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NEW SOLUTIONS TO A LINEAR ANTENNA SYNTHESIS PROBLEM ACCORDING TO THE GIVEN AMPLITUDE PATTERN

Отримано явну форму розв'язку задачі амплітудно-фазового синтезу лінійної антени за заданою амплітудною діаграмою спрямованості. Наведено числові результати, які демонструють залежність точності одержаного розв'язку від вхідних параметрів задачі.

Ключові слова: амплітудно-фазовий синтез антен, амплітудна діаграма, явна форма розв'язку.

Closed form of solution to the problem of linear antenna amplitude-phase synthesis according to the given amplitude pattern is obtained. The computational results, related to dependence of the exactness of solution on the problem's input parameters, are discussed.

Key words: antenna amplitude-phase synthesis, amplitude pattern, closed form of solution.

Introduction

The big number of papers (e.g. [1, 2, 4–8, 13]) is devoted to finding the current $j(x)$ if the radiation pattern (RP)

$$f(k) = \int_{-l}^{l} e^{ikx} j(x)dx := Aj, \; -k_0 \le k \le k_0 \tag{1}$$

is given. Here $k_0 > 0$ and $l > 0$ are fixed numbers. The RP $f(k)$ is an entire function of k, $|f(k)| \le ce^{l|k|}$, $k \in \mathrm{C}$. By $c > 0$ we denote various estimation constants. Equation (1) for j is an integral equation with compact operator $A: L^2(-l,l) \to L^2(-k_0,k_0)$. The operator A is injective, its range $R(A) := \{f \;:\; Aj,\; j \in L^2(-l,l)\}$ is not closed. Thus, if $\| f_\delta - f \|_{L^2(-k_0,k_0)} < \delta$, then f_δ may be not in $R(A)$.

In this paper the following problem is discussed:

Given $h(k) \ge 0$, $h(k) \in L^2(-k_0,k_0)$, and $\delta > 0$, find $j \in L^2(-l,l)$ such that

$$\| h(k) - | f_\delta(k) | \|_{L^2(-k_0,k_0)} < \delta, \tag{2}$$

where

$$f_\delta(k) = \int_{-l}^{l} e^{ikx} j_\delta(x)dx. \tag{3}$$

This is a nonlinear problem. There was no closed form solution to this problem. The problem has been discussed in [1], where the approach was based on a numerical solution of a corresponding nonlinear minimization problem. Some theoretical aspects of antenna synthesis problem were discussed in [4–11], [13]. During the last decade, the approach, based on the polynomials representation of solution to the non-linear antenna synthesis problem according to the prescribed amplitude and power radiation patterns, was developed too (see, e. g., [14, 15]).

Closed form solution

The approach proposed is quite different from the one in [1]. It reduces problem (2)–(3) to a linear problem which is solved in closed form. Main theoretical result was formulated in [16] as respective theorem.

Denote

$$f = Fj := \int_{-l}^{l} e^{ikx} j(x)dx, \qquad j \in L^2(-l,l), \;\; j = 0, \; |x| > l, \tag{4}$$

and

$$F^{-1} f = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx} f(k)dk. \tag{5}$$

A solution to problem (2)-(3) is

$$j_\delta(x) = 2\pi(F^{-1}G_n(\delta) \cdot (F^{-1}h) , \tag{6}$$

where

$$G_n(k) = \left(\frac{1}{2l} \int_{-l}^{l} e^{\frac{ikx}{2n+p}} \right)^{2n+p} \left(\frac{n}{\pi k_1^2} \right)^{1/2} \left(1 - \frac{k^2}{k_1^2} \right)^n , \;\; n = n(\delta), \tag{7}$$

$k_1 > 2k_0$, $p > \frac{1}{2}$ is fixed, and $n = n(\delta)$ is chosen so that

$$\| \int_{-k_0}^{k_0} G_n(k-s)h(s)ds - h(k) \|_{L^2(-k_0,k_0)} < \delta . \tag{8}$$

The formulas (6), (7) give the explicit form of solution to problem (2), (3). The proposed method is valid if the linear segment $(-l,l)$ is replaced by a multidimensional bounded domain D. In this case the origin has to be chosen at the gravity center of D, that is, at the point such that $\int_D xdx = 0$. In this case the function $G_n(k)$ is

$$G_n(k) = \left(\frac{1}{|D|} \int_D e^{\frac{ik \cdot x}{2n+p}} dx \right)^{2n+p} \cdot c_{n,N} \left(1 - \frac{k \cdot k}{k_1^2} \right)^n , \tag{9}$$

where $k \cdot k$ is the dot product of vectors, $|D|$ is measure (volume) of D, N is the dimension of the space, and $c_{n,N}$ is the normalizing constant:

$$\lim_{n \to \infty} c_{n,N} \int_{\substack{|k|<k_0 \\ 2k_0<k_1}} \left(1 - \frac{k \cdot k}{k_1^2} \right)^n dk = 1 . \tag{10}$$

