МЕТОДИ І АЛГОРИТМИ СУЧАСНИХ ІНФОРМАЦІЙНИХ ТЕХНОЛОГІЙ

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SOME GENERALIZATION OF GOLDBACH'S CONJECTURE AND BERTAND'S POSTULATE

In the article the elementary proofs of some generalizations of Goldbach's Conjecture, Bertand's postulate and some corollarys from this are presented.

Key words: prime number, Goldbach's conjecture, Bertrand's postulate.

The Goldbach's conjecture is worded this way: every even number, ≥ 6, can be expanded into the sum of two odd prime numbers [1].

Definition 1. If two prime numbers p and q are the 'twin'-numbers, (the equality $p - q = 2$ is executable) [2], so we'll call p the greater and q – the smaller prime. We'll mark set of all 'twin'-numbers as $\mathbf{B_2}$**.**

Definition 2. If the equality $r - s = 4$ is executable for two prime numbers r and s, we'll call r the greater and s – the smaller prime. We'll mark set of those prime numbers as $\mathbf{B_4}$**.**

Definition 3. If the equality $u - v = 6$ is executable for two prime numbers u and v, we'll call u the greater and v – the smaller prime. We'll mark set of those prime numbers as $\mathbf{B_6}$**.**

Definition 4. If the equality $s - t = 8$ is executable for two prime numbers s and t, we'll call s the greater and t – the smaller prime. We'll mark set of those prime numbers as $\mathbf{B_8}$.

Theorem 1. Every even natural number, beginning with 6, can be represented by the sum of two odd primes, one of which belongs to set $\mathbf{B_4}$ and another one – to set $\mathbf{B_2}$.

Proof. Let's mark the n-th even number as a_n, $a_n = 2n$, $n \geq 3$. We'll prove this theorem by using the method of mathematical induction by the number $n \geq 3$, beginning with $a_3 = 3 + 3$. Let's suppose, that all even numbers that are less than or equal to a_n, can be represented by the sum of $q_n + p_n$, where $q_n \in \mathbf{B_4}$, $p_n \in \mathbf{B_2}$**.** Then a_{n+1} will look like

$$a_{n+1} = a_n + 2 = p_n + q_n + 2. \qquad (1)$$

If in (1) $p_n \in \mathbf{B_2}$ is the smaller prime, then the proof is concluded, as then for $p'_{n+1} \in \mathbf{B_2}$, $p'_{n+1} = p_n + 2$ is the greater prime, $a_{n+1} = q_n + p'_{n+1}$.

But let's suppose, that, $p_n \in \mathbf{B_2}$ is the greater prime in $\mathbf{B_2}$**.** Let's present a_{n+1} in form of

$$a_{n+1} = a_{n-1} + 4 = p_{n-1} + q_{n-1} + 4, \qquad (2)$$

where $q_{n-1} \in \mathbf{B_4}$ and $p_{n-1} \in \mathbf{B_2}$ **.**

For the primes in (2) the following opportunities are possible:

1) p_{n-1} is smaller in $\mathbf{B_2}$ **,** q_{n-1} is smaller in $\mathbf{B_4}$ **,** $(q_{n-1} + 2) \in \mathbf{B_4}$ **;**
2) q_{n-1} is smaller in $\mathbf{B_4}$ **,** $p_{n-1} \in \mathbf{B_2}$**;**
3) p_{n-1} is greater in $\mathbf{B_2}$ **,** q_{n-1} is greater in $\mathbf{B_4}$ **;**
4) q_{n-1} is greater in $\mathbf{B_4}$ **,** p_{n-1} is smaller in $\mathbf{B_2}$ **.**

If condition (1) is executable, then theorem 1 is proved, as $a_{n+1} = q'_{n+1} + p'_{n+1}$, $p'_{n+1} \in \mathbf{B_2}$, $q'_{n+1} \in \mathbf{B_4}$, $p'_{n+1} = p_{n-1} + 2$, $q'_{n+1} = q_{n-1} + 2$.

