COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 5, No. 1, 2015

ANALYSIS OF POSITIVITY AND STABILITY OF DISCRETE-TIME AND CONTINUOUS-TIME NONLINEAR SYSTEMS

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Abstract: The positivity and asymptotic stability of discrete-time and continuous-time nonlinear systems are addressed. Sufficient conditions for the positivity and asymptotic stability of the nonlinear systems are established. The proposed stability tests are based on an extension of the Lyapunov method to the positive nonlinear systems. The effectiveness of the tests are demonstrated on examples.

Key words: positive, discrete-time, asymptotic stability, nonlinear, Lyapunov method.

1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative-initial-condition state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in the positive system theory was given in monographs [8, 9] and in papers [15–18]. Systems having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

The Laypunov, Bohl and Perron exponents and stability of time-varying discrete-time linear systems were comprehensively investigated [1–7]. Furthermore, positive standard and descriptor systems and their stability were also analyzed [9, 15–19]. Positive linear systems of different fractional orders [16, 20] and descriptor discrete-time linear systems [17] were addressed by the author in previous publications. Descriptor positive discrete-time and continuous-time nonlinear systems [10, 13] were likewise analyzed as well as the positivity and linearization of nonlinear discrete-time systems by state-feedbacks were investigated [15]. The problem of minimum energy control of positive linear systems was adequately addressed [11, 12, 14]. The stability and robust stabilization of discrete-time switched systems were analyzed [21, 22].

In this paper, the positivity and asymptotic stability of discrete-time and continuous-time nonlinear systems will be investigated.

The paper is organized as follows. In the section 2, the definitions and theorems concerning the positivity and stability of positive discrete-time and continuous-time linear systems are recalled. Necessary and sufficient conditions for the positivity of discrete-time nonlinear systems are established in the section 3. The asymptotic stability of positive nonlinear systems is addressed in the section 4, with conditions for their stability being proposed. The conditions for the positivity of continuous-time nonlinear systems are given in the section 5, and those for the stability of continuous-time positive nonlinear systems are presented in the section 6. Concluding remarks are given in the section 7.

The following notations will be used: $\Re$ denotes the set of real numbers, $\Re^{n\times m}$ represents the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ stands for the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, Z_+ is the set of nonnegative integers, M_n represents the set of $n\times n$ Metzler matrices (with nonnegative off-diagonal entries), identity matrix, I_n stands for the $n\times n$ identity matrix.

2. Positive discrete-time and continuous-time linear systems and their stability

Consider a discrete-time linear system

$$x_{i+1} = Ax_i + Bu_i\,,\ i\in Z_+ = \{0,1,...\} \tag{2.1a}$$

$$y_i = Cx_i + Du_i \tag{2.1b}$$

where $x_i \in \Re^n$, $u_i \in \Re^m$, $y_i \in \Re^p$ are the state, input and output vectors, respectively, and $A\in \Re^{n\times n}$, $B\in \Re^{n\times m}$, $C\in \Re^{p\times n}$, $D\in \Re^{p\times m}$.

Definition 2.1. [8, 9] A discrete-time linear system (2.1) is called (internally) positive if $x_i \in \Re_+^n$, $y_i \in \Re_+^p$, $i\in Z_+$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u_i \in \Re_+^m$, $i\in Z_+$.

Theorem 2.1. [8, 9] A discrete-time linear system (2.1) is positive if and only if

$$A\in \Re_+^{n\times n},\ B\in \Re_+^{n\times m},\ C\in \Re_+^{p\times n},\ D\in \Re_+^{p\times m}. \tag{2.2}$$

Definition 2.2. [8, 9] A positive discrete-time linear system (2.1) is called asymptotically stable if

$$\lim_{i\to\infty} x_i = 0 \ \text{ for any } \ x_0 \in \Re_+^n. \tag{2.3}$$

Theorem 2.2. A positive discrete-time linear system (2.1) is asymptotically stable if and only if one of the following equivalent conditions is satisfied:

1) All coefficients of the polynomial

$$p_n(z) = \det[I_n(z+1) - A] = \\ = z^n + a_{n-1}z^{n-1} + ... + a_1 z + a_0 \quad (2.4)$$

are positive, i.e. $a_i > 0$ for $i = 0,1,...,n-1$.