If D is smooth and strictly convex then the Fourier transform of the characteristic function of D is $O\left(\frac{1}{k}\right)$. Therefore $G_n(k) = O\left(\frac{1}{k^p}\right) \in L^2(\mathrm{R}^N)$ if $p > \frac{N}{2}$. The information related to the rate of decay of the Fourier transform of the characteristic function of a bounded domain D in R^N is given, for example, in [3, 11].

Numerical modeling

The numerical results related to investigation of the role of the number n on the quality of approximation of the given RP $h(k)$ are shown in Table 1. The parameters of the problem are the following: $l = 2.0$, $k_0 = 3.0$, $k_1 = 2k_0 + 1.5$, $p = 1.0$. The errors of the estimate (8) are given in the second column of Table 1. The mean-square deviation (MSD) $\| h(k) - | f(k) | \|$, obtained in the process of solving the nonlinear synthesis problem by approach in [1], is presented in the third column, and the square of the norm $\| j \|^2$ is given in the last column. One can see that the value of n influences strongly the accuracy of the approximation of the desired diagram. In order to get the error δ of the approximation which is less than 10^{-3} it is sufficient to chose n around 4000.

The quality of the approximation of the given RP $h(k)$ by $|f(k)|$ for small n is shown in Fig. 1. The given RP $h(k)$ is plotted by thick solid line. The moduli $|f(k)|$ for $n = 10, 20, 50, 100$ are shown with the thin lines, the respective currents $j(x)$ are shown in Fig. 2. So, the error of estimate (8) for $n = 100$ is equal to 0.0295. The error of the approximation decreases if n increases, and its minimal value in our computations is equal to 0.7146×10^{-4} and is attained at $n = 4000$.

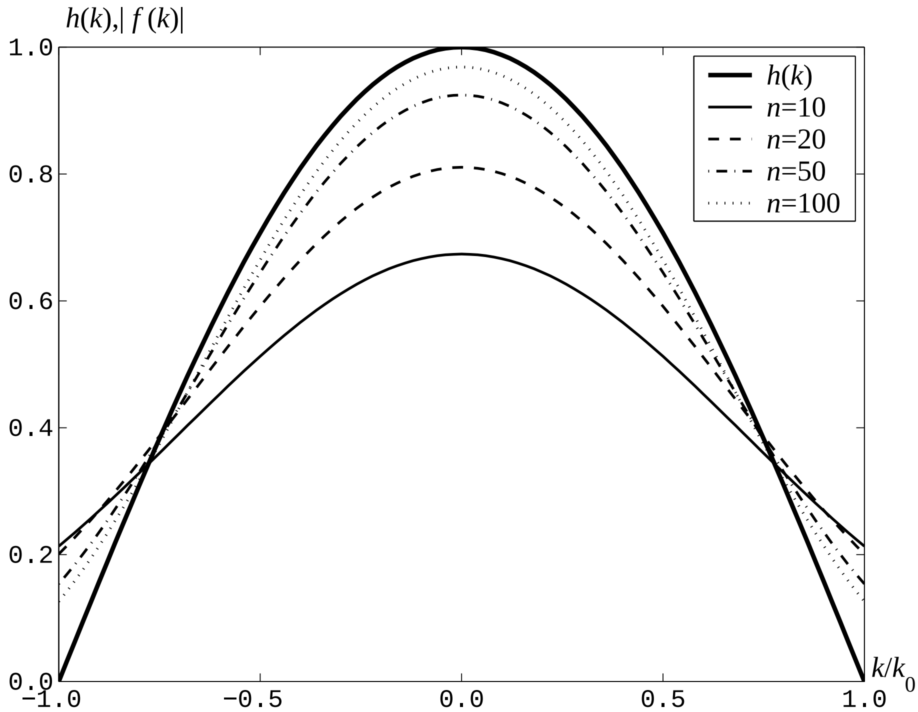

Fig. 1. The given $h(k) = \cos(\pi k / 2k_0)$ and the obtained $f(k)$ RPs

The values of the MSD, presented in the third column, are of the same order as the errors of estimate (8). For the given RP $h(k)$, the increase of the accuracy of the approximation does not force the growth of the norm $\| j \|$ of the current which would cause practical difficulties (column 4 in Table 1).