If condition (2) is executable, then theorem 1 is also proved, as $a_{n+1} = q'_{n+1} + p_{n-1}$, $q'_{n+1} \in \mathbf{B_4}$, $q'_{n+1} = q_{n-1} + 4$.

By carrying out condition 3) from (1) and (2) we'll get $p_n + q_n + 2 = p_{n-1} + q_{n-1} + 4$ as a result, or

$$a_{n-1} = p_{n-1} + q_{n-1} = p'_n + q_n, \tag{3}$$

where $p'_n = p_n - 2$ is the smaller prime in $\mathbf{B_2}$, $p'_n \in \mathbf{B_2}$.

If 4) is executable, then it follows from (1) and (2) that:

$$a_n = p_n + q_n = p'_{n-1} + q_{n-1}, \tag{4}$$

where $p'_{n-1} = p_n + 2$ is the greater prime in $\mathbf{B_2}$, $p'_{n-1} \in \mathbf{B_2}$.

It now follows from (3) and (4) that each even number can be represented by the sum of two prime numbers, one of which belongs to $\mathbf{B_4}$, and another one-- to $\mathbf{B_2}$. But this statement is incorrect, because for the numbers 6 = 3 + 3, 8 = 5 + 3, 12 = 5 + 7, 94 = 71 + 23 these expansions are the only ones possible.

This contradiction proves, that the assumption, that $p_n \in \mathbf{B_2}$ is the greater prime in $\mathbf{B_2}$ is incorrect. To end the proof, let's show, that in (3) and (4) $q_n \neq q_{n-1}$. But let's suppose that in (3) $q_n = q_{n-1}$. Then $p_n - p_{n-1} = 2$, that contradicts the assumption 3). If $q_n = q_{n-1}$ in (4), than according to the assumption 4) q_n is the greater prime in $\mathbf{B_4}$. That is, each even number a_n can be represented by the sum of two greater primes, one of which is in $\mathbf{B_2}$ and another one – in $\mathbf{B_4}$, not less than twice, that contradicts, for example, such expancions as 14 = 11 + 3, 16 = 13 + 3 = 5 + 11, where at least one of the primes is the smaller one. We got the contradictions, which prove that in (3) and (4) $q_n \neq q_{n-1}$. Theorem 1 is proved.

Even numbers, that are less than or equal to 120 and that can be represented by the sum $q + p$, $p \in \mathbf{B_2}$, $q \in \mathbf{B_4}$, are presented in Table 1.

Table 1

p \ q	3	7	11	13	17	19	23	37	41	43	47	67
3	6	10	14	16	20	22	26	40	44	46	50	70
5	8	12	16	18	22	24	28	32	46	48	52	72
7	10	14	18	20	24	26	30	44	48	50	54	74
11	14	18	22	24	28	30	34	48	52	54	58	78
13	16	20	24	26	30	32	36	50	54	56	60	80
17	20	24	28	30	34	36	40	54	58	60	64	84
19	22	26	30	32	36	38	42	56	60	62	66	86
29	32	36	40	42	46	48	52	66	70	72	76	96
31	34	38	42	44	48	50	54	68	72	74	78	98
41	44	48	52	54	58	60	64	78	82	84	88	108
43	46	50	54	56	60	62	66	80	84	86	90	110
59	62	66	70	72	76	78	82	96	100	102	106	126
61	64	68	72	74	78	80	84	98	102	104	108	128
71	74	78	82	84	88	90	94	108	112	114	118	138
73	76	80	84	86	90	92	96	110	114	116	120	140

Theorem 2. Every odd number $4n + 1$, beginning with 9, can be represented by the sum $q + 2p$, where $q \in \mathbf{B_4}$, $p \in \mathbf{B_2}$,

Proof. Let's mark the n-th odd number as a_n, $a_n = 4n + 1$. We'll prove this theorem by using the method of mathematical induction by the number $n \geq 2$, beginning with $a_2 = 3 + 2 \cdot 3$. Let's suppose, that all odd numbers $4n + 1$, that are less than or equal to a_n, can be represented by the sum $q_n + 2p_n$, where $q_n \in \mathbf{B_4}$, $p_n \in \mathbf{B_2}$.