2) All principal minors of the matrix $\bar{A} = I_n - A = [\bar{a}_{ij}]$ are positive, i.e.

$$M_1 = |\bar{a}_{11}| > 0, \\ M_2 = \begin{vmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{vmatrix} > 0, \quad ..., M_n = \det \bar{A} > 0 \quad (2.5)$$

The proof was given in [9].

Let us consider a continuous-time linear system

$$\dot{x} = Ax + Bu, \quad (2.6a)$$

$$y = Cx + Du \quad (2.6b)$$

where $x = x(t) \in \Re^n$, $u = u(t) \in \Re^m$, $y = y(t) \in \ \in \Re^p$ are the state, input and output vectors and $A \in \Re^{n \times n}$, $B \in \Re^{n \times m}$, $C \in \Re^{p \times n}$, $D \in \Re^{p \times m}$.

Definition 2.3. [8, 9] A continuous-time linear system (2.6) is called (internally) positive if $x \in \Re_+^n$, $y \in \Re_+^p$, $t \ge 0$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u \in \Re_+^m$, $t \ge 0$.

Theorem 2.3. [8, 9] A continuous-time linear system (2.6) is positive if and only if

$$A \in M_n, B \in \Re_+^{n \times m}, C \in \Re_+^{p \times n}, D \in \Re_+^{p \times m}. \quad (2.7)$$

Definition 2.4. [8, 9] A positive continuous-time linear system (2.6) is called asymptotically stable if

$$\lim_{t \to \infty} x = 0 \text{ for any } x_0 \in \Re_+^n. \quad (2.8)$$

Theorem 2.4. A positive continuous-time linear system (2.6) is asymptotically stable if and only if one of the following equivalent conditions is satisfied:

1) All coefficients of the polynomial

$$p_n(s) = \det[I_n s - A] = \\ = s^n + \hat{a}_{n-1}s^{n-1} + ... + \hat{a}_1 s + \hat{a}_0 \quad (2.9)$$

are positive, i.e. $\hat{a}_k > 0$ for $k = 0,1,...,n-1$.

2) All principal minors of the matrix $\hat{A} = -A = [\hat{a}_{ij}]$ are positive, i.e.

$$\hat{M}_1 = |\hat{a}_{11}| > 0, \\ \hat{M}_2 = \begin{vmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{vmatrix} > 0, \quad ..., \hat{M}_n = \det \hat{A} > 0 \quad (2.10)$$

The proof was given in [9].

3. Positivity of discrete-time nonlinear systems

Following previously set reasoning [18], let us consider a discrete-time nonlinear system

$$x_{i+1} = Ax_i + f(x_i, u_i), \ i \in Z_+ = \{0,1,...\}, \quad (3.1a)$$

$$y_i = g(x_i, u_i), \quad (3.1b)$$

where $x_i \in \Re^n$, $u_i \in \Re^m$, $y_i \in \Re^p$, $i \in Z_+$ are the state, input and output vectors, respectively; $f(x_i, u_i) \in \Re^n$, $g(x_i, u_i) \in \Re^p$ are continuous vector functions of x_i and u_i satisfying the conditions $f(0,0) = 0$, $g(0,0) = 0$ and $A \in \Re^{n \times n}$.

Definition 3.1. A discrete-time nonlinear system (3.1) is called (internally) positive if $x_i \in \Re_+^n$, $y_i \in \Re_+^p$, $i \in Z_+$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u_i \in \Re_+^m$.

Theorem 3.1. A discrete-time nonlinear system (3.1) is positive if and only if

$$A \in \Re_+^{n \times n} \text{ and } f(x_i, u_i) \in \Re_+^n, \ g(x_i, u_i) \in \Re_+^p \text{ for all } x_i \in \Re_+^n \text{ and } u_i \in \Re_+^m, \ i \in Z_+. \quad (3.2)$$

Proof. *Sufficiency*. From (3.1) for $i = 0$ we have

$$x_1 = Ax_0 + f(x_0, u_0) \in \Re_+^n, \\ y_0 = g(x_0, u_0) \in \Re_+^p, \quad (3.3)$$

since (3.2) holds and $x_0 \in \Re_+^n$, $u_0 \in \Re_+^m$.

