{l}\cos\tilde{b}_1$, and s is the module of reflection factor

[1] The abstract of the first report was submitted to 7th WSE, Universitat Leipzig, March, 5-7, P.107, 2012.

$\tilde{r}_{01,12} = \rho_{01,12}\exp(-i\phi_{01,12})$, d is geometric thickness of film, β is refraction angle. In the approach the analytical envelope functions are the following: for amplitude

$$R_{M,m} = \left(\frac{\rho_{01} \pm \Theta}{1 \pm \rho_{01}\Theta}\right)^2 \tag{2}$$

and for phase spectra

$$\phi_{M,m} \cong 2\pi \pm \left(\frac{\rho_{01}\left(1-\rho_{12}\Theta\right)\sin\phi_{01} + \rho_{12}(1-\rho_{01}^2)\Omega}{\rho_{01}\left(1+\Theta^2\right)\cos\phi_{01}}\right). \tag{3}$$

Formuls (2) and (3) satisfactory describes the envelope functions for absorptive film.

The analysis by the method of computer simulation showed that taking into consideration the peculiarities of Fabry-Perot interferometry, functions $\tan\Psi_m = \sqrt{\frac{R_{pm}}{R_{sm}}}$, $\tan\Psi_M = \sqrt{\frac{R_{pM}}{R_{sM}}}$ and $\phi_{pm} - \phi_{sm}$, $\phi_{pM} - \phi_{sM}$, in the general case, describe the envelope functions of extremum spectra for polarization angles Ψ and Δ. The peculiarities are the following:

1. Brewster's type of reflection for ***p***-polarized wave leads to breakdown of synchronous character of oscillations in accordance with s-polarized wave that causes problems with determination of analytical form of oblique spectrum envelope function $\Delta(\alpha)$.

2. In extremum points of Fabry-Perot interference $\sin^2 F^{\pm} = 0$ (or $\cos^2 F^{\pm} = 0$) the amplitude of ellipsometric angle Ψ oscillations will be limited by the value interval of energetic coefficient ratios $\mathrm{atan}\sqrt{\frac{R_{pM}}{R_{sM}}}$ and $\mathrm{atan}\sqrt{\frac{R_{pm}}{R_{sm}}}$.

3. The region of contour R_m antireflection forms in the points of spectrum of contour ρ_{01} and $\rho_{12}\Omega$ intersection**.** Layer antireflection takes place when equalities $b^2\cos^2 F^{+} = 0$ are satisfied simultaneously. The condition is equivalent to wave phase shift $F^{\pm} = (2\mathbf{l} \pm 1)\frac{\pi}{2}$, $\mathbf{l} = 0,1,2,3,\ldots$.

4. At $\alpha \to \alpha_p$ and $\alpha > \alpha_p$ the inversion of the reflection contour for $p-$polarized wave takes place.

As the basic structure we choose transparent structure with parameters $SiO_2 - Si$ [1]. According to known Vlasov's equation [4] $\left(\frac{1}{n_0^2} + \frac{1}{n_1^2}\right)\sin^2\alpha_{p\,01} = \frac{1}{n_0^2}$, for external interface 01 the Brewster's angle is real and equals $\alpha_{p01} \approx 55.6^0$, and for internal interface 12 the angle is complex. As it follows from Fig.1a, in the interval of incidence angles interval $0 \div \alpha_{p01}$ the functions Ψ_m satisfactorily describe the envelope functions for contour $\Psi(\alpha)$ maxima and Ψ_M satisfactorily describe for minima, that is connected with inequality $R_p \langle R_s$. For structures $SiO_2 - Si$ the envelope functions for maxima Ψ_m coincide with polarization contour Ψ_{psub} for substrate. Contour Ψ_{psub} coincides with envelope function Ψ_M for some ratio between refraction indexes of the structure (Fig.2).