Then a_{n+1} will look like

$$a_{n+1} = a_n + 4 = q_n + 2p_n + 4. \tag{5}$$

If at least one prime in (5) is the smaller one, then the proof is concluded, as or $a_{n+1} = q_n + 2p'_{n+1}$, $p'_{n+1} = p_n + 2$, $p'_{n+1} \in \mathbf{B_2}$ if p_n is the smaller prime, join $a_{n+1} = q'_{n+1} + 2p_n$, $q'_{n+1} = q_n + 4$, $q'_{n+1} \in \mathbf{B_4}$, if q_n is the smaller prime. Let's suppose, that $q_n \in \mathbf{B_4}$, $p_n \in \mathbf{B_2}$ are both greater primes. If $q_{n-1} \in \mathbf{B_4}$, $p_{n-1} \in \mathbf{B_2}$, then a_{n+1} will look like

$$a_{n+1} = a_{n-1} + 8 = q_{n-1} + 2p_{n-1} + 8\ , \qquad (6)$$

For primes in (6) the following opportunities are possible:

1) both numbers are the smaller primes;
2) $p_{n-1} \in \mathbf{B_2}$ is the smaller prime, $q_{n-1} \in \mathbf{B_4}$ is the greater prime;
3) $p_{n-1} \in \mathbf{B_2}$ is the greater prime, $q_{n-1} \in \mathbf{B_4}$ is the smaller prime;
4) both numbers are the greater primes.

If condition 1) is executable, then the proof is concluded, as $a_{n+1} = q'_{n+1} + 2p'_{n+1}$, $q'_{n+1} \in \mathbf{B_4}$, $p'_{n+1} \in \mathbf{B_2}$ and $q'_{n+1} = q_{n-1} + 4$, $p'_{n+1} = p_{n-1} + 2$ are the greater primes.

By carrying out condition 2) from (6), we'll get:

$$a_{n+1} = q_{n-1} + 2p'_n + 4\ , \qquad (7)$$

where $p'_n \in \mathbf{B_2}$, $p'_n = p_{n-1} + 2$ is the greater prime.

As a result of assumption of the induction, it follows from (7), that

$$a_n = q_n + 2p_n = q_{n-1} + 2p'_n\ . \qquad (8)$$

If condition 3) is executable, then it follows from (6) that

$$a_n = q_n + 2p_n = q'_n + 2p_{n-1}\ , \qquad (9)$$

where $q'_n \in \mathbf{B_4}$, $q'_n = q_{n-1} + 4$ is the greater prime.

Correlations (8) and (9) signify, that every odd number $a_n = 4n + 1$ can be represented in form of the sum of the greater prime from set $\mathbf{B_4}$ and the doubled greater prime from set $\mathbf{B_2}$, that contradicts the statement that $9 = 3 + 2{\cdot}3$, and $3 = 7 + 2{\cdot}3 = 3 + 2{\cdot}5$, where the prime 3 is the smaller prime in any case.

If condition 4) is executable, then it means, that all odd numbers a_{n-1}, a_n can be represented in form of the sum of the greater prime number from $\mathbf{B_4}$ and the doubled greater prime from set $\mathbf{B_2}$, that contradicts, for example, such expantions as: $9 = 3 + 2{\cdot}3$ and $13 = 3 + 2{\cdot}5$, where the numbers 3 and 5 are the smaller primes, or the expantions: $33 = 11 + 2{\cdot}11$ and $37 = 13 + 2{\cdot}11$, where 11 is the smaller prime in $\mathbf{B_2}$. Theorem 2 is proved.