Similarly, for $i = 1$ we obtain

$$x_2 = Ax_1 + f(x_1, u_1) \in \Re_+^n, \\ y_1 = g(x_1, u_1) \in \Re_+^p, \quad (3.4)$$

since (3.2) and (3.3) holds.

Repeating the procedure for i = 2,3,… we obtain $x_i \in \Re_+^n$ and $y_i \in \Re_+^p$ for $i \in Z_+$; therefore, by Definition 3.1 the system is positive.

Necessity. Assuming that the system (3.1) is positive, we shall show that (3.2) holds. From (3.3) for $f(x_0, u_0) = 0$ we have $x_1 = Ax_0$ and this implies that $A \in \Re_+^{n \times n}$ since by assumption $x_1 \in \Re_+^n$ and, additionally, $x_0 \in \Re_+^n$ can be arbitrary. In other case, if $Ax_0 = 0$, then from (3.3) we have $x_1 = f(x_0, u_0)$ and this implies that $f(x_0, u_0) \in \Re_+^n$ since by assumption $x_1 \in \Re_+^n$. From (3.3) we also have that $y_0 = g(x_0, u_0)$ and $x_0 \in \Re_+^n$, $u_0 \in \Re_+^m$ since by assumption $y_0 \in \Re_+^p$. Continuing the procedure, we can show that (3.2) holds if the system is positive.

From Theorem 3.1 we have the following:

Corollary 3.1. A discrete-time nonlinear system (3.1) is positive only if the linear system

$$x_{i+1} = Ax_i\,,\ i \in Z_+ = \{0,1,...\} \qquad (3.5)$$

is positive.

Example 3.1. Let us consider the following discrete-time nonlinear system (3.1) with

$$x_i = \begin{bmatrix} x_{1,i} \\ x_{2,i} \end{bmatrix},\quad A = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix},$$
$$f(x_i,u_i) = \begin{bmatrix} x_{1,i}x_{2,i} + e^{-i} \\ x_{2,i}^2 + 1 - e^{-i}\cos i \end{bmatrix}, \qquad (3.6)$$
$$g(x_i,u_i) = \begin{bmatrix} x_{1,i}^2 + 0.1e^{-i} \\ x_{2,i} + 2 + \cos i \end{bmatrix}.$$

As follows from (3.6), the matrix A has nonnegative entries and the vector functions $f(x_i,u_i)$, $g(x_i,u_i)$ are also nonnegative for all $x_i \in \Re_+^n$ and $u_i \in \Re_+^m$, $i \in Z_+$. Therefore, by Theorem 3.1, the system is positive. The linear part of the system is also asymptotically stable since the coefficients of the polynomial

$$\det[I_2(z+1) - A] = \begin{vmatrix} z+0.8 & -0.1 \\ -0.3 & z+0.6 \end{vmatrix} = \qquad (3.7)$$
$$= z^2 + 1.4z + 0.45$$

are positive, i.e. $a_0 = 0.45$, $a_1 = 1.4$.

The same result follows from the condition 2 of Theorem 2.2 since

$$\bar{A} = I_2 - A = \begin{bmatrix} 0.8 & -0.1 \\ -0.3 & 0.6 \end{bmatrix} \quad \text{and} \quad M_1 = 0.8\,,$$
$$M_2 = \det \bar{A} = 0.45\,. \qquad (3.8)$$

4. Stability of positive discrete-time nonlinear systems

Consider a positive discrete-time nonlinear system

$$x_{i+1} = Ax_i + f(x_i)\,,\ x_0 \in \Re_+^n\,, \qquad (4.1)$$

where $x_i \in \Re_+^n$, $A \in \Re_+^{n\times n}$, $f(x_i) \in \Re_+^n$ is a continuous and bounded vector function.