Table 1

The quality of approximation for $h(x) = \cos(\pi k / 2k_0)$ at various n

n	Est. (8)	MSD	$\| j(x) \|^2$
10	0.2784	0.2951	0.5060
20	0.1463	0.1495	0.5858
50	0.0591	0.0587	0.6432
100	0.0295	0.0287	0.6653
200	0.0147	0.0135	0.6772
500	0.0059	0.0044	0.6847
1000	0.0029	0.0014	0.6873
4000	0.7146×10^{-4}	0.8135×10^{-4}	0.6892

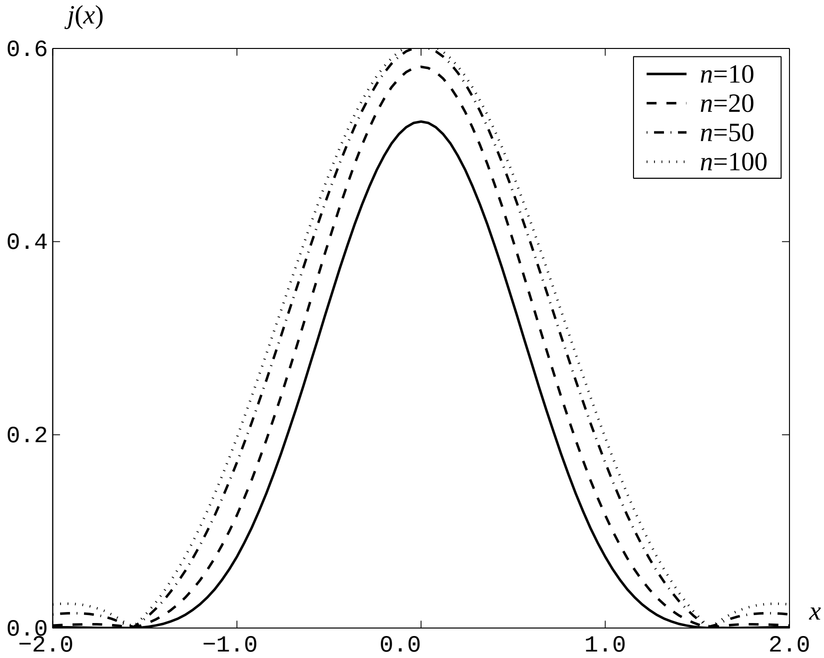

Fig. 2. The currents $j(x)$ for various n, $h(k)=\cos(\pi k/2k_0)$

The quality of approximation to the given RP $h(k)=1$ is shown in Table 2. For this $h(k)$ the error of the approximation is larger than the error for $h(k)=\cos(\pi k/2k_0)$ at the same values of n. The value of δ in estimate (8) at $n=4000$ is two orders greater than that for the RP $h(k)=\cos(\pi k/2k_0)$. Although the error of estimate (8) and the MSD is small, but the difference of the shapes of $h(k)$ and $f(k)$ is visible. In the four last rows of Table 2 the results are presented for $k_0=6.0$ and $k_0=9.0$. The error of estimate (8) is almost the same as for $k_0=3.0$, but the value of the MSD is lower. This means an improvement of the approximation to the given RP by the shape (compare the dash dot and dot curves in Fig. 3). The mean-square deviation at $n=1000$ and $n=4000$ for $k_0=9.0$ is almost two times less than for $k_0=3.0$.