The numbers in form of $4n + 1$, that can be represented by the sum $q + 2p$, where $q \in \mathbf{B_4}$, $p \in \mathbf{B_2}$, are given in Table 2.

Table 2

p\q	3	7	11	19	23	43	47	67	71	79	83
3	9	13	17	25	29	49	53	73	77	85	89
5	13	17	21	29	33	53	57	77	81	89	93
7	17	21	25	33	37	57	61	81	85	93	97
11	25	29	33	41	45	65	69	89	93	101	105
13	29	33	37	45	49	69	73	93	97	105	109
17	37	41	45	53	57	77	81	101	105	113	117
19	41	45	49	57	61	81	85	105	109	117	121
29	61	65	69	77	81	101	105	125	129	137	141
31	65	69	73	81	85	105	109	129	133	141	145
41	85	89	93	101	105	125	129	149	153	161	165
43	89	93	97	105	109	129	133	153	157	165	169
59	121	125	129	137	141	161	165	185	189	197	201
61	125	129	133	141	145	165	169	189	193	201	205
71	145	149	153	161	165	185	189	209	213	221	225
73	149	153	157	165	169	189	193	213	217	225	229

Theorem 3. Every even number $6n + 2$, beginning with 14**,** can be represented by the sum $q + 3p$, where $q \in \mathbf{B_6}$, $p \in \mathbf{B_2}$.

Proof. Let's mark the n-th even number as a_n, $a_n = 6n + 2$. We'll prove this theorem by using the method of mathematical induction by the number $n \geq 2$, beginng with $a_2 = 14$. Let's suppose, that all even numbers in form of $6n + 2$, that are less than or equal to a_n, can be represented by the sum $q_n + 3p_n$, where $q_n \in \mathbf{B_6}$, $p_n \in \mathbf{B_2}$. Then

$$a_{n+1} = a_n + 6 = q_n + 3p_n + 6\ . \qquad (11)$$

If at least one prime in (11) is the smaller one, then the proof is concluded, as or $a_{n+1} = q_n + 3p'_{n+1}$, $p'_{n+1} = p_n + 2$, $p'_{n+1} \in \mathbf{B_2}$, if p_n is the smaller prime, join $a_{n+1} = q'_{n+1} + 3p_n$, $q'_{n+1} = q_n + 6$, $q'_{n+1} \in \mathbf{B_6}$, if q_n is the smaller prime.

Let's suppose, that $q_n \in \mathbf{B_6}$, $p_n \in \mathbf{B_2}$ are both greater primes. Let's present a_{n+1} in form of

$$a_{n+1} = a_{n-1} + 12 = q_{n-1} + 3p_{n-1} + 12 , \tag{12}$$

where $q_{n-1} \in \mathbf{B_6}$, $p_{n-1} \in \mathbf{B_2}$.

For the primes in (12) the following opportunities are possible:

1) both numbers are the smaller primes;
2) $p_{n-1} \in \mathbf{B_2}$ is the smaller prime, $q_{n-1} \in \mathbf{B_6}$ is the greater prime;
3) $p_{n-1} \in \mathbf{B_2}$ is the greater prime, $q_{n-1} \in \mathbf{B_6}$ is the smaller prime;
4) both numbers are the greater primes.

If condition 1) is executable, then the proof is concluded, as $a_{n+1} = q'_{n+1} + 3p'_{n+1}$, $q'_{n+1} \in \mathbf{B_6}$, $p'_{n+1} \in \mathbf{B_2}$ and $q'_{n+1} = q_{n-1} + 6$, $p'_{n+1} = p_{n-1} + 2$ are the greater primes.

By carrying out condition 2) from (12), we'll get:

$$a_{n+1} = q_{n-1} + 3p'_n + 6 , \tag{13}$$

where $p'_n \in \mathbf{B_2}$, $p'_n = p_{n-1} + 2$ is the greater prime.