Definition 4.1. A positive discrete-time nonlinear system (4.1) is called asymptotically stable in the region $D \in \Re_+^n$ if $x_i \in \Re_+^n$, $i \in Z_+$ and

$$\lim_{i\to\infty} x_i = 0 \text{ for any finite } x_0 \in D \in \Re_+^n. \qquad (4.2)$$

To test the asymptotic stability of the positive system (4.1), the Lyapunov method is used. As a candidate of Lyapunov function we choose

$$V(x_i) = c^T x_i > 0 \text{ for } x_i \in \Re_+^n \qquad (4.3)$$

where $c \in \Re_+^n$ is a vector with strictly positive components $c_k > 0$ for $k = 1,...,n$.

Using (4.3) and (4.1), we obtain

$$\begin{aligned} &\Delta V(x_i) = V(x_{i+1}) - V(x_i) = \\ &= c^T x_{i+1} - c^T x_i = c^T\{[A - I_n]x_i + f(x_i)\} < 0 \end{aligned} \qquad (4.4)$$

for

$$[I_n - A]x_i - f(x_i) < 0\,,\ x_i \in D \in \Re_+^n \qquad (4.5)$$

since $c \in \Re_+^n$ is a strictly positive vector.

Therefore, the following theorem has been proved.

Theorem 4.1. A positive discrete-time nonlinear system (4.1) is asymptotically stable in the region $D \in \Re_+^n$ if the condition (4.5) is satisfied.

Example 4.1. Let us consider the following nonlinear system (4.1) with

$$x_i = \begin{bmatrix} x_{1,i} \\ x_{2,i} \end{bmatrix}, A = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, f(x_i) = \begin{bmatrix} x_{1,i}x_{2,i} \\ x_{2,i}^2 \end{bmatrix}. \qquad (4.6)$$

The nonlinear system is positive since $A \in \Re_+^{2\times 2}$ and $f(x_i) \in \Re_+^2$ for all $x_{1,i} \ge 0$ and $x_{2,i} \ge 0$, $i \in Z_+$.

In this case, the condition (4.5) is satisfied in the region D defined by

$$\begin{aligned} D &:= \{x_{1,i}, x_{2,i}\} = [I_2 - A]x_i - f(x_i) = \\ &= \begin{bmatrix} 0.9x_{1,i} - 0.2x_{2,i} - x_{1,i}x_{2,i} \\ 0.7x_{2,i} - 0.2x_{1,i} - x_{2,i}^2 \end{bmatrix} \in \Re_+^2 \end{aligned}\ . \qquad (4.7)$$

The region D is shown in Fig. 4.1.

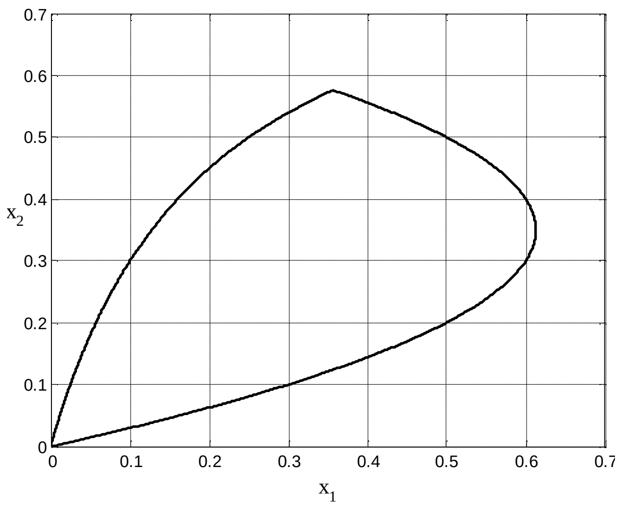

Fig. 4.1. Stability region (inside the curved line).

By Theorem 4.1, the positive nonlinear system (4.1) with (4.6) is asymptotically stable in the region (4.7).

5. Positivity of continuous-time nonlinear system

Consider a continuous-time linear system

$$\dot{x} = Ax + f(x,u), \tag{5.1a}$$

$$y = g(x,u) \tag{5.1b}$$

where $x = x(t) \in \Re^n$, $u = u(t) \in \Re^m$, $y = y(t) \in \ \in \Re^p$ are the state, input and output vectors, respectively; $A \in \Re^{n\times n}$; $f(x,u)$ and $g(x,u)$ are continuous and bounded vector functions of x and u, respectively, satisfying $f(0,0) = 0$ and $g(0,0) = 0$.