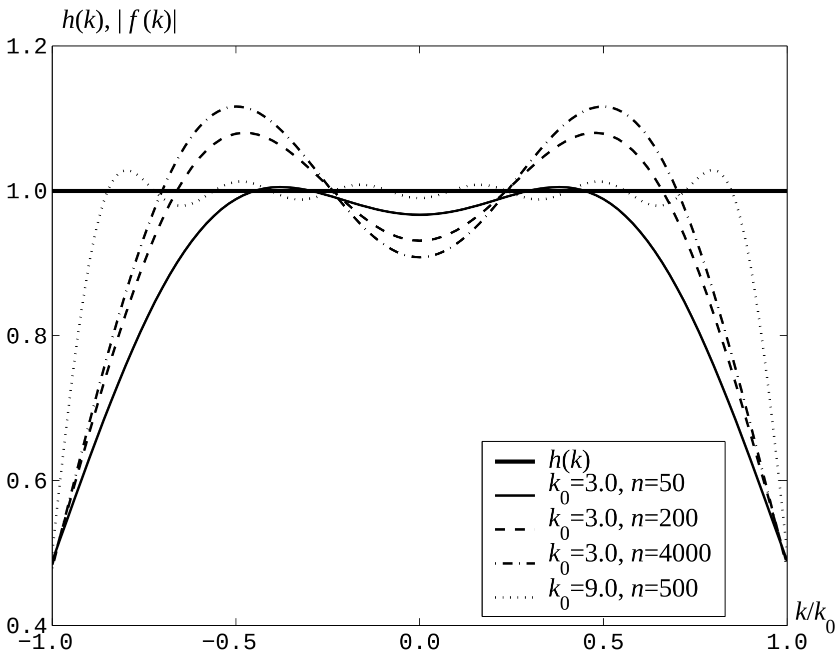

Fig. 3. The given $h(k)=1$ and the obtained $|f(k)|$ RPs

The corresponding distributions of the current $j(x)$ are shown in Fig. 4. For larger k_0 the norm of the current grows. This agrees with the numerical results in [1], namely the better approximation of the given RP leads to the larger norm of the current.

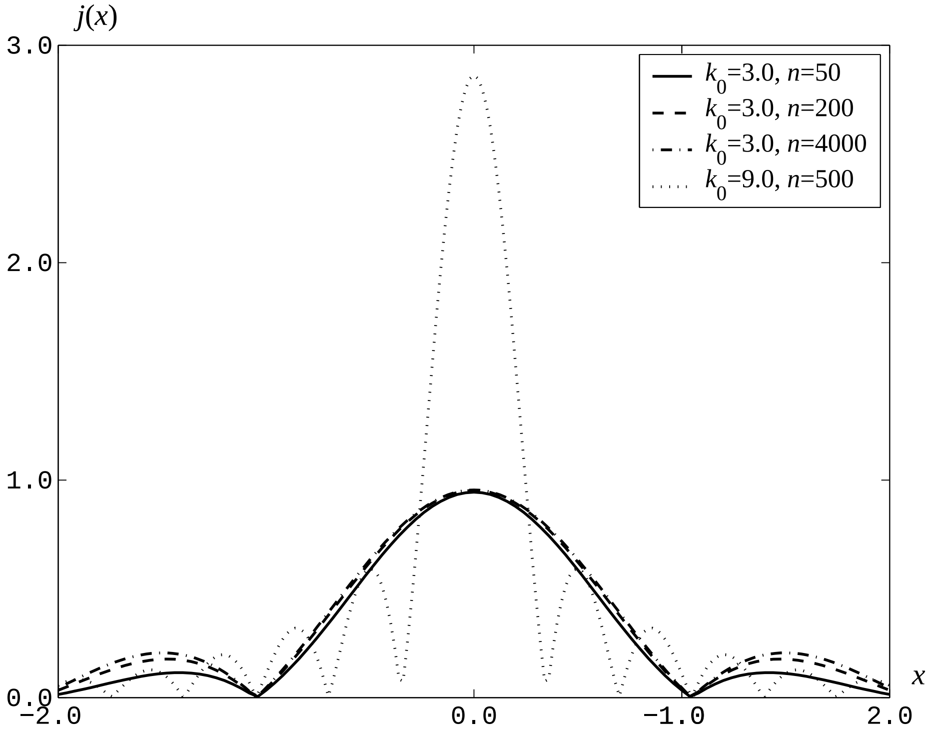

Fig. 4. The currents $j(x)$ for several parameters k_0, $h(k)=1$

The number of n, which is sufficient for obtaining the desired error δ, is shown in Table 3 for $l=2.0$, $k_0=6.0$, $k_1=2k_0+1.5$, and $p=1.0$. The results are presented for the given RPs $h(k)=(\cos(\pi k/2k_0))^{2q}$, with $q=1,4,8,16,32$. To obtain a higher accuracy of approximation of h it is necessary to increase the number n for all q. The quantity n for the prescribed δ varies for different q.