As a result of assumption of the induction, it follows from (13), that

$$a_n = q_n + 3p_n = q_{n-1} + 3p'_n . \tag{14}$$

By carrying out condition 3) from (6) we'll get, if $q'_n \in \mathbf{B_6}$, $q'_n = q_{n-1} + 6$ is the greater prime:

$$a_n = q_n + 3p_n = q'_n + 3p_{n-1} , \tag{15}$$

Correlations (14) and (15) signify, that every even number $a_n = 6n + 2$ can be presented in form of the sum of the greater prime from set $\mathbf{B_6}$ and the greater prime from set $\mathbf{B_2}$, that is multiplied by three, that contradicts the expantion $14 = 5 + 3\cdot 3$, where both numbers are the smaller primes.

By carrying out proof step 4) from (11) and (12), we'll get the equality $q_n + 3p_n + 6 = q_{n-1} + 3p_{n-1} + 12$. Taking into account the assumption of the induction, it follows from this equality, that

$$a_{n-1} = q_{n-1} + 3p_{n-1} = q'_{n-1} + 3p_n = q_n + 3p'_{n-1} , \tag{16}$$

where $q'_{n-1} \in \mathbf{B_6}$, $p'_{n-1} \in \mathbf{B_2}$, $q'_{n-1} = q_n - 6$, $p'_{n-1} = p_{n-1} - 2$ are the smaller primes.

Correlation (16) means, that every even number $a_{n-1} = 6(n-1) + 2$ can be presented in form of the sum of the prime from set $\mathbf{B_6}$ and the prime from set $\mathbf{B_2}$, that is multiplied by three, where at least one of the primes is the greater one, that contradicts the statement, that even number $20 = 5 + 3\cdot 5 = 11 + 3\cdot 3$ has an expantion, in which both primes are the smaller ones.

We got the contradictions, which prove, that or one of the primes is the smaller one in the correlation (11), join both primes are the smaller ones in the correlation (12) Theorem 3 is proved.

The numbers, that look like $6n + 2$ and can be represented by the sum $q + 3p$, where $q \in \mathbf{B_6}$, $p \in \mathbf{B_2}$, are shown in Table 3.

Table 3

p/q	5	11	17	29	41	47	53	59	67	73
3	14	20	26	38	50	56	62	68	76	82
5	20	26	32	44	56	62	68	74	82	88
7	26	32	38	50	62	68	74	80	88	94
11	38	44	50	62	74	80	86	92	100	106
13	44	50	56	68	80	86	92	98	106	112
17	56	62	68	80	92	98	104	110	118	124
19	62	68	74	86	98	104	110	116	124	130
29	92	98	104	116	128	134	140	146	154	160
31	98	104	110	122	134	140	146	152	160	166
41	128	134	140	152	164	170	176	182	190	196
43	134	140	146	158	170	176	182	188	196	202
59	182	188	194	206	218	224	230	236	244	250
61	188	194	200	212	224	230	236	242	250	256
71	218	224	230	242	254	260	266	272	280	286
73	224	230	236	248	260	266	272	278	286	292

Theorem 4. Every odd number $8n + 1$, beginning with 17, can be represented by the sum $q + 4p$, where $q \in \mathbf{B_8}$, $p \in \mathbf{B_2}$.

Proof. Let's mark the n-th odd number as b_n, $b_n = 8n + 1$.We'll prove this theorem by using the method of mathematical induction by the number $n \geq 2$, beginning with $a_2 = 5 + 4 \cdot 3$.

Let's suppose, that all odd numbers $8n + 1$, that are less than or equal to b_n, can be represented by the sum $q_n + 4p_n$, where $q_n \in \mathbf{B_8}$, $p_n \in \mathbf{B_2}$. Then

$$b_{n+1} = b_n + 8 = q_n + 4p_n + 8 . \quad (17)$$

If at least one of the primes in (17) is the smaller one, then the proof is concluded, as or $b_{n+1} = q_n + 4p'_{n+1}$, $p'_{n+1} = p_n + 2$, $p'_{n+1} \in \mathbf{B_2}$, if p_n is the smaller prime, join, if q_n is the smaller prime, then $b_{n+1} = q'_{n+1} + 4p_n$, $q'_{n+1} = q_n + 8$, $q'_{n+1} \in \mathbf{B_8}$.