The Brewster's character of reflection of $p-$polarized wave forms changing phase shift between $p-$ and $s-$polarized waves. Since, contour $\Delta(\alpha)$ became oscillating with increasing amplitude at $\alpha \to \alpha_{p01,12}$ and differential functions $\mathrm{atan}\phi_{pm,M} - \mathrm{atan}\phi_{sm,M}$ not always reflect the character of amplitude $\Delta(\alpha)$ increase. As it shown in Fig. 1b, it can be useful to choose differential functions $\Delta\Psi(\alpha) = \left|\Psi_m(\alpha) - \Psi_M(\alpha)\right|$, which reflect direct relationship between measured ellipsometry angles. An important property of envelope functions is the fact, that line AB, as geometric positions of values

$\Psi_{m,M}(a)$ and $\pm\Delta\Psi(a)$, passes through the centre of $\Delta-\Psi$ spiral trajectory and intersects the trajectory in the points corresponding to extremums of contours of polarization spectra (Fig. 1c).

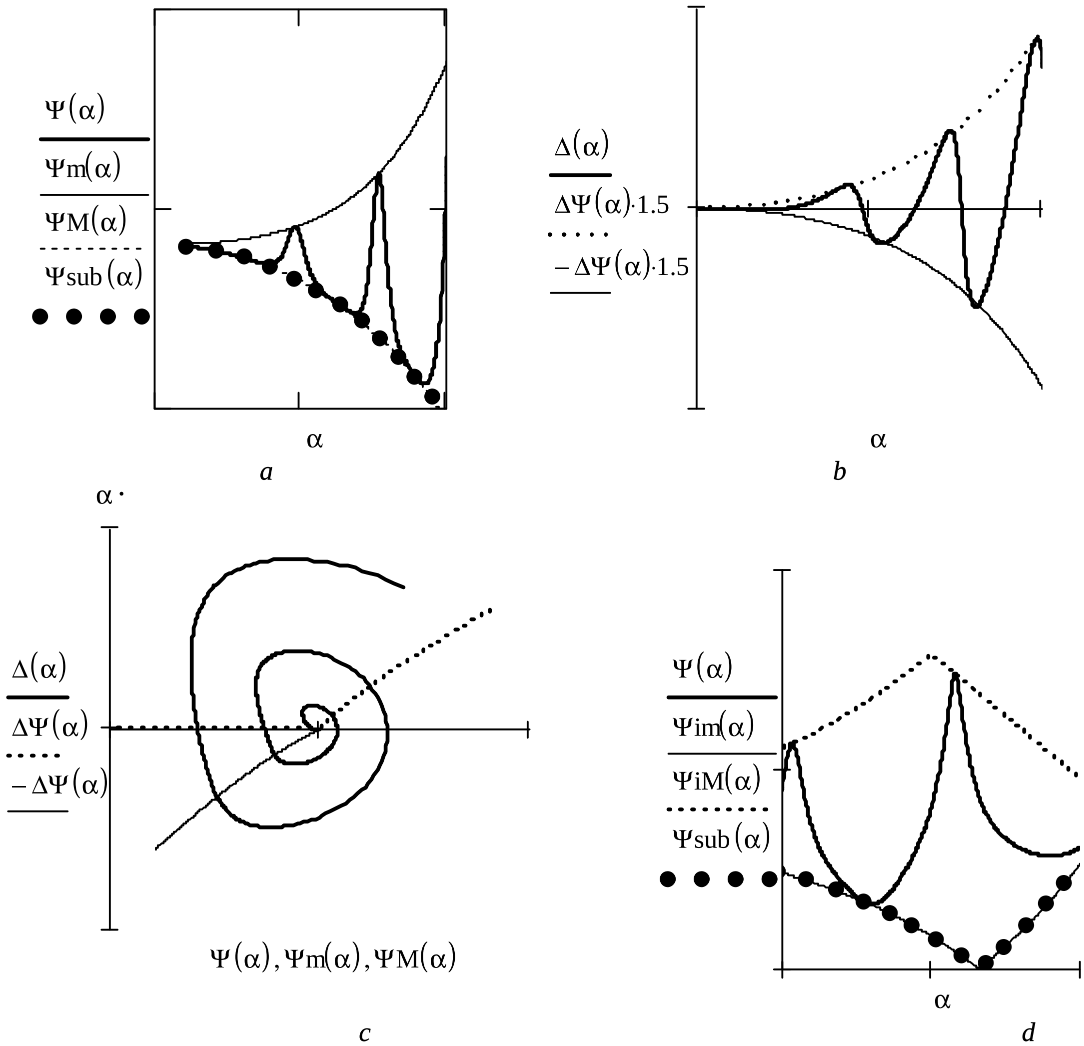

Fig. 1. Calculated dependences of envelope functions of polarization angle extremums for transparent structure with parameters SiO_2-Si $n_{Si}=3{,}86$, $k_{Si}=0{,}03$ *[1]*