Table 2

The quality of approximation for $h(x)=1$ at various n

k	n	Est, (8)	MSD	$\| j(x)\|^2$
$k_0=3,0$				
	10	0,5592	0,5885	0,7287
	20	0,3654	0,3712	0,8314
	50	0,2161	0,2219	0,8967
	100	0,1487	0,1650	0,9218
	200	0,1034	0,1338	0,9361
	500	0,0645	0,1141	0,9456
	1000	0,0433	0,1073	0,9458
	4000	0,0225	0,1021	0,9515
$k_0=6,0$				
	500	0,0584	0,0715	1,3506
	4000	0,0203	0,0537	1,3624
$k_0=9,0$				
	500	0,0565	0,0607	1,6575
	4000	0,0195	0,0380	1,6749

The power q in the function $(\cos(\pi k/2k_0)^{2q}$ corresponds to the RPs with different widths at the level $h(k)=0.5$. The value of $|h(k)-|f(k)||_{h(k)=0.5}$ is minimal at $q=16$. Also, this $f(k)$ has the smallest side lobes outside of the interval $|k|\leq k_0$ in comparison with the values of the side lobes at other values of q. Such RP is called as optimal [2], and it can be created easier in comparison with other RPs. This leads to the minimal value of n which is necessary to obtain the desired error δ.

Table 3

Number of n necessary to attain the given value of δ for various $h(k)$

$h(k)$	$\delta = 0{,}1$	$\delta = 0{,}01$	$\delta = 0{,}001$
$h(k) = (\cos(\pi k/2k_0)^2$	$n = 24$	$n = 86$	$n = 320$
$h(k) = (\cos(\pi k/2k_0))^8$	$n = 15$	$n = 50$	$n = 240$
$h(k) = (\cos(\pi k/2k_0))^{16}$	$n = 12$	$n = 44$	$n = 165$
$h(k) = (\cos(\pi k/2k_0))^{32}$	$n = 9$	$n = 46$	$n = 150$
$h(k) = (\cos(\pi k/2k_0))^{64}$	$n = 13$	$n = 55$	$n = 182$

Conclusion

The closed form of solution of the linear antenna synthesis problem by the amplitude RP is given. The initial nonlinear problem is reduced to a linear one. The solution of this linear problem is presented in closed form, see formula (2). The numerical results demonstrate the high accuracy of the proposed method. The approach can be used also for solving the multidimensial antenna synthesis problems.

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References

1. M. Andriychuk Antenna synthesis by amplitude radiation pattern / M. Andriychuk, N. Voitovich, P. Savenko, V. Tkach. – K.: Nauk. Dumka, 1993. 2. Balanis C. A. Antenna theory: analysis and design / C. A. Balanis. – 3rd ed., Hoboken. – New Jersey: Wiley-Interscience, 2005. 3. Iosevich A. Decay of the Fourier transform: analytic and geometric aspects / A. Iosevich, E. Liflyand. – Birkhäuser, Basel, 2014. 4. Minkovich B. Theory of antenna synthesis / B. Minkovich, V. Yakovlev. – M.: Sov. Radio, 1969. 5. Ramm A. G. Optimal solution of the problem of linear antenna synthesis / A. G. Ramm // Soviet Physics – Doklady. – 1968. – Vol. 13, № 6. – 546–548. 6. Ramm A. G. On antenna synthesis theory / A. G. Ramm // Collection "Antennas". – M.: Izd. Svjaz, 1969. – № 5. – 35–46. 7. Ramm A. G. Some nonlinear problems in the theory of antenna synthesis / A. G. Ramm // Soviet Physics – Doklady. – 1989. – Vol. 14, № 6. – 532–535. 8. Ramm A. G. Optimal solution of the linear antenna synthesis problem / A. G. Ramm // Radiofisika. – 1969. – 12. – 1842–1848. 9. Ramm A. G. Approximation by entire functions / A. G. Ramm // Mathematics, Izv. Vusov. – 1978. – 10. – 72–76. 10. Ramm A. G. Signal estimation from incomplete data / A. G. Ramm // J. Math. Anal. Appl. – 1987. – 125. – 267–271. 11. Ramm A. G. The Radon transform and local tomography / A. G. Ramm, A. I. Katsevich // CRC Press, Boca Raton 1996. 12. Rudin W. Real and complex analysis / W. Rudin // McGraw Hill. – New York, 1988. 13. Stutzman W. Antenna theory and design / W. Stutzman, G. Thiele, Wiley, London, 2013. 14. Voitovich N. N. Antenna synthesis by amplitude radiation pattern and modified phase problem, in book B. Z. Katsenelenbaum / N. N. Voitovich // Electromagnetiv fields: restriction and approximation, Wiley-WCH, Berlin, 2002. 15. Bulatsyk O. O. Phase optomization problems – application in wave field theory / O. O. Bulatsyk, B. Z. Katsenelenbaum, Y. P. Topolyuk, N. N. Voitovich, Wiley-WCH, Berlin, 2010. 16. Ramm A. G. Antenna synthesis by the modulus of the diagram / A. G. Ramm, M. I. Andriychuk. – (to be published).