Let's suppose, that $q_n \in \mathbf{B_8}$, $p_n \in \mathbf{B_2}$ are both greater primes. Let's write b_{n+1} in form of

$$b_{n+1} = a_{n-1} + 16 = q_{n-1} + 4p_{n-1} + 16 , \quad (18)$$

where $_{n-1} \in \mathbf{B_8}$, $p_{n-1} \in \mathbf{B_2}$.

For the primes in (18) the following opportunities are possible:

1) both numbers are the smaller primes;
2) $p_{n-1} \in \mathbf{B_2}$ is the smaller prime, $q_{n-1} \in \mathbf{B_8}$ is the greater prime;
3) $p_{n-1} \in \mathbf{B_2}$ is the greater prime, $q_{n-1} \in \mathbf{B_8}$ is the smaller prime;
4) both numbers are the greater primes.

If condition 1) is executable, then the proof is concluded, as $a_{n+1} = q'_{n+1} + 4p'_{n+1}$, $q'_{n+1} \in \mathbf{B_8}$, $p'_{n+1} \in \mathbf{B_2}$ and $q'_{n+1} = q_{n-1} + 8$, $p'_{n+1} = p_{n-1} + 2$ are the greater primes.

By carrying out 2) from (18) we'll get:

$$b_{n+1} = q_{n-1} + 4p'_n + 8 , \quad (19)$$

where $p'_n \in \mathbf{B_2}$, $p'_n = p_{n-1} + 2$ is the greater prime.

As a result of asumption of the induction, it follows from (19), that

$$b_n = q_n + 4p_n = q_{n-1} + 4p'_n . \quad (20)$$

If condition 3) is executable, then

$$b_n = q_n + 4p_n = q'_n + 4p_{n-1} , \quad (21)$$

where $q'_n \in \mathbf{B_8}$, $q'_n = q_{n-1} + 8$ is the greater prime.

It arises from (20) and (21), that every odd number $b_n = 8n + 1$ can be represented by the sum of the greater prime from set $\mathbf{B_8}$ and the greater prime from set $\mathbf{B_2}$, that is multiplied by four, that contradicts such expantions as $15 = 3 + 4 \cdot 3$, $17 = 5 + 4 \cdot 3$, $25 = 5 + 4 \cdot 5$, $35 = 23 + 4 \cdot 3$, in which both primes are the smaller ones.

By carrying out condition 4) from (17) and (18) we'll get: $q_n + 4p_n + 8 = q_{n-1} + 4p_{n-1} + 16$., Taking into account assumption of the induction , it follows from this equality, that

$$b_{n-1} = q_{n-1} + 4p_{n-1} = q'_{n-1} + 4p_n = q_n + 4p'_{n-1} \quad (22)$$

where $q'_{n-1} \in \mathbf{B_8}$, $p'_{n-1} \in \mathbf{B_2}$, $q'_{n-1} = q_n - 8$, $p'_{n-1} = p_{n-1} - 2$ are the smaller primes.

Equality (22) means, that every odd number $b_{n-1} = 8(n - 1) + 1$ can be represented by the sum of the prime from set $\mathbf{B_8}$ and the greater prime from set $\mathbf{B_2}$ that is multiplied by four, but that contradicts the fact, that the odd numbers have the expantions in form of the sum $23 = 3 + 4 \cdot 5 = 11 + 4 \cdot 3$, $49 = 5 + 4 \cdot 11 = 29 + 4 \cdot 5$, in which both primes are the smaller ones.

We got the contradictions, which prove that or in the equality (17) one of the primes is the smaller one, join in the equality (18) both primes are the smaller ones. Theorem 4 is proved.