Definition 5.1. [8, 9] A continuous-time linear system (5.1) is called (internally) positive if $x \in \Re_+^n$, $y \in \Re_+^p$ (for $t \geq 0$) for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u \in \Re_+^m$, $t \geq 0$.

Theorem 5.1. [8, 9] A continuous-time linear system (5.1) is positive if and only if

$$A \in M_n,\quad f(x,u) \in \Re_+^n,\quad g(x,u) \in \Re_+^n \ \text{ for all } \ x \in \Re_+^n,$$
$$u \in \Re_+^m,\ t \geq 0. \tag{5.2}$$

Proof. The solution to the equation (5.1a) for a given A and $f(x,u)$ has the form

$$x(t) = \Phi_0(t)x_0 + \int_0^t \Phi(t-\tau) f[x(\tau),u(\tau)]d\tau \tag{5.3}$$

where

$$\Phi(t) = e^{At}. \tag{5.4}$$

Using the Picard method, we obtain from (5.3a) the following:

$$x_{k+1}(t) = \Phi_0(t)x_0 + \int_0^t \Phi(t-\tau) f[x_k(\tau),u(\tau)]d\tau,$$
$$k = 1, 2, \ldots \tag{5.5}$$

As follows from (5.4), if the conditions (5.2) are satisfied, then $x_k(t) \in \Re_+^n$ (for $t \geq 0$, $k = 1, 2, \ldots$) since for $A \in M_n$ the inclusion holds $\Phi(t) \in \Re^{n\times n}$ (for $t \geq 0$) [9].

From (5.1b) we have $y \in \Re_+^p$ (for $t \geq 0$) since by the assumption (5.2) $g(x,u) \in \Re_+^n$ for $x \in \Re_+^n$, $u \in \Re_+^m$, $t \geq 0$.

6. Stability of continuous-time nonlinear systems

Consider a positive continuous-time nonlinear system

$$\dot{x} = Ax + f(x), \tag{6.1}$$

where $x = x(t) \in \Re^n$, $A \in M_n$, $f(x) \in \Re_+^n$ is a continuous and bounded vector function and $f(0) = 0$.

Definition 6.1. A positive continuous-time nonlinear system (6.1) is called asymptotically stable in the region $D \in \Re_+^n$ if $x(t) \in \Re_+^n$, $t \geq 0$ and

$$\lim_{t\to\infty} x(t) = 0 \ \text{ for any finite } \ x_0 \in D \in \Re_+^n. \tag{6.2}$$

To test the asymptotic stability of the positive system (6.1), the Lyapunov method is used. As a candidate of Lyapunov function we choose

$$V(x) = c^T x > 0 \ \text{ for } \ x = x(t) \in \Re_+^n,\ t \geq 0 \tag{6.3}$$

where $c \in \Re_+^n$ is a vector with strictly positive components $c_k > 0$ for $k = 1, \ldots, n$.

Using (6.3) and (6.1), we obtain

$$\dot{V}(x) = c^T \dot{x} = c^T [Ax + f(x)] < 0 \tag{6.4}$$

for

$$Ax + f(x) < 0 \ \text{ for } \ x \in D \in \Re_+^n,\ t \geq 0 \tag{6.5}$$

since $c \in \Re_+^n$ is the strictly positive vector.

Therefore, the following theorem has been proved.

Theorem 6.1. A positive continuous-time nonlinear system (6.1) is asymptotically stable in the region $D \in \Re_+^n$ if the condition (6.5) is satisfied.

Example 6.1. Let us consider the following nonlinear system (6.1) with

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\quad A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix},\quad f(x) = \begin{bmatrix} x_1 x_2 \\ x_2^2 \end{bmatrix}. \tag{6.6}$$

The nonlinear system (6.1) with (6.6) is positive since $A \in M_2$ and $f(x) \in \Re_+^2$ for all $x \in \Re_+^2$, $t \geq 0$.

In this case, the condition (6.4) is satisfied in the region D defined by

$$D := \{x_1, x_2\} = \begin{bmatrix} -2x_1 + x_2 + x_1 x_2 \\ x_1 - 3x_2 + x_2^2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{6.7}$$

From (6.7) we have

$$x_1(2 - x_2) > x_2 > 0 \ \text{ and } \ 0 \leq x_1 < (3 - x_2)x_2. \tag{6.8}$$

The region D is shown in Fig. 6.1.