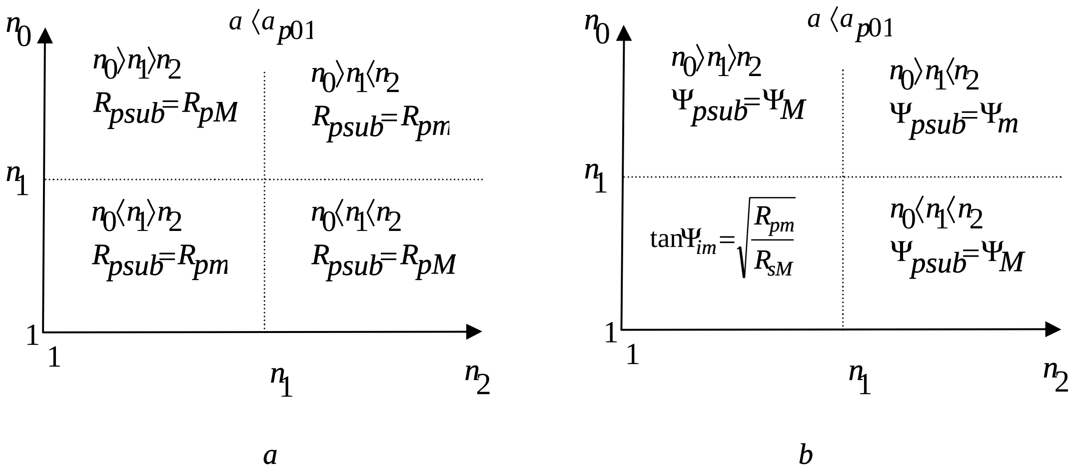

Fig. 2. Illustration of correlation of reflection contour for $p-$*polarized wave reflected by pure substrate with envelope functions of Fabry-Perot interference* $R_{pm,M}$ *extremums (a) and with phase shift at interfaces (b)*

At $\alpha \to \alpha_{p01}$ and $\alpha > \alpha_{p01}$ the oscillations of Fabry-Perot interference in the contour of $p-$polarized wave are damped and reflection contour inverts. Since, at $\alpha \rangle \alpha_{p01}$, the envelope functions for the curve $\Psi(\alpha)$ will be determined as $\tan\Psi_{im} = \sqrt{\frac{R_{pm}}{R_{sM}}}$ and $\tan\Psi_{iM} = \sqrt{\frac{R_{pM}}{R_{sm}}}$ (Fig.1d). But, in contrast to reflection contour, oscillations of phase contour are undamped and when transition through Brewster's angle takes place the phase contour does not invert. Therefore, differential values of phases $f_{pm} - f_{sm}$ in the region of minima and $f_{pM} - f_{sM}$ for maxima of multibeam interference not always reflect the shape of envelope functions of $\Delta(\alpha)$ contour. With approaching to Brewster's angles, wave phase for both polarization of waves reflected of transparent structures is multiple to $2p$ and reflectance does not depend on layer thickness [5]. In spite of problematic simulation of envelope functions of $\Delta(\alpha)$ contour by differential functions $\operatorname{atan} f_{pm,M} - \operatorname{atan} f_{sm,M}$ in the fixed region of incidence angles, the corresponding problem is, in principle, also solvable.

Conclusion

The essence of the main conclusion from the work: envelope function method correctly describes ellipsometric spectra for monolayer structures and allows formulation of important equations which connect exptremum values of polarization angles Ψ and Δ and media parameters. The results essentially improved the functionality of ellipsometry method in the whole.

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