Theorem 5. (Bertand's postulate). For every natural number $n > 2$ between n and $2n$ there is at least one prime number $p \geq 3$.

Proof. According to theorem 1 for every even number $2n = p + q$, where p, q are the primes, there is a prime (let's call it p), that $p < 2n$. The following opportunities are possible: or $p > n$, and the proof is concluded, join $p < n$. Let's suppose, that $p < n$. Then p doesn't divide or divides n.

Let's take up the first case. Then $n - p = a$, $2n - p = q$. By subtracting and adding these two equalities, we'll get, that $n = q - a$, $3n - 2p = q + a$, and it follows from them, that $a^2 = q^2 + 2pn - 3n^2$.

The last equality is executable for every n number. And as its left part is a perfect square, so the discriminant of its right part is $4(p^2 + 3q^2) = 0$, but that is impossible, as $p \neq 0$, $q \neq 0$. So, if $p < n$, then p divides n, $n = mp$, $m < 1$. Then $2mp = p + q$, that is $p(2m - 1) = q$, but that is impossible, as q is a prime number. We came to the contradictions, which prove, that the unequality $p < n$ is impossible.

Theorem 6. (The generalization of Bertand's postulate). For every $n > 2$ and k, $0 \leq k < n/2$ between n i $2(n - k)$ there is at least one prime number $p \geq 3$.

Proof. According to theorem 1 for every even number $2(n - k)$ the equality $2(n - k) = p + q$ is executable, where p and q are the odd primes, so there is a prime number (let's call it p), which $p < 2(n - k)$. So, or $p > n$ and the proof is concluded, join $p \leq n$. Let's suppose, that $p \leq n$. Then p doesn't divide n or p divides n. Let's take up the first case. Then $n - p = a$, $2n - p = q + 2k$. By subtracting and adding these two equalities, we'll get, that: $n = q - a + 2k$ and $3n - 2p = q + a + 2k$. It arises from these equalities, that for every natural n: $-a^2 = 3n^2 - 2(p + 4k)n + 4k(p + k) - q^2$. The last part of the last equality is a perfect square for every n. Then the discriminant of its right part is equal to zero. It means, that the following equality is correct: $(p - 2k)^2 + 3q^2 = 0$, but it's impossible, because $p \neq 0$ i $q \neq 0$. So, if $p \leq n$, then p divides n. Let's suppose, that $n = mp$. Then the equality $(2m - 1)p - 2k = q$ is executable, and, according to the fact, that $q > 2$ is a prime, it arises from this statement, that $(p, k) = 1$. Let's suppose, that $p < n/2$. Then $n - 2p = a$, $2n - p = q + 2k$. By subtracting and adding these two equalities, we'll get, that $n + p = q + 2k - a$, $3n - 3p = q + 2k + a$. It arises from these equalities, that forevery natural n the following equality is executable: $-a^2 = 3n^2 - [3p^2 + (q + 2k)^2]$. The left part of the last equality is a perfect square for every n. Then the discriminant of its right part is equal to zero. It means, that the following equality is correct: $(q + 2k)^2 + 3p^2 = 0$, but it's impossible, because $p \neq 0$ and $q > 0$. So, $p \geq n/2$, so $n \leq 2p$. When the last inequality is divided by p, we'll get, that $m \leq 2$. If $m = 1$ then $p - q = 2k$, $n = p$ and theorem 6 is proved. If $m = 2$ than $n = 2p$ and by theorem 5 the theorem 6 also is proved.

Corollary 1. Every even number $2k$ can be represented in form of $p - q = 2k$, where p, q are some primes, $(p, k) = 1$.

The proof arises from the proof of theorem 6, because for $m = 1$ $n = p$, then $p - q = 2k$.

Corollary 2. Every even number $2k$ can be represented in form of $3p - q = 2k$, where p, q are some primes, $(p, k) = 1$.

The proof also arises from the proof of theorem 6, because if $m = 2$, $n = p$ and $3p - q = 2k$.

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