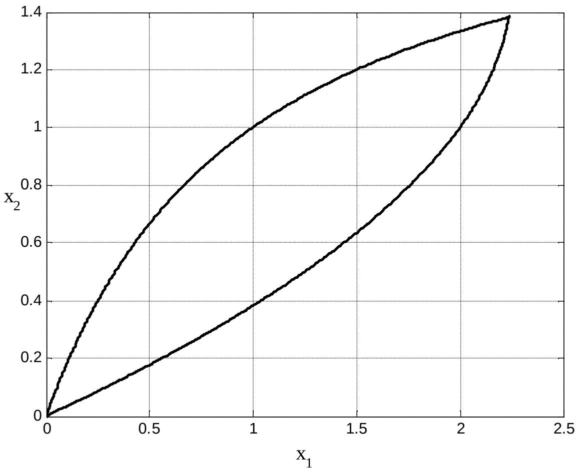

Fig. 6.1. Stability region (inside the curved line).

By Theorem 6.1, the positive nonlinear system (6.1) with (6.6) is asymptotically stable in the region (6.7).

7. Concluding remarks

The positivity and asymptotic stability of the discrete-time and continuous-time nonlinear systems have been addressed. The necessary and sufficient conditions for the positivity of the discrete-time nonlinear systems have been established (Theorem 3.1). Using the Lyapunov direct method, the sufficient conditions for asymptotic stability of the discrete-time nonlinear systems have been proposed (Theorem 4.1). The effectiveness of the conditions has been demonstrated on Example 4.1. The sufficient conditions for the positivity of continuous-time nonlinear systems have been established in section 5 (Theorem 5.1) and for the asymptotic stability in section 6 (Theorem 6.1). The stability conditions for continuous-time nonlinear systems are illustrated on Example 6.1. The considerations can be extended to fractional discrete-time nonlinear systems. An open problem is an extension of the conditions to the descriptor fractional discrete-time and continuous-time nonlinear systems.

Acknowledgment

This work has been supported by National Science Centre in Poland under work No. 2014/13/B/ST7/03467.

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АНАЛІЗ ПОЗИТИВНОСТІ ТА СТІЙКОСТІ ДИСКРЕТНИХ ТА НЕПЕРЕРВНИХ В ЧАСІ НЕЛІНІЙНИХ СИСТЕМ

Тадеуш Качорек

Досліджено позитивність та асимптотична стійкість нелінійних систем, часові залежності яких є дискретними або неперервними. Встановлено достатні умови позитивності та асимптотичної стійкості нелінійних систем. Запропоновані тести на стійкість базуються на розширенні методу О. Ляпунова для позитивних нелінійних систем. Ефективність цих тестів демонструється на прикладах.

Tadeusz Kaczorek – D. Sc., Professor, born in 1932 in Poland, received his M.Sc., Ph.D. and D.Sc. degrees in Electrical Engineering from Warsaw University of Technology, Poland, in 1956, 1962 and 1964, respectively. In the period 1968–1969 he was the dean of Electrical Engineering Faculty and in the period 1970–1973 – the prorector of Warsaw University of Technology, Poland. Since 1971 he has been a professor and since 1974 a full professor at Warsaw University of Technology, Poland. In 1986 he was elected a corresponding member and in 1996 full member of Polish Academy of Sciences. In the period 1988 - 1991 he was the director of the Research Centre of Polish Academy of Sciences in Rome. In June 1999 he was elected the full member of the Academy of Engineering in Poland. In May 2004 he was elected to the honorary member of the Hungarian Academy of Sciences. He was awarded by the University of Zielona Gora, Poland (2002) by the title doctor honoris causa, the Technical University of Lublin, Poland (2004), the Technical University of Szczecin, Poland (2004), Warsaw University of Technology, Poland (2004), Bialystok University of Technology, Poland (2008), Lodz University of Technology, Poland (2009), Opole University of Technology, Poland (2009), Poznan University of Technology, Poland (2011), and Rzeszow University of Technology, Poland (2012